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SPATIAL SCALES OF WAVE HEIGHT IN THE NORTH ATLANTIC AND NORTH SEA FROM RADAR ALTIMETER DATA

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SPATIAL SCALES OF WAVE HEIGHT IN THE NORTH ATLANTIC AND NORTH SEA FROM RADAR ALTIMETER DATA

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SUMMARY

This report looks at spatial scales of significant wave height in the North Atlantic and the North Sea. If we could establish the spatial scales involved there are a number of possible applications. In fact any aspect of ocean engineering or ocean science which uses wave information in a spatial context would benefit. For example when setting up a wave model it is necessary to specify the size of the computational grid. If it could be shown that there were preferred scales on which significant wave height changed, these grids could be designed in an optimal manner. Similarly when measuring waves it could be that certain buoys are giving redundant information because of their proximity to other measurement sites.

The data used are taken from the radar altimeter on the ERS-1 satellite during August to December 1991. This was during the commissioning phase of the satellite when the instrument was flown over the same track every three days. Two approaches are used to look at spatial scales. One is a Fourier method where along track spectral density functions are produced. In the other autocorrelation functions in the space domain are calculated and used to derive space scales.

It is found that the along track spectrum of significant wave height in the North Atlantic can be modelled as a power law with power approximately equal to -5/3. This value is consistent across the entire region studied. In addition to calculating the spectrum two length scales are also computed for each track. It is found that there is little consistency between tracks, apart from the fact that longer tracks give larger length scales. It is postulated that this lack of consistency is due to changing meteorological conditions.

When data in the North Sea are analysed an immediate problem is found because of the orbit phasing. The three day repeat of ERS-1 has tracks following the British and European coasts with no data in the middle of the North Sea. However when the data are analysed in the same way as in the North Atlantic very similar results are obtained. There is no evidence of a different set of spatial scales in the semi-enclosed North Sea compared to similar length tracks in the North Atlantic.

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GLOSSARY

h	Height of a satellite above the sea surface
Hs	Significant wave height = 4 . standard deviation of sea surface elevation
L1, L2	Length scales (see page 26 for definition)
$r(x)$	Autocorrelation function (see page 19 for definition)
$S(\omega)$	Measured wavenumber spectrum
$T(\omega)$	Transfer function relating the measured to true wavenumber spectra
$\dot{x}$	Derivative of x
z_c	First zero-crossing of the autocorrelation function
$\Phi(\omega)$	True wavenumber spectrum
σ^0	Radar backscatter coefficient

1. INTRODUCTION

This report is concerned with the spatial scales of the wave field in the North Atlantic and the North Sea. We know a lot about the time scales over which significant wave height varies but very little about variation in space. This is because until recently the only data available were from buoys or ships which are either stationary or moving at slow speeds. With immobile measuring systems it is possible to examine temporal variation, but with data from slowly moving platforms it is impossible to look at either spatial or temporal variability as we cannot tell one from the other. Thus for a ship travelling at 5 ms^{-1} (10 knots) a signal with a period of one hour is indistinguishable from one with a wavelenth of 60 km. This is one reason for the unpopularity of such measurements. The introduction of methods of measuring ocean waves by remote sensing techniques has, for the first time, given us the ability to look at spatial variability. This is because remote sensing either looks at a large area simultaneously, e.g. HF radar, or the instrument is mounted on a very fast moving platform, e.g. a satellite or aircraft.

Why should we be interested in space scales? An interest in time scales is natural as off-shore structures, in general, stay in one place and 'move' through time. Therefore knowing how long a storm will last is a natural question. Knowing how large it is spatially has less obvious application. However some thought will reveal some important applications. We will now briefly describe two.

A common method of ascertaining design wave conditions is to hindcast data over a large period of time. Wave modelling is now very good and both accurate hindcasts and forecasts (for about 3 – 12 hours ahead)can be made. However the grid spacing for the models is chosen arbritrarily, at best it is related to the grid scales of the meteorological models which generate the wind fields. A knowledge of the spatial scales of the real ocean will enable us to chose the grid spacing in an optimal, or near optimal, manner, improving the efficiency of the wave models. At present wave model results are compared with time series collected from buoys, we need to know if they are also producing storms with the correct spatial dimensions. If not there may be something wrong with, say, swell propogation.

It is not only modelling that could gain from a knowledge of spatial scales. When design waves are estimated it is common to add two nearby series together to make a single longer series and analyse that. Knowing what the spatial scales are will enable us to do this in a more rigorous manner. For instance a question we might want to answer is how far apart can stations be before they are independent? Anyone deploying instrumentation might also want the answer to this question. Is it worth deploying two buoys x kilometres apart, say, if the waves recorded at these two sites will have a 90% correlation.

The information on time scales is normally presented in the form of persistence or storm duration. Similar statistics could be developed in the space domain; in which case for example the equivalent of duration would be storm extent. Parameters such as storm duration have evolved over the years into those which are useful for the off-shore industry. It is by no means certain that by simply converting these into their spatial analogues other useful parameters will be generated. For this reason this report will be concerned with more basic parameters such as the spectrum and autocorrelation function of significant wave height (H_S).

There are a number of possible applications of measures of spatial variability. If you assume that all sites are independent then if you measure waves at fifty sites in UK waters one of those sites will record the fifty year return value every year! This is obviously nonsense and is a consequence of the unwarranted assumption of independence. In the real world spatial dependence means that we have less than fifty independent observations, but we do not know how many less than fifty. Similarly at present wave models are validated by comparison with measurements taken from single sites, their gross characteristics are never checked. Knowledge of the spatial statistics should allow such comparisons to be made. Another application is also to do with wave models. In the past wave models have been run purely using analysed meteorological wind fields, at the present time there is a lot of interest in assimilating wave data, in particular but not exclusively satellite data. This will constrain the model and allow deficiencies in the wind field or modelled physics to be minimised (Lionello et al., 1992). Although assimilation is normally talked about in the context of forecasting the same methods would improve wave hindcasts even with the limited amount of historical wave data available. In order to apply assimilation schemes a knowledge of the spatial (and temporal) scales of variation is required. At present values are assumed which have little or no justification either empirically or theoretically.

Most previous publications on spatial scales have been concerned with the problem of calibrating and validating satellite sensors against buoys which are not necessarily under the satellite tracks. These papers include Challenor et al. (1986), Monaldo (1988) and Monaldo (1990). Two papers which do look at the problem from the perspective of spatial variability for its own sake are Challenor (1983) and Tournadre (1993).

This report will look at the analysis of spatial correlation by considering along track spectra and autocorrelation functions of significant wave height from satellite altimeter data. No attempt will be made to answer the kind of questions raised above. These must wait until we have a much better understanding of the statistics of spatial correlation.

2. DATA AVAILABLE AND THE DIFFERENCES BETWEEN THE DATA SETS

2.1 DATA FOR SPATIAL CORRELATION STUDIES

In order to study spatial correlation we need data sets which satisfy a number of conditions. Obviously significant wave height must be measured over a large area or at least along a line. Secondly all the data must be taken as near as possibly simultaneously. These conditions imply that remotely sensed data are particularly suitable for this problem. In this study radar altimeter data from polar orbiting satellites is used. The principles of how altimeters measure waves is explained in Tucker (1990) and Chelton et al., (1989). The technical details of how altimeters work are not necessary for this report, apart from one aspect, the method of tracking the sea surface, which is dealt with in some detail below. The radar altimeter conventionally measures three parameters: these are the surface roughness (normally expressed as significant wave height; H_s), the height of the satellite above the sea surface (h) and the radar backscatter coefficient (σ^0). This last parameter is normally used to compute an estimate of wind speed using, for example, the algorithm given by Witter and Chelton (1991). Measurements are taken over a small patch of ocean directly below the satellite, the size of this patch depends on sea state and varies from 2 to 10km in diameter (Chelton et al., 1989). In practice this makes little difference to the measurements and is usually ignored. Because the satellite orbits the Earth in about 100 minutes the speed of the sub-satellite point across the ocean is large; about 7 km s^{-1}.

2.2 ALTIMETER DATA AVAILABLE

So far there have been five satellite missions which have produced usable wave data from radar altimeters. These are in chronological order : GEOS-3, Seasat, Geosat, ERS-1 and Topex/Poseidon. GEOS-3 and Seasat flew in the late 1970's and should be considered as experimental missions. For instance GEOS-3 did not have an on-board tape recorder so data could only be recorded within range of a ground station while Seasat only lasted 100 days. Because of the six year gap between the end of these two missions and the launch of Geosat in 1985 it has so far proved impossible to calibrate these data to the same standard as the more recent data. For these reasons these data were not considered in this study. A two and a half year data set was collected by the US Navy's satellite Geosat during 1986-1989 during its so called Exact Repeat Mission. During the earlier part of the mission the satellite ground track was not repeated. This makes analysis more difficult. Currently flying are the European Space Agency's satellite ERS-1 and the US/French mission Topex/Poseidon. This latter satellite was only launched during the summer of 1992 and the data have not been validated yet.

ERS-1 data come in two versions: fast delivery data and off-line data. The fast delivery data are produced very rapidly and delivered to operational users within three hours of collection. Because of this need for haste data are sometimes not processed. In particular one orbit a day is collected at the Prince Albert receiving station in Canada which does not have facilities for processing data. The off-line data on the other hand are re-processed at the French Processing and Archiving Facility (F-PAF) in Brest. These data are processed using what are expected to be more accurate algorithms, they are not constrained by computer limitations and operational requirements in the same way as those on board the satellite are, and processing can be delayed until all relevant corrections have been assembled. However these methods are as yet untested and it is important to validate any satellite measurements. Significant wave height from this product has been validated at least to first order and can be regarded as good (but see comments in the conclusions), wind speed on the other hand has not yet been validated and for the present is regarded as suspect. Thus only wave height from this product will be used in the report.

2.3 THE 'TRACKER' PROBLEM

The radar altimeter works by transmitting a very narrow pulse of radar energy towards the ocean surface. This is reflected back and the return echo power is measured back at the satellite. The returned power is integrated over a number of contiguous very short times of order nanoseconds. These short intervals are commonly known as 'gates'. The gates are very accurately timed so that the distance to the sea surface can be estimated to the order of a few cm. Wave height is calculated from the slope of the return echo and the radar backscatter from the total power returned. Since there are a limited number of gates it is important to have a prediction of where the sea surface will be for the next pulse and so set the instrument accordingly. This is a common problem in radar technology and the standard method is to use a so-called α-β tracker (Chelton et al., 1989). At each stage a new value is predicted and the difference between this prediction and the measured value is used to generate a new prediction. In mathematical form an α-β tracker is defined by the following set of equations:

$$\dot{x}(n) = \dot{x}(n-1) + \beta\Delta x(n-1)$$
$$x(n+1) = x(n) + \alpha\Delta x(n-1) + \dot{x}(n)$$

where x is the parameter being predicted, $\dot{}$ denotes a derivative and Δ the difference between the measured value at time n and the prediction $x(n)$. For the next step $\Delta x(n+1)$ would be calculated as the difference between the measured value at time $n+1$ and $x(n+1)$ and the same equations used with n replaced by $n+1$. The results transmitted by the satellite are the predictions rather than the measured values.

The effect of an α-β tracker is to smooth or filter the data. As an example consider the data shown in figure 2.1. This shows a portion of a 'white noise' signal, every point is uncorrelated with every other. If we apply an α-β tracker to this signal we obtain the trace shown in figure 2.2. The values of α and β are 1/4 and1/64 respectively as suggested in Chelton et al. (1989). Even to the naked eye it is apparent that the frequency/wavenumber characteristics of the signal have been changed.

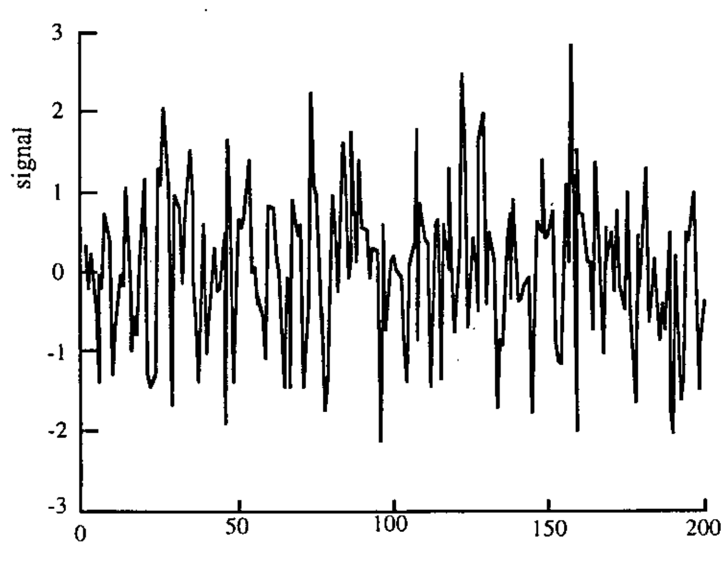

Figure 2.1 Simulated Gaussian white noise. Because this is a simulation the scales are arbitrary.

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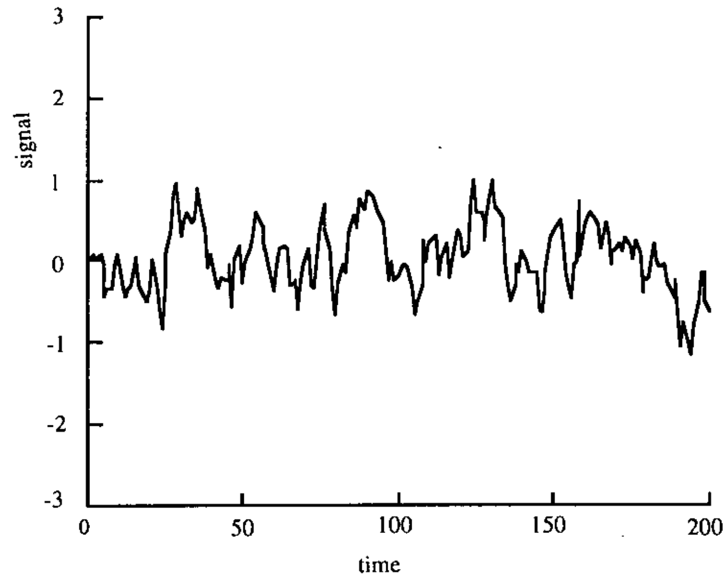

Figure 2.2 The same white noise signal as figure 2.1 after being through an α-β tracker with α=1/4 and β=1/64. Because this is a simulation the scales are arbitrary.

One way of expressing the effect of a filter, such as an α-β tracker is through a transfer function. If the true spectrum is given by $\Phi(\omega)$, where ω is the wavenumber, and the measured one by $S(\omega)$ then they are related by the transfer function $T(\omega)$through the equation

$$S(\omega) = T(\omega)\,\Phi(\omega)$$

the transfer function is therefore given by

$$T(\omega) = \frac{S(\omega)}{\Phi(\omega)}$$

Ideally the transfer function of the tracker loops would be computed theoretically given the α-β tracker and the relevant constants. However the methods used to track the ocean surface are very different for different satellites. Geosat, for example, uses a split gate tracker (Chelton et al., 1989) while ERS-1, on the other hand, uses an SMLE tracker (Levrini and Rubertone, 1984). These 'trackers' are really estimation algorithms and should not be confused with the α-β trackers that smooth the resulting estimates. ERS-1, for example, has three tracker loops: one for σ^0, one for height and one for the inverse of H_S. These interact in a complex way. The smoothing constants in the α–β trackers are also different and in most satellite missions the values of α and β are changed from time to time to improve instrument performance. It is therefore difficult if not impossible to calculate the transfer function; even if one could find the values of all the appropriate constants.

The along track spectrum computed from the altimeter data will include the effect of these filters and therefore will not give the correct spectrum. Since we cannot compute the transfer function we need another method to estimate the unfiltered spatial spectrum. One way to do this is to recompute new estimates of H_S etc. on the ground where there are no constraints imposed by the operation of the instrument. For ERS-1 this has been done in the off-line data. It could be argued that the α–β tracker only affects data a very short distance apart and thus will not influence the shape of the spectrum in the region of interest (10's to 100's km). However the results presented by Challenor (1993) show that the spectra calculated from Geosat, ERS-1 Fast Delivery and ERS-1 Off Line data have very different shapes. The only explanation forthcoming for this difference at present is the $\alpha-\beta$ trackers. This is the reason only the off-line data, which are unaffected by any α-β tracker problems, are used in this report.

At present only the first part of the ERS-1 mission has been analysed in this way. This is the so-called commissioning phase. Data are available from September to December 1991. The satellite was in a three day repeat which gives good temporal coverage but means that the satellite tracks are a large distance apart. This has serious implications for looking at data in the North Sea as will be explained in chapter 4. Since these data are from the very beginning of the ERS-1 mission there are a number of gaps in the data, including some large ones. Later ERS-1 data are much more reliable.

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3. SCALES OF VARIATION IN THE OPEN OCEAN

3.1 INTRODUCTION

Before moving on to look at the more complex situation of spatial variation of the North Sea, which is semi-enclosed and in parts shallow, we will consider variability in the open ocean. Previous work using Geosat data has shown that there is little variation from ocean basin to ocean basin (Challenor, 1992). Therefore only the North Atlantic will be considered here, although it is believed that the conclusions hold for all the world's main oceans. The area chosen for investigation is from 20°N to 70°N and from 60°W to 10°W. Figure 3.1 shows a map of the area with the ERS-1 ground tracks overlaid. The small numbers by each track will be used to identify that track later in this section. Some tracks (35, 6, 10, 39, 18 and 27) are relatively short and have been discarded. The tracks can be divided into two classes: those running approximately from NW to SE and those from SW to NE. The satellite travels westwards and therefore the NW-SE tracks are ascending passes where ERS-1 is moving northwards and the NE-SW tracks are descending tracks. The latitude where the satellite ceases to move north and starts south again is 82°N for ERS-1, well north of our region. The wide spacing of the tracks of a satellite in a 3-day repeat is apparent. However it should be noted that in the period September to December ERS-1 traversed each track thirty times. Figure 3.2 shows the same area with the coverage by ERS-1 in its 35-day repeat. The spatial coverage is much better, but temporally each track is only sampled one twelfth as often. As stated in chapter 2 only data from the initial 3-day repeat have been re-analysed by the F-PAF and because of the superior processing scheme these are the only data that will be considered in this report.

Figure 3.3 shows the data along one of these tracks: 5. Each successive pass is offset by four metres. There are some unavoidable gaps. These data are from the initial four months of ERS-1 and there is a higher than normal percentage of missing data. Looking at the individual tracks it is apparent that in some the spatial properties are constant across the area while in others they are not. Storms are visible which have a length scale of between 2° and 10° of latitude.

3.2 ALONG TRACK SPECTRA

When dealing with temporal data one very important statistic is the frequency spectrum. With data in space the equivalent is the wave number spectrum. Spectra of the data along individual tracks have been computed. Note these are spectra on significant wave height every 7 km along track not wave spectra in the conventional sense where we are looking at surface elevation data at frequencies around 2 Hz. This point is important. A conventional wave measuring device, such as a Waverider buoy, measures sea surface elevation every 0.5 s or so. Significant wave height is then computed by taking 4 times the standard deviation of these elevations. The altimeter directly measures the surface roughness or equivalently the significant wave height. The along track spectra of significant wave height will bear no relationship to the frequency spectra of surface elevation over 15 - 30 minutes. For details of the definition of the spectrum see Chatfield(1978)

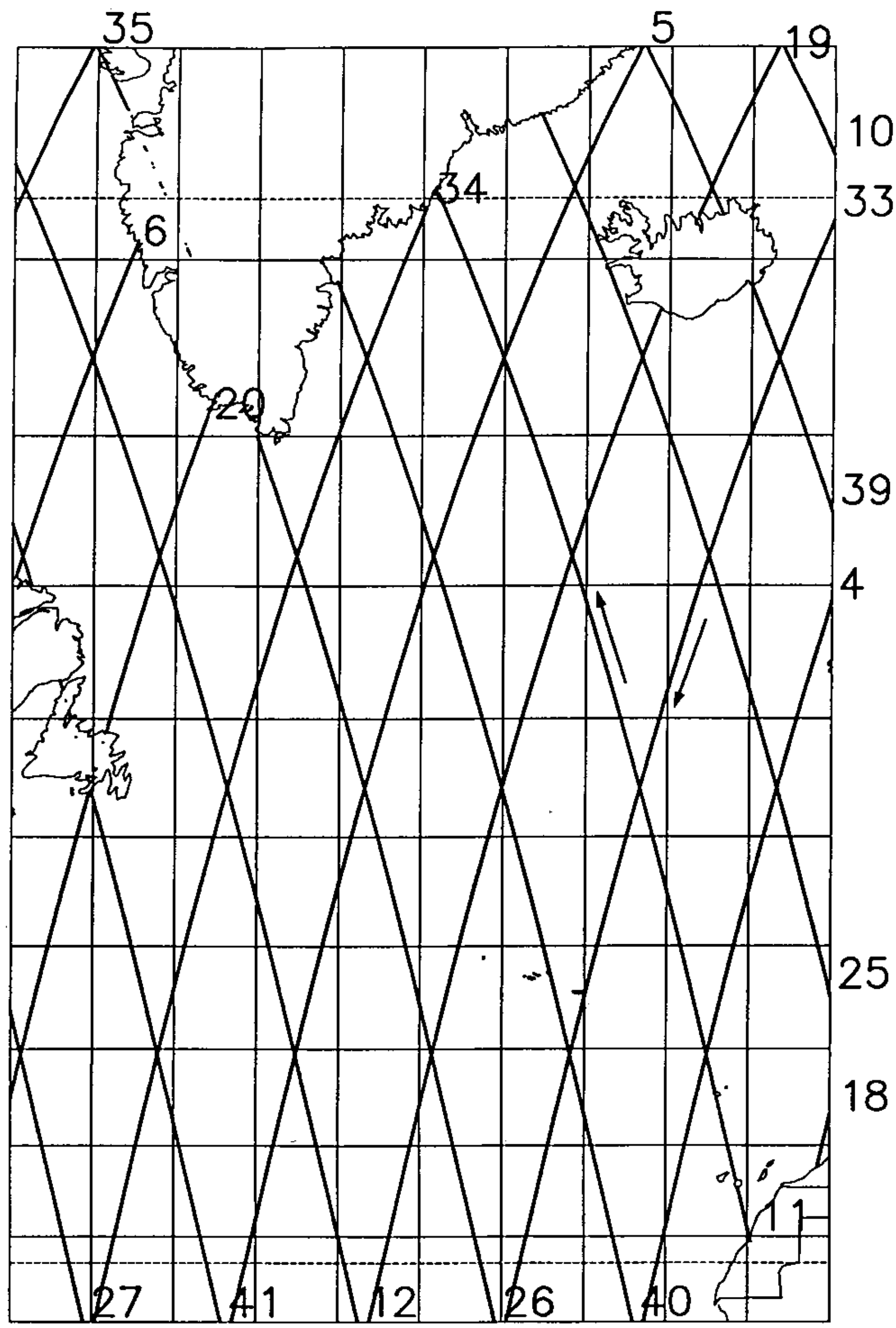

Figure 3.1 The area of the North Atlantic studied in this report. The superimposed lines show the ERS-1 tracks used with identification numbers. The arrows show the direction in which the satellite is moving

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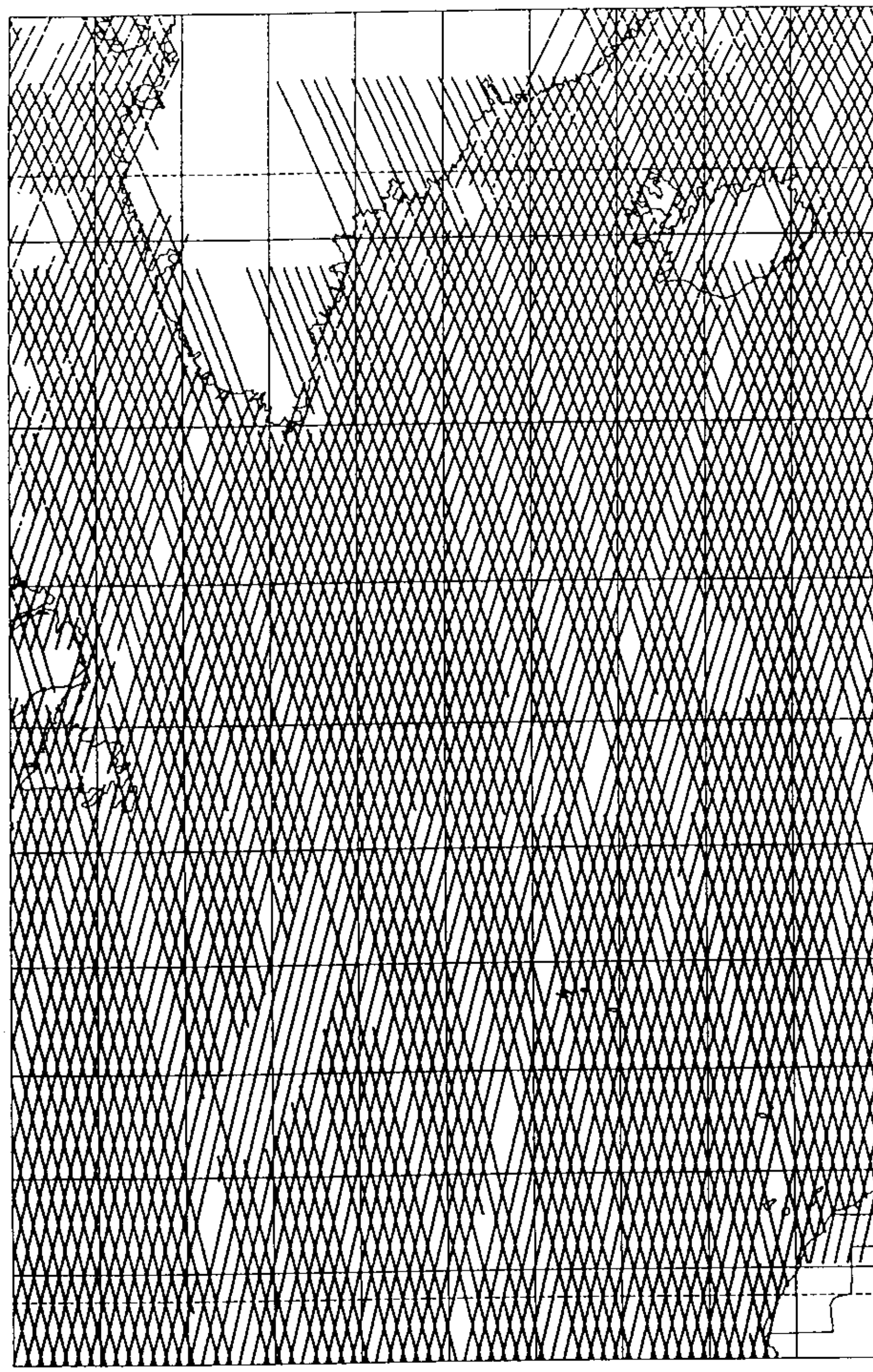

Figure 3.2 The coverage of the North Atlantic from ERS-1 in the 35 day repeat.

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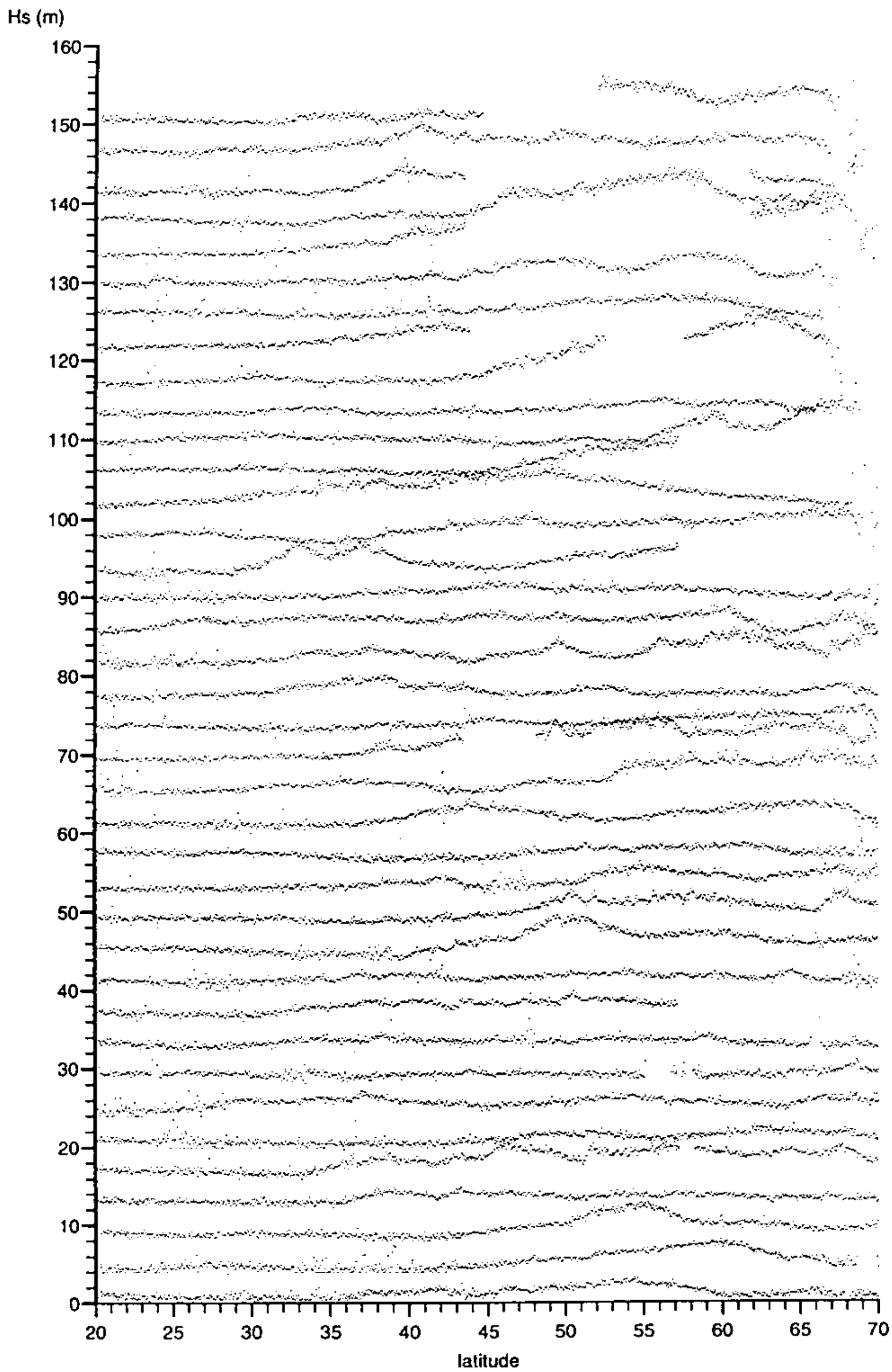

Figure 3.3 Significant wave height along track 5 from August to December 1991. Each successive pass is offset by 4m.

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In order to look at the scales of variation in space spectra were taken along track. In contrast to Challenor (1983) and Tournadre (1993) no effort was made to separate 'stationary' portions of the wave field from those where the properties were changing in space. This has a number of consequences. A positive result is that the spectra are a true representation of the spatial scales present along the satellite track, sampling for stationarity could bias the results in some way. In particular selecting the stationary portions can act as a form of filter. In practical applications, for example assimilating data into models, selecting stationary regions and choosing the appropriate statistics would be impractical. However it has to be borne in mind that non-stationarity could influence the results as well. The region under consideration was chosen, subjectively, to be consistent in its properties.

Because of gaps in the data a technique to deal with missing data is required. Each pass is examined starting from the north with descending passes and the south with the ascending passes and gaps less than five data points long are filled by linear interpolation. Any gap longer than five points causes the examination to cease. The resulting data set is padded with zeros to make it 2048 points long and a Fast Fourier Transform used to produce spectral estimates at 512 wavenumber values. A Daniel (rectangular) window is used to smooth the spectrum.

Since we are interested in the scales of along track variability rather than in the size of the waves all the data have been standardised. The mean H_S is removed from each pass before the spectrum is calculated and after the spectrum has been computed its area is set equal to one. This means that the along track variance has been set to unity.

Figure 3.4(a-n) show the mean spectrum for each track across the area, identified by the track number as shown in figure 3.1. *A priori* one may have expected differences between the western and eastern Atlantic caused by the generally shorter fetches with the prevailing South-westerly winds in the western half of the ocean. However there is great variation about these shapes in the individual tracks. Figure 3.5 shows every third spectrum along a single track, number 5. This should be compared with the mean in figure 3.4. Figure 3.6 shows the mean of the means and in view of the very small differences between the mean spectra along individual tracks this can be regarded as the mean along track wave spectrum for the entire North Atlantic. (As an aside work on Geosat data reported in Challenor (1993) implies that this could be regarded as the along track spectrum for all the open ocean.)

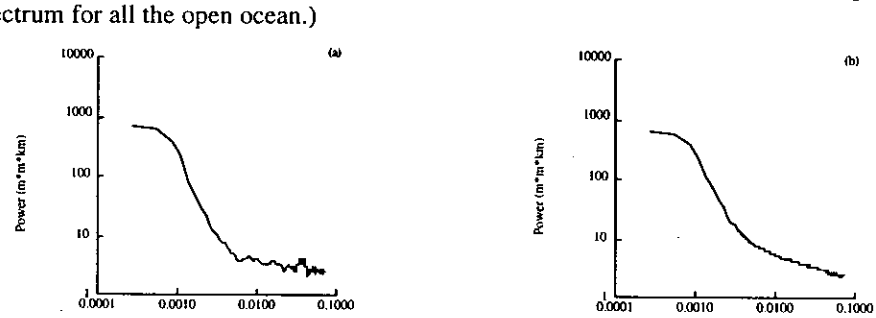

Figure 3.4 Average standardised along track spectrum of H_S for (a) track 4, (b) track 5. The track numbers are shown in figure 3.1

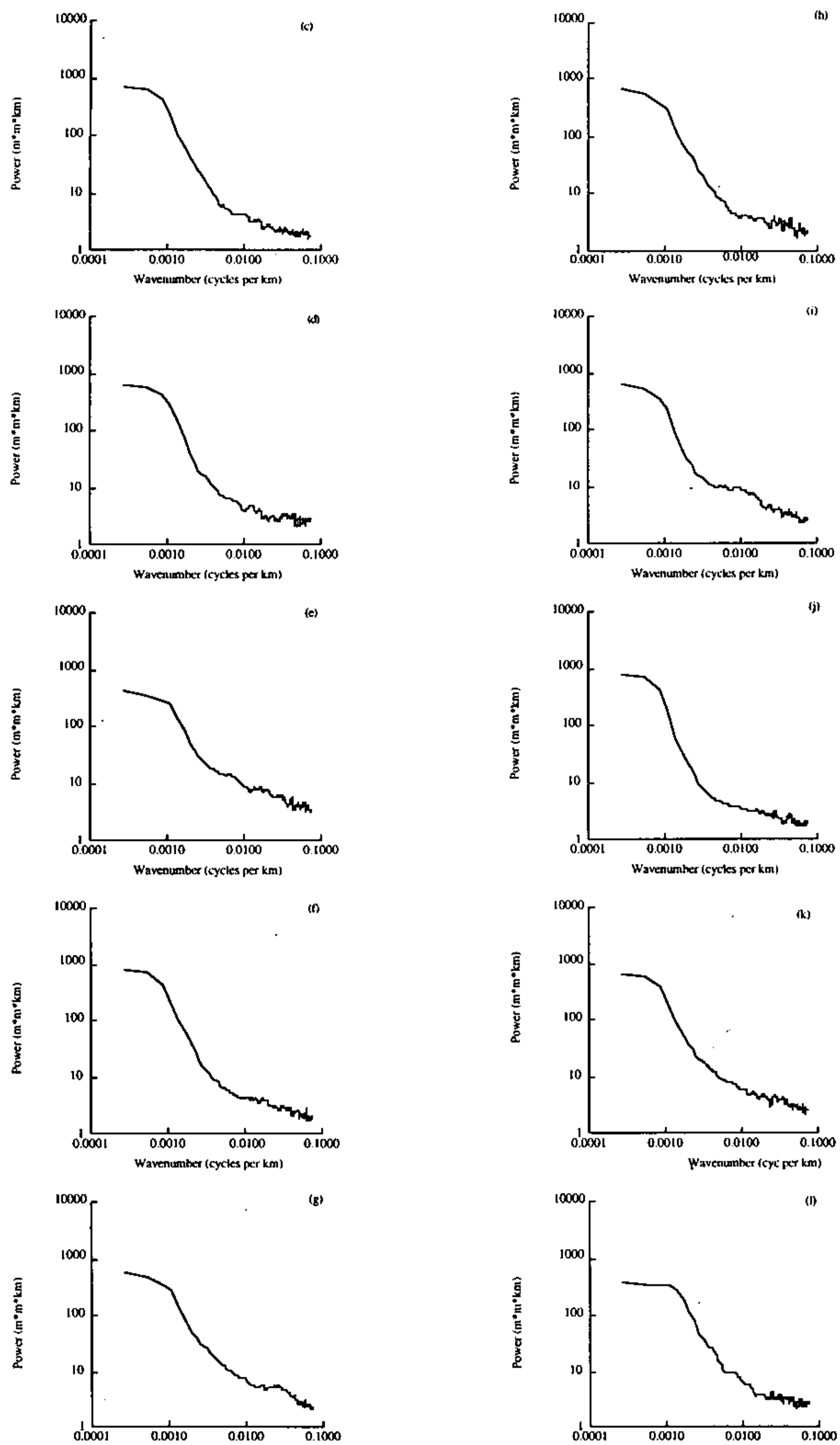

Figure 3.4 (continued) Average standardised along track spectrum of H_S for (c) track 11, (d) track 12, (e) track 13 , (f) track 19, (g) track 20, (h) track 25, (i) track 26, (j) track 33, (k) track 34, (l) track 39. The track numbers are shown in figure 3.1

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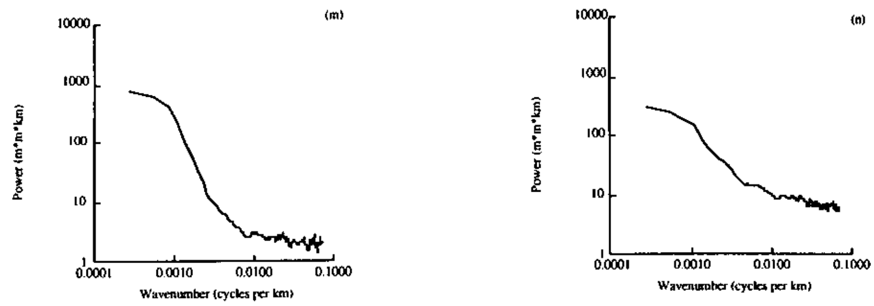

Figure 3.4 (continued) Average standardised along track spectrum of H_S for (m) track 40, (n) track 41. The track numbers are shown in figure 3.1

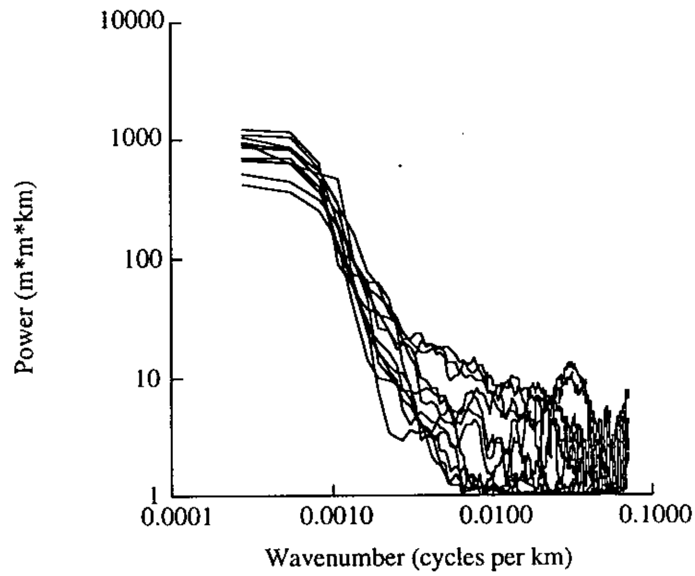

Figure 3.5 Individual standardised along track spectra of H_S from track 5 (Only every third spectrum is plotted to aid clarity)

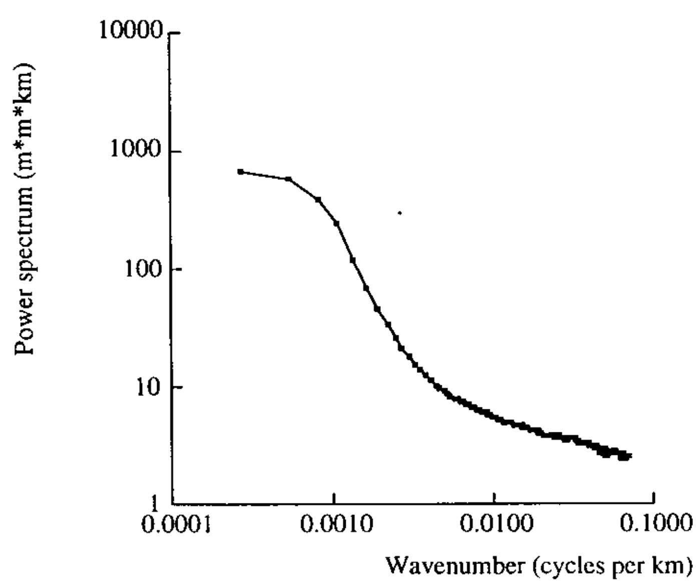

Figure 3.6 The average standardised along track spectrum of H_S for the North Atlantic.

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Examining this mean spectrum we can see that plotted on a log-log scale as in figure 3.6 it could be modelled as comprising two straight lines. (The flat plateau at very low wave numbers is an artefact of the processing and should be ignored. Because the mean was removed from the data, the value at zero wavenumber is zero; leakage and the effect of windowing means that the value of the spectrum at low wavenumbers is therefore reduced). At high wave numbers the spectrum appears to be flat and can therefore be regarded as white noise. Note that the plots are on a log scale which exaggerates variability in these high wavenumber regions. The more interesting part of the spectrum shown in figure 3.6 is the straight line linking the two plateaux. On log-log plots such as these a straight line implies a power law relationship with the slope equal to the power. This slope has been calculated by fitting a straight line to that portion of the spectrum lying between wavenumbers 0.0017 km^{-1} and 0.0050 km^{-1} (wavelengths of approximately 200 - 600 km). For the mean case this slope is -1.67. The means of the slopes of the individual tracks are shown in table 3.1.

Track number	number of passes	mean spectral slope
4	37	-2.08
5	38	-1.80
11	33	-2.05
12	27	-1.82
13	16	-1.14
19	36	-1.82
20	35	-1.56
25	30	-2.09
26	34	-1.43
33	33	-1.88
34	33	-1.64
39	30	-2.29
40	34	-2.01
41	24	-1.33

Table 3.1 Mean spectral slope between wavenumbers 0.0017 km^{-1} and 0.0050 km^{-1}

3.3 AUTOCORRELATION FUNCTIONS AND SCALES OF VARIATION

An alternative to looking at the wavenumber spectrum is to look at the autocorrelation function (acf). This is the Fourier transform of the spectrum so contains the same information but presented in a different way. In general it is easier to extract information on scales from the autocorrelation function.

The autocorrelation function gives the correlation between a measurement here and another at a distance, x km say, away. It is defined by

$$r(x) = \frac{\sum_t y(t)y(t+x)}{\sum_t y^2(t)}$$

where $r(x)$ is the autocorrelation function at lag x, $y(t)$ is the value of H_S at position t. For details see Chatfield (1978). Figure 3.7 (a-m) shows the average autocorrelation functions for each track in figure 3.1. In all 200 lags were calculated, for passes with less than 200 valid data values the acf was not calculated. No pass along track 39 satisfied this criterion and therefore no acf data are presented.

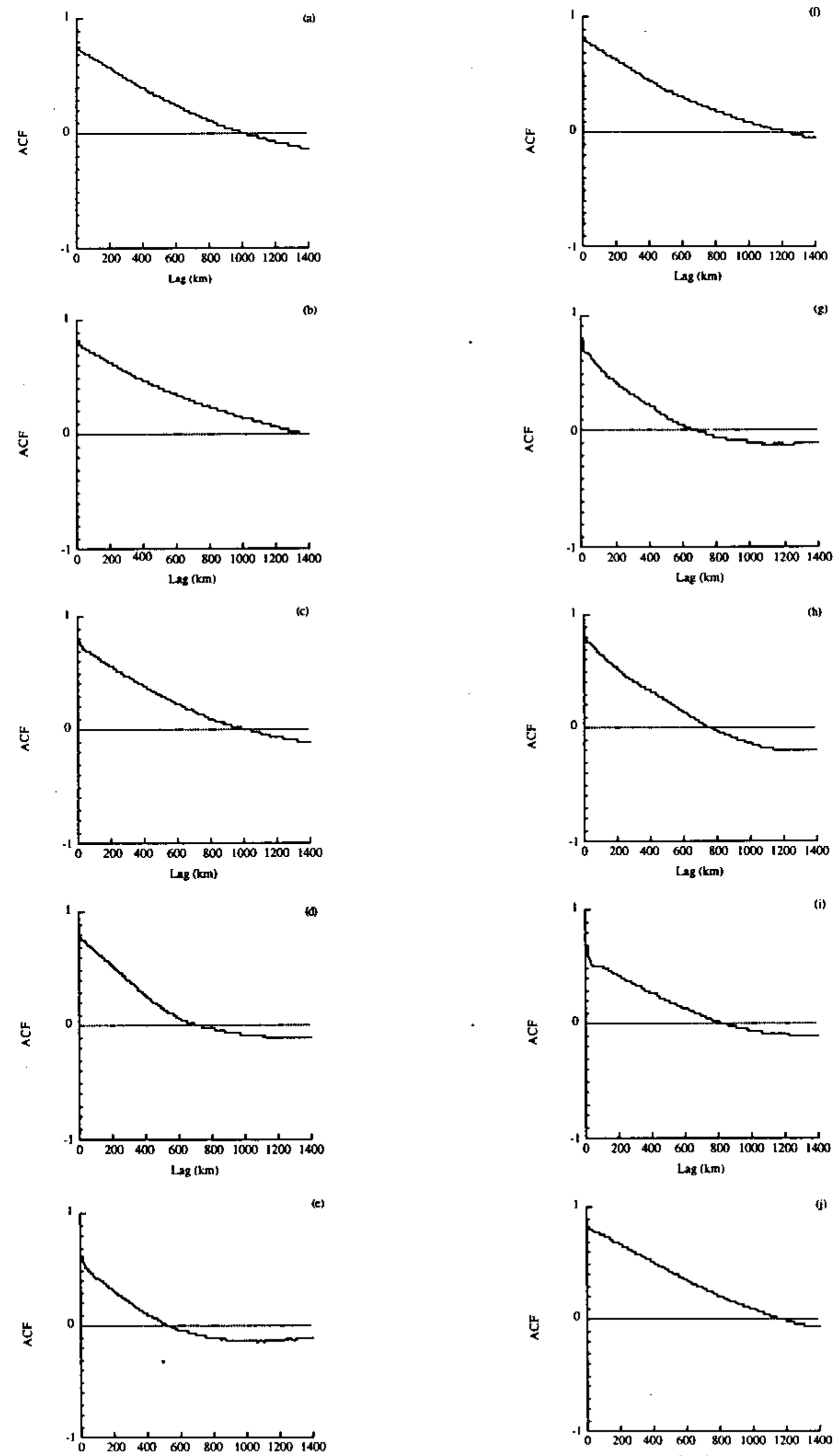

Figure 3.7 Along track autocorrelation function of H_S for (a) track 4 and (b) track 5, (c) track 11 and (d) track 12 , (e) track 13 and (f) track 19, (g) track 20 and (h) track 25, (i) track 26 and (j) track 33. Track numbers are identified in figure 3.1.

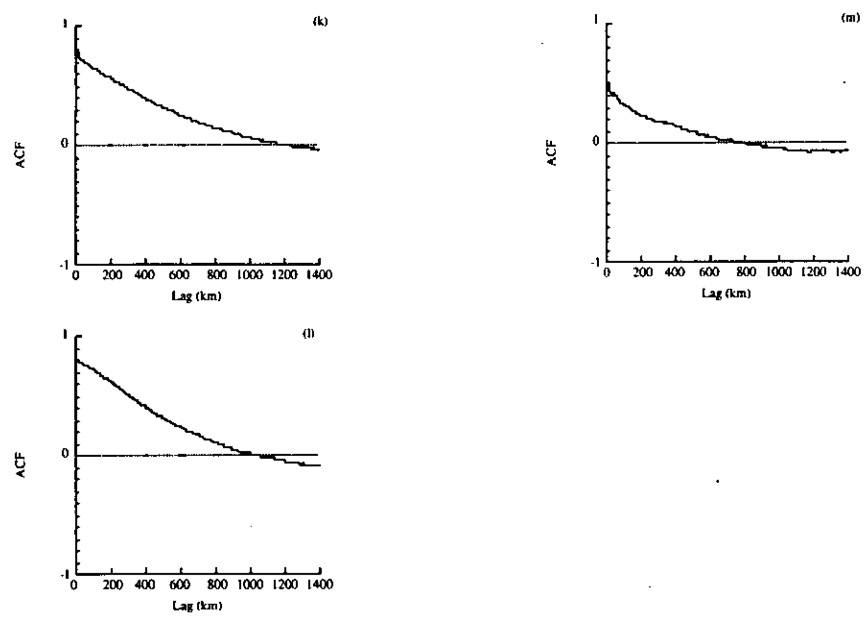

Figure 3.7 (continued) Along track autocorrelation function of H_S for (k) track 34 and (l) track 40, and (m) track 41. Track numbers are identified in figure 3.1.

Since we are mainly interested in the autocorrelation function to look at length scales we need to consider how to calculate such a scale. A number of definitions of a length scale have been given in the literature. Some of these are discussed in Le Traon and Rouquet (1990). The two measures used here are *L1* and *L2* defined by

$$L1 = \int_0^{z_c} r(x)dx$$

$$L2 = \int_0^{\infty} r^2(x)dx$$

where $r(x)$ is the autocorrelation function and z_c is the first zero-crossing of $r(x)$. Although *L2* is defined as an integral between 0 and ∞ only 200 lags were computed so the integral is truncated at 200. Similarly if z_c is greater than 200 the integral for *L1* is also truncated.

The results of applying these measures of length scale to the along track acf's is shown in table 3.2. The first thing to note is the amount of variation in the length scales given by these definitions. In particular there is considerable variation between the scales computed from the same tracks at different times as shown by the size of the standard deviations in the table. In general there appears to be an increase in length scale as one goes further east, however the evidence for this is flimsy. One important point is that the length scale increases as the track becomes longer. Given the shape of the spectrum this is not a surprise. As greater distances are included longer wavelengths can be resolved and these have greater energy than the shorter wavelengths giving the longer length scales.

The conclusion that must be drawn from this section is that the length scales of significant wave height are highly variable in time and that the four months data used in this report is insufficient to average out this variability. Some of this variation may be due to non-stationarity in the data. It may be fruitful to repeat the work using some method of detecting changes in the statistical properties of H_S along track, as used, for example, by Challenor (1982) or Tournadre (1993). However the application of such results is more difficult. As time progresses much more reprocessed ERS-1 data will become available

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and it should be possible to average out the variability and better define both means and variances. Other advantages of more data will be the ability to look at possible seasonal and interannual variations in spatial scales.

Track no.	Average L1 (km)	St. Dev. L1 (km)	Average L2 (km)	St. Dev L2 (km)	Average length of track (km)
4	357	194	215	138	3983
5	472	185	259	135	5656
11	354	200	215	140	3976
12	275	151	175	109	3815
13	139	91	90	79	2569
19	433	168	243	121	5173
20	206	123	127	90	3087
25	289	131	188	97	3409
26	248	169	140	111	3787
33	477	201	289	149	5208
34	372	179	202	123	5040
39	-	-	-	-	<1400
40	389	201	239	144	4333
41	135	128	63	81	3388

Table 3.2 Length scale along each altimeter track shown in figure 3.1

4. SCALES OF VARIATION IN THE NORTH SEA

4.1 INTRODUCTION

The sampling of the ERS-1 validation phase 3-day repeat over the North Sea is shown in figure 4.1. This orbit phasing was chosen to allow the instrument to pass over the Acqua Alta tower off the Adriatic coast near Venice. As can be seen in the figure the resulting pattern of data collection over the North Sea is less than ideal. There are four tracks that are of interest. These are numbered 3, 24, 32 and 38. Two (3 and 38) lie close to the Danish coast at the far east of the North Sea and the other two (24 and 32) are near the British coast on the western side. Track 38 continues north of Norway to give some data in the northern North Sea. Since we are trying to look for differences between the semi-enclosed North Sea and the open Atlantic we will concentrate on data collected south of 60°N where these differences would be expected to be greatest.

Figure 4.2 a-d shows significant wave height along track for tracks 3, 24, 32 and 38. Looking at these figures there are regions of obviously incorrect data, for instance between 58.2°N and 60°N on track 32 (fig 4.2 c). This is when the altimeter is passing over land and instead of measuring wave height is computing some measure of land surface roughness. A similar region which is not over land is apparent on track 38 south of 55°N. This is over the ocean but is giving very poor data. The reason for this is not clear but there are numerous islands and sand banks in this region. Sand banks could have a severe effect on the altimeter giving signals similar to other areas of still water such as sea ice and wetlands (Guzkowska et al., 1990). In order to avoid the problems caused by these anomalous signals the tracks in the North Sea have been limited by the bounds given in table 4.1. These are also show on figure 4.1. Track 32 has been arbitrarily truncated in the north. At this end of this track the situation is much more akin to the open ocean than the semi-enclosed North Sea so it was decided to cease processing at 60°N.

Track no	Southern Limit	Northern Limit
3	53.6°N	58.3°N
24	53.2°N	59.0°N
32	55.0°N	60.0°N
38	55.0°N	58.2°N

Table 4.1. Northern and Southern bounds on tracks used for analysis. The track positions are shown in figure 4.1

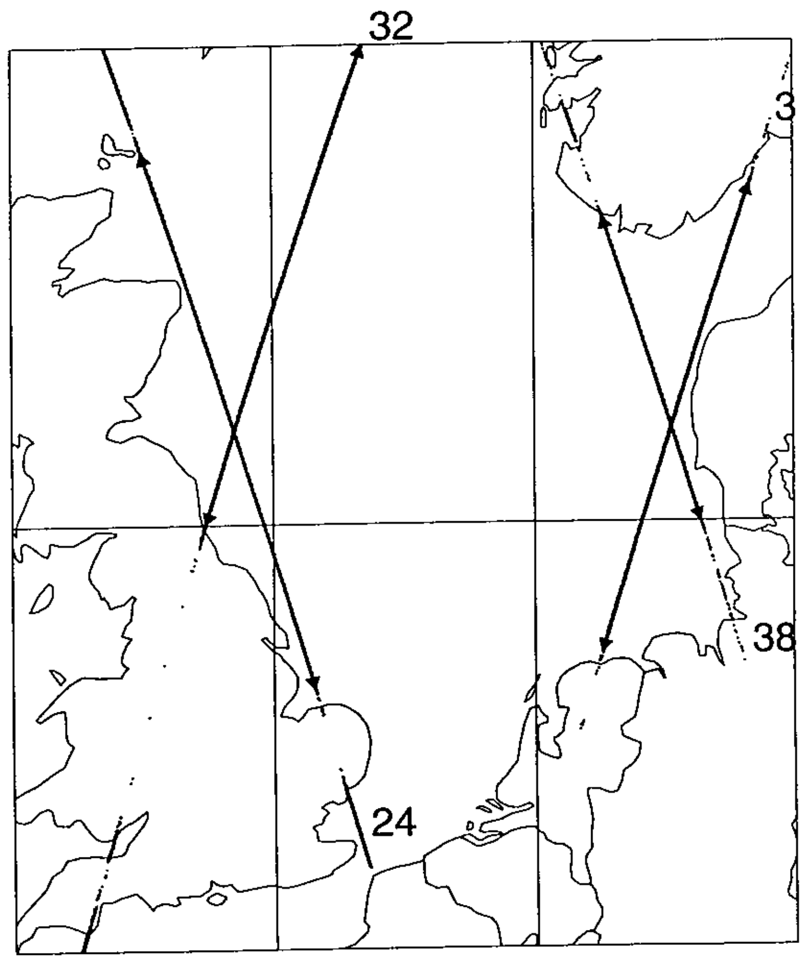

Figure 4.1 Map of the North Sea showing coverage by ERS-1 in the commissioning phase with track identification numbers. The ◄–► show the limits on each track used in this report.

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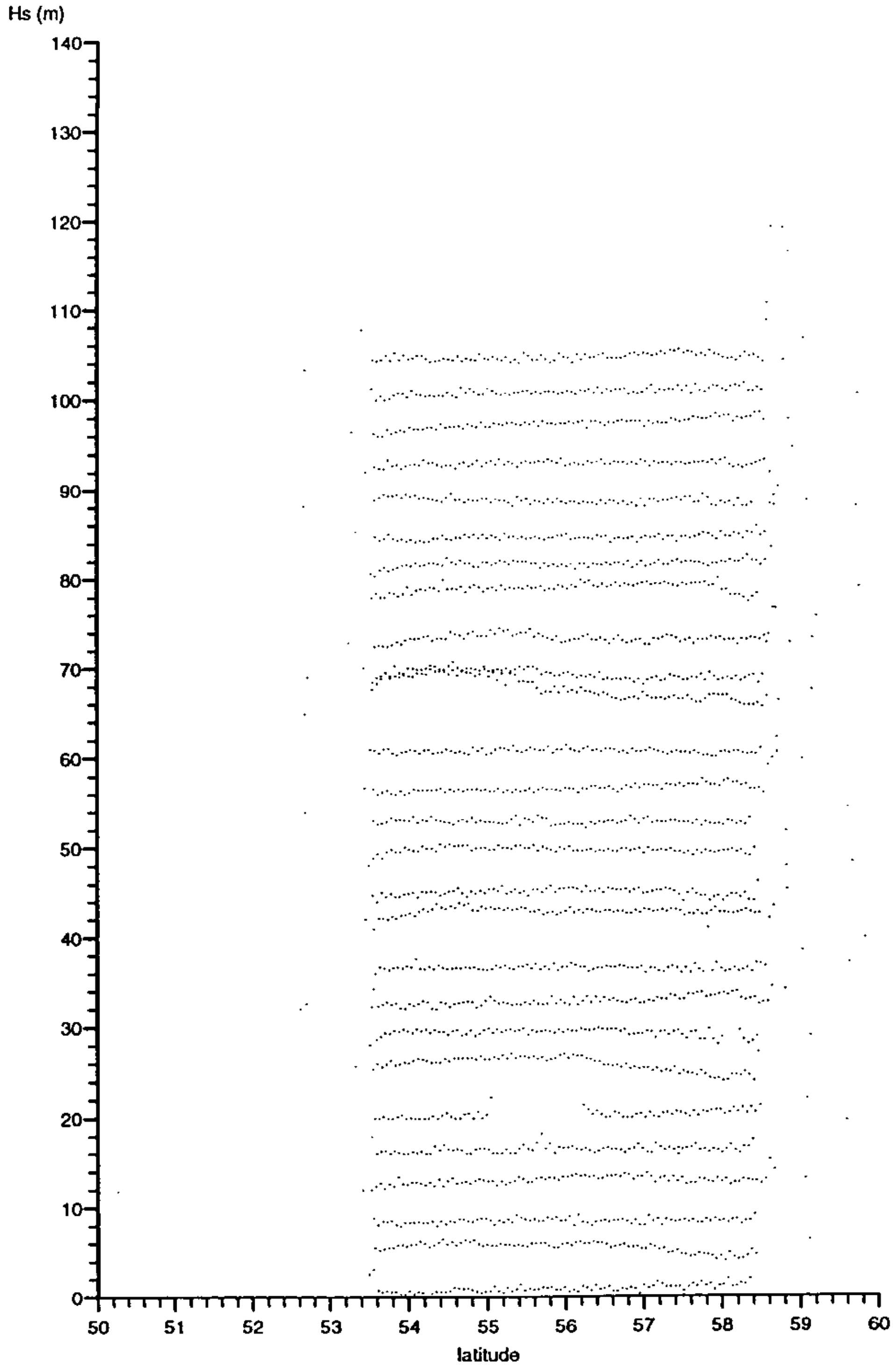

Figure 4.2(a) H_S along track for track 3 . Each pass is offset by 4m. Note the bad data over land

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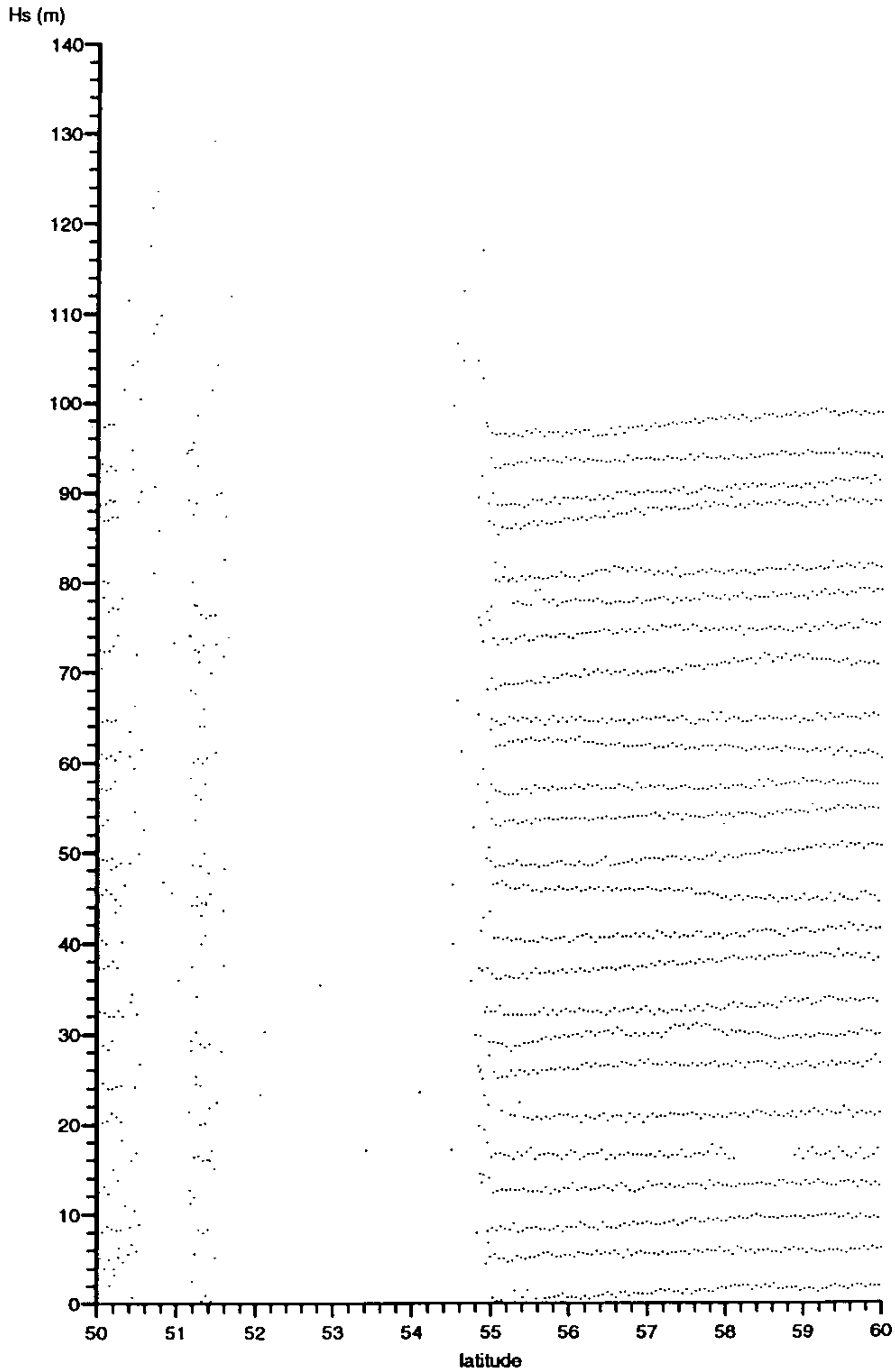

Figure 4.2(b) H_S along track for track 24 . Each pass is offset by 4m. Note the bad data over land

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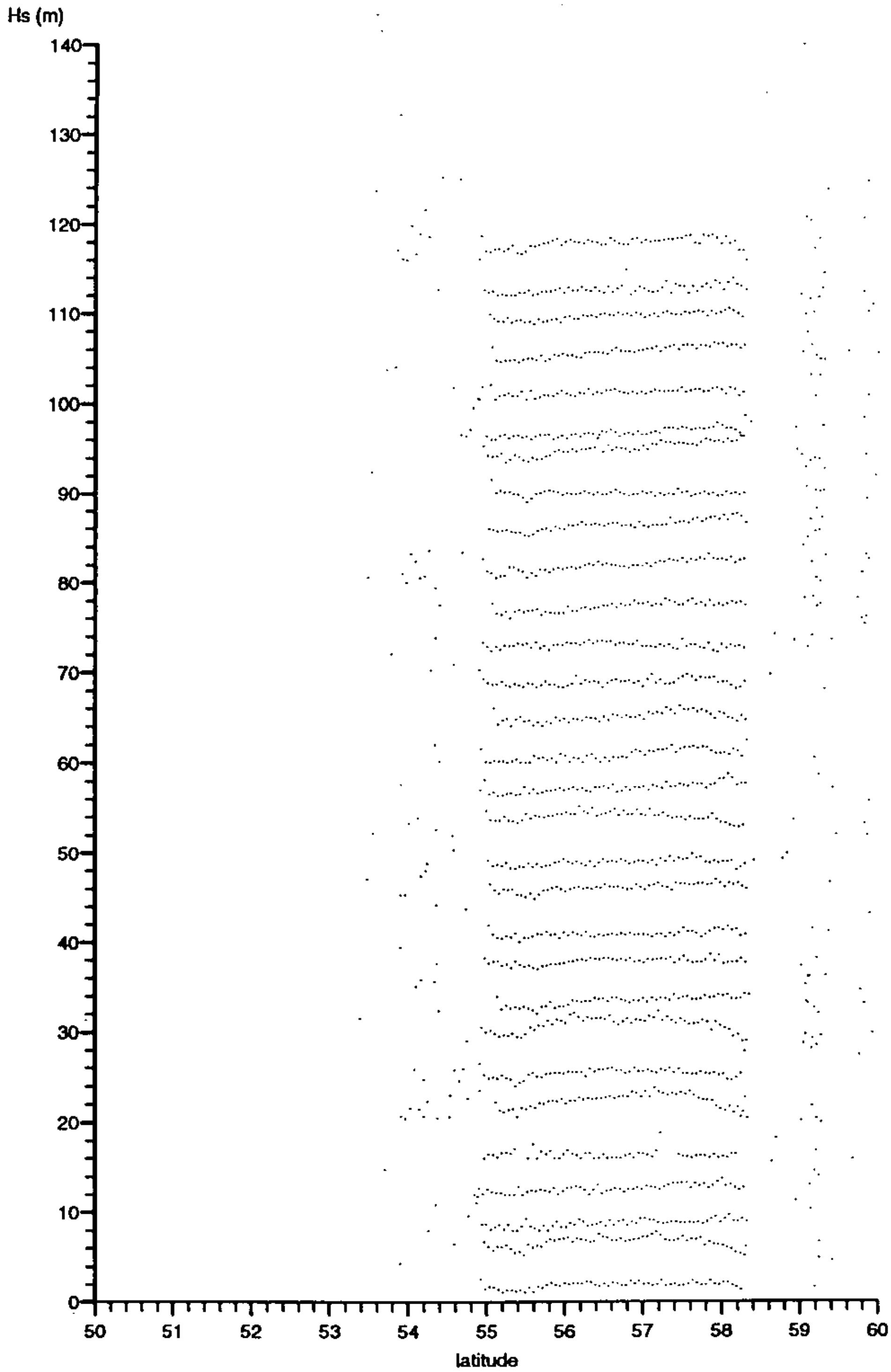

Figure 4.2(c) H_S along track for track 32 . Each pass is offset by 4m. Note the bad data over land.

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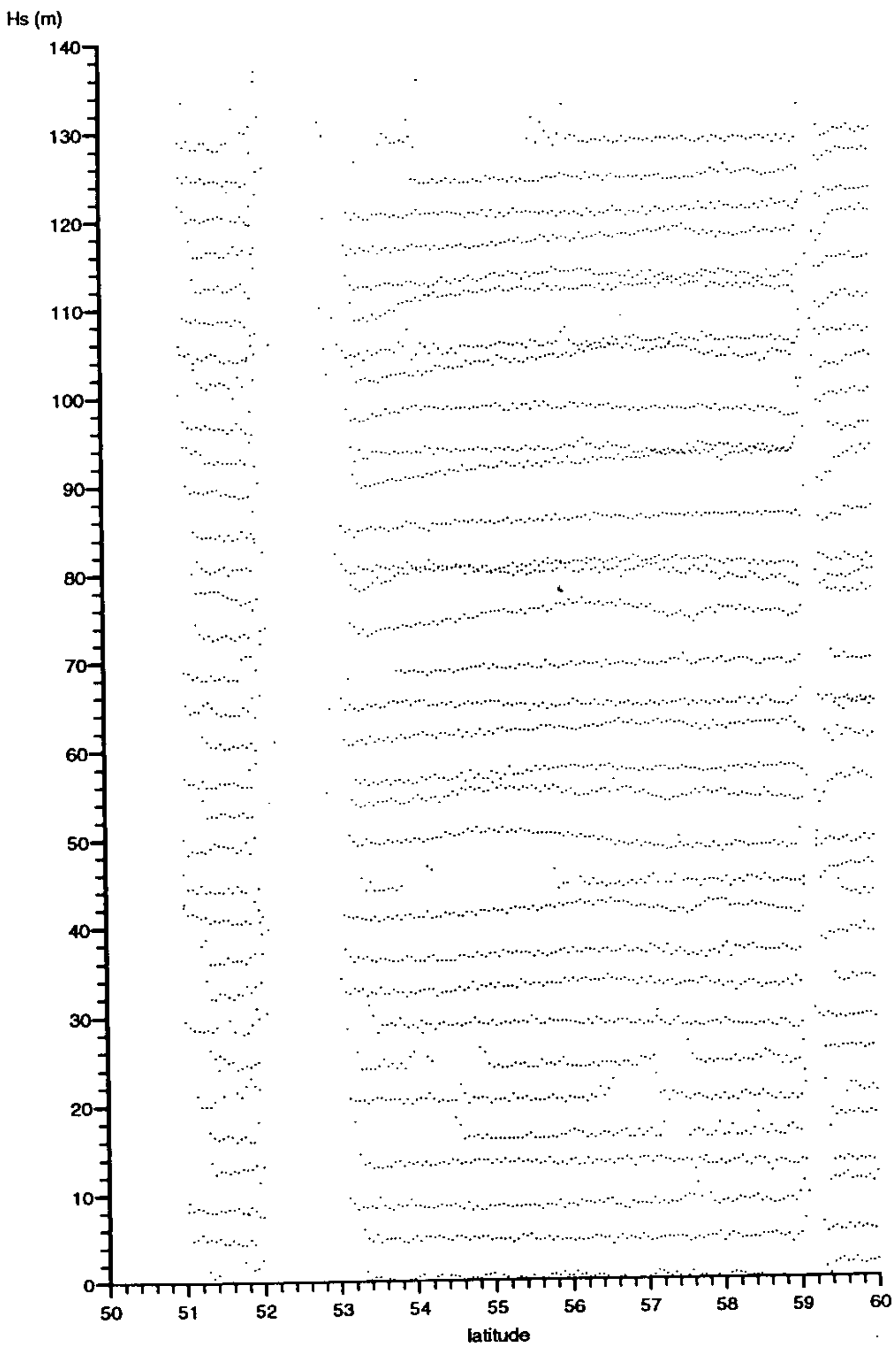

Figure 4.2(d) H_S along track for track 38 . Each pass is offset by 4m. Note the bad data over land

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4.2 ALONG TRACK SPECTRA

The spectra along track were calculated using the same methods as in the previous section. The major difference here is that because of the much shorter tracks we have many fewer points to analyse. This means that it is impossible to resolve the lower wavenumbers. Instead of 2048 points each series was padded to 256 and the spectrum was produced at only 64 wavenumbers. Figure 4.3(a-d) shows the mean spectrum from each of the four tracks considered. Again the degree of uniformity is surprising. The individual spectra are much more variable. The shapes of the spectra are in general similar to those in the North Atlantic. A power law leading to a 'noise' floor. This noise floor appears at higher wavenumbers. One possibility is that this flat region of the spectrum is in fact caused by instrumental noise. However if this were the case one would expect it to appear at similar 'power' levels in each region. Because of the standardisation applied to the spectra it is not possible to check this.

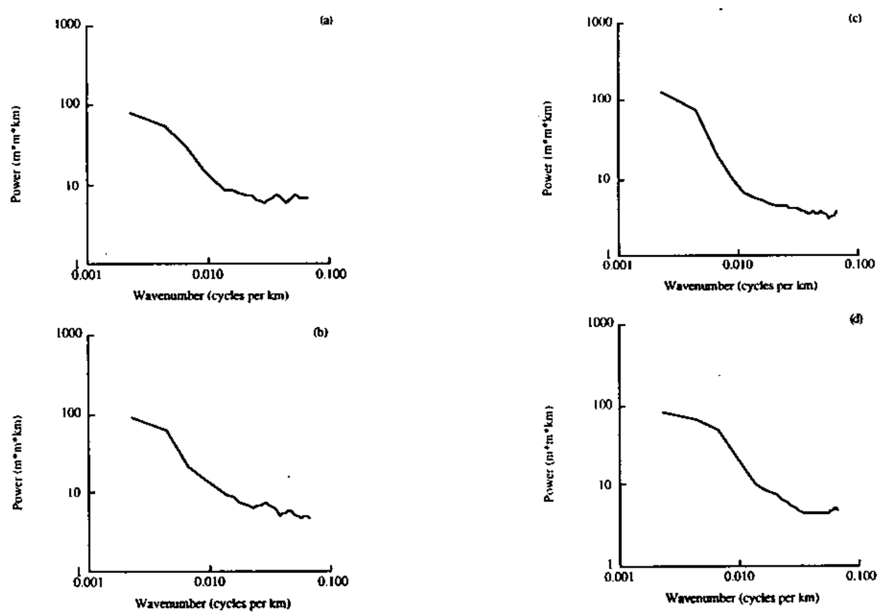

Figure 4.3 Average standardised along track spectra of H_S for (a) track 3 and (b) track 24., (c) track 32 and (d) track 38. The track numbers are identified in figure 4.1.

Of more interest is the power law region. The slope of each spectrum has been calculated over the same range of wavenumbers, 0.0017 - 0.005 km^{-1} (200-600 km wavelength). The results are given in Table 4.2. The results are consistent with the slopes being the same as in the open ocean.

Track number	no of passes	slope
3	27	-1.60
24	33	-2.06
32	25	-2.74
38	30	-1.14

Table 4.2. The slope of the average spectrum between 0.004-0.007 km^{-1} for each track in figure 4.1.

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4.3 AUTOCORRELATION FUNCTIONS AND SCALES OF VARIATION

When considering the very long tracks of data across the North Atlantic it was decided to calculate 200 lags for the autocorrelation function. The data in the North Sea cover a much shorter distance, the shortest track (38) is only some 55 points long. Therefore it is necessary to limit the number of lags calculated. The number of lags used in this chapter is 45. This number was chosen as a compromise between maximising the number of lags and hence the accuracy of *L1* and *L2* and the number of data points going into the calculation of the long lags. Figure 4.4 (a-d) shows the average autocorrelation functions for the four tracks across the North Sea and table 4.3 shows the length scales, defined as in chapter 3. At first sight it appears that the length scales of the two tracks near the Danish coast (3 and 38) are significantly shorter than those near the UK coast (24 and 32). However if all tracks are restricted to the same length of track as the shortest (track 38; 55°N - 58.15°N) the results given in table 4.4 are obtained. Here there is no discernible pattern showing that the difference in scales across the North Sea was probably due to the difference in the track length rather than any geophysical change across the basin. Thus we would have to conclude that there is no evidence for any difference in length scale between the west edge and eastern edge of the North Sea.

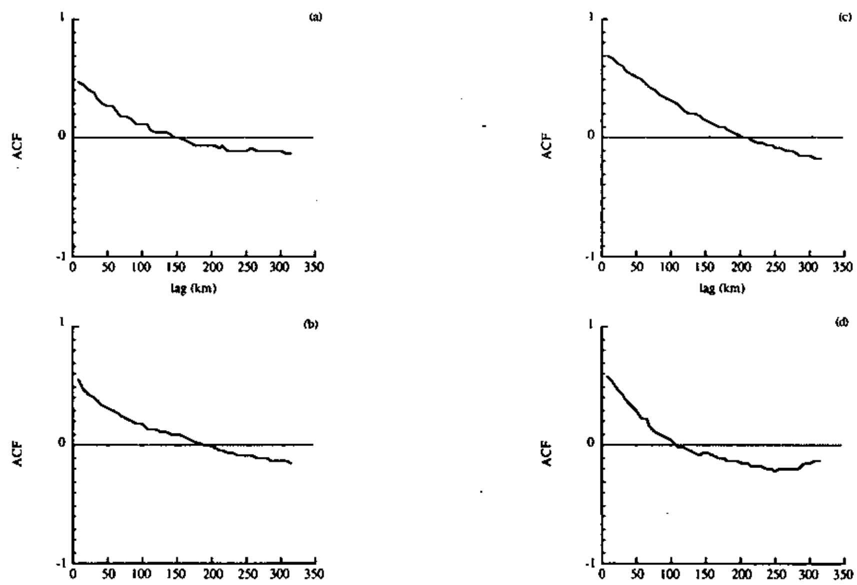

Figure 4.4 Average autocorrelation function for (a) track 3, (b) track 24 and (c) track 33, (d) track 38. Track numbers are identified in figure 4.1.

Track no	Average L1 (km)	St. dev. L1 (km)	Average L2 (km)	St. dev. L2 (km)
3	31	.25	18	25
24	40	34	23	22
32	64	32	37	22
38	28	14	21	12

Table 4.3. Length scales for the tracks shown in figure 4.1

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Track no	Average L1 (km)	St. dev. L1 (km)	Average L2 (km)	St. dev. L2 (km)
3	20	19	14	15
24	16	15	13	13
32	32	19	23	16
38	28	14	21	12

Table 4.4. Length scales for the tracks in figure 4.1 where the calculation has been restricted to the latitude band 55°N - 58.15°N

4.4 CONTRAST WITH THE OPEN OCEAN

We have computed two sets of statistics for the North Atlantic and the North Sea namely spectral slopes and length scales. How do the two regions compare?

Looking at the spectral slopes first the slopes from the North Sea appear to be steeper than those of the open ocean. However because of the unfortunate track phasing over the North Sea only very short tracks are available for analysis. This means that a higher range of wavenumbers has had to be used to calculate the slopes. Because the noise floor becomes apparent at higher wavenumbers in the Atlantic data it is not possible to calculate slopes over a similar range. If one accepts that the difference is real then the length scales in the North Sea are shorter than in the open ocean. As an aside Tournadre (1993) gets a spectral slope of 1.6 in the North Sea using Geosat data (which is identical to the slope we have computed for the mean spectrum in the Atlantic). Because of the differences in the instruments and analytical techniques described in Chapter 2 these values are not directly comparable.

The length scales derived from the acf's show much longer length scales in the Atlantic than in the North Sea. However as we have seen above the length of track used to derive the length scale influences the values. Table 4.5 shows the values for *L1* and *L2* for the open ocean passes when the latitude is restricted to the same limits as used for Table 4.4 (55° - 58.15°). The values are remarkably similar. There is no evidence in these data for any difference in length scales over these relatively short distances (<400 km). Both the spectral data and the length scales have shown that the longer the track (or smaller the wavenumber) the longer the scale of variation. When we restrict the distance over which we are considering data, either artificially as in Table 4.5, or because of the size of the basin as in the North Sea, we lower the length scale. The reason for this is simply the shape of the spectrum as the distances considered get longer smaller wavenumber signals which are more energetic are included in the calculation. An alternative hypothesis would be that the wave field consists of a number of short regions over which the wave field is stationary, these are linked by areas of rapid change in wave conditions (see Tournadre, 1993). The author believes that the evidence presented here supports the former rather than the latter conclusion.

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Track no	Average L1 (km)	St. dev. L1 (km)	Average L2 (km)	St. dev. L2 (km)
4	-	-	-	-
5	20	19	14	14
11	21	18	15	14
12	23	20	16	17
13	-	-	-	-
19	23	18	16	14
20	20	187	15	13
25	21	16	13	11
26	25	20	18	17
33	11	14	8	9
34	22	19	16	14
39	-	-	-	-
40	21	17	14	11
41	-	-	-	-

Table 4.5 Length scales for the North Atlantic data restricted to 55°N to 58.15°N

5. CONCLUSIONS

This report has looked at the spatial scales of variability of significant wave height in the North Atlantic and North Sea using off-line data from the ERS-1 altimeter. Although there is now a number of years data from the altimeters on Geosat and ERS-1 only the off-line data (at present the last four months of 1991) has been used because it is free from any distortion caused by the operation of the instrument. It has been assumed that the ERS-1 measurements of H_S are unbiased. Some work has been done (Carter, pers. comm.) which suggests that this is not the case. There are two ways in which a bias could influence the data. The first would be through an additive factor (such as ERS-1 always measures H_S as 0.5m, say, too low) alternatively there could be a multiplicative factor (for instance the 13% correction to Geosat wave heights reported in Carter et al., 1992). The former correction would not affect the results given here since when the along track mean wave height is removed (section 3.2) any additive factor would also be taken into account. A multiplicative factor is more difficult to cope with and it could influence the results. Currently opinion is that the correction for ERS-1 may be a combination of the two, i.e. a linear function. However work on the final calibration of ERS-1 significant wave heights is at an early stage and it would be premature to apply any of the results at this time.

The major result from this study has been to show that the spectrum of significant wave height in space can be represented by a 'red' spectrum. A 'red' spectrum is one where energy increases as one moves towards lower wavenumber (or frequency), apparently without limit. Such spectra are common in geophysics. For example Le Traon and Roquet (1990) produce very similar spectra for sea level variation in the North Atlantic. Further it has been shown that these spectra are well represented by a power law with an exponent between -1.1 and -2.0. In some ways these results are surprising as *a priori* one would have expected the scale of atmospheric depressions to be significant. Since we are dealing with very large areas is it possible that we are simply seeing a change in wave climate from north to south across the Atlantic. The consistency of the power law relationship over a large number of scales would imply not. A simple trend would show up as a peak in the spectrum at very low wavenumbers.

What are the consequences of a power law relationship? A power law implies that significant wave height is self similar. This means that if we were to expand the scale it would be impossible to tell the original signal from the blown up version as the spatial properties are the same. Such behaviour can be indicative of a fractal signal (Mandelbrot, 1982). Fractals are curves whose dimension is not an integer and exhibit more and more structure as the scale increases. The presence of a fractal may imply that the underlying dynamics is non-linear and even chaotic. In the case of significant wave height we know that the power law does not apply at very short scales where individual waves become important and thus it is not a true fractal. However over those regions where it is self similar it may be worth while to look in future to see if significant wave height can be regarded as fractal (or even chaotic) in this region.

This report has also looked at differences between spatial scales in the open ocean and in the North Sea. It has proved impossible to discern any significant differences. This seems surprising as it would be expected that the limited fetch conditions in the North Sea would limit the swell being produced. However it should be noted that very limited amounts of data were used in this study and the results need to be treated with caution. Much larger volumes of data are now becoming available not only from ERS-1 but also from the joint US/French satellite TOPEX/POSEIDON. Analysis of this larger data set will enable a much more conclusive study to be carried out in future.

One result of this study has been the extreme variability of both along track spectra and autocorrelation functions. This variability could be due to sampling variability but is more likely simply indicative of the natural variability of the ocean. Again more work on a

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larger data set is required to show which of these two possibilities is correct. If it is due to natural variability this could have some unpleasant implications. For instance if data are being assimilated into a wave model (a practice which is now common in forecasting, and as the amount of satellite data increase will probably be used in hindcasting in the future) it is usual to use some form of the autocorrelation structure, or a parameter derived from it, to weight by distance the data that are to be included. If as this study would suggest the autocorrelation function cannot be regarded as a constant then such methods could be seriously in error.

In conclusion it must be stated that although there have been some interesting results from this study the amount of data was too small to make any firm conclusions, and in the case of the North Sea data had very poor geographic coverage. When there are several years data available it would be worthwhile to repeat the exercise to allow firmer conclusions to be drawn. Problems such as seasonal variation could also be included in such a larger study.

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