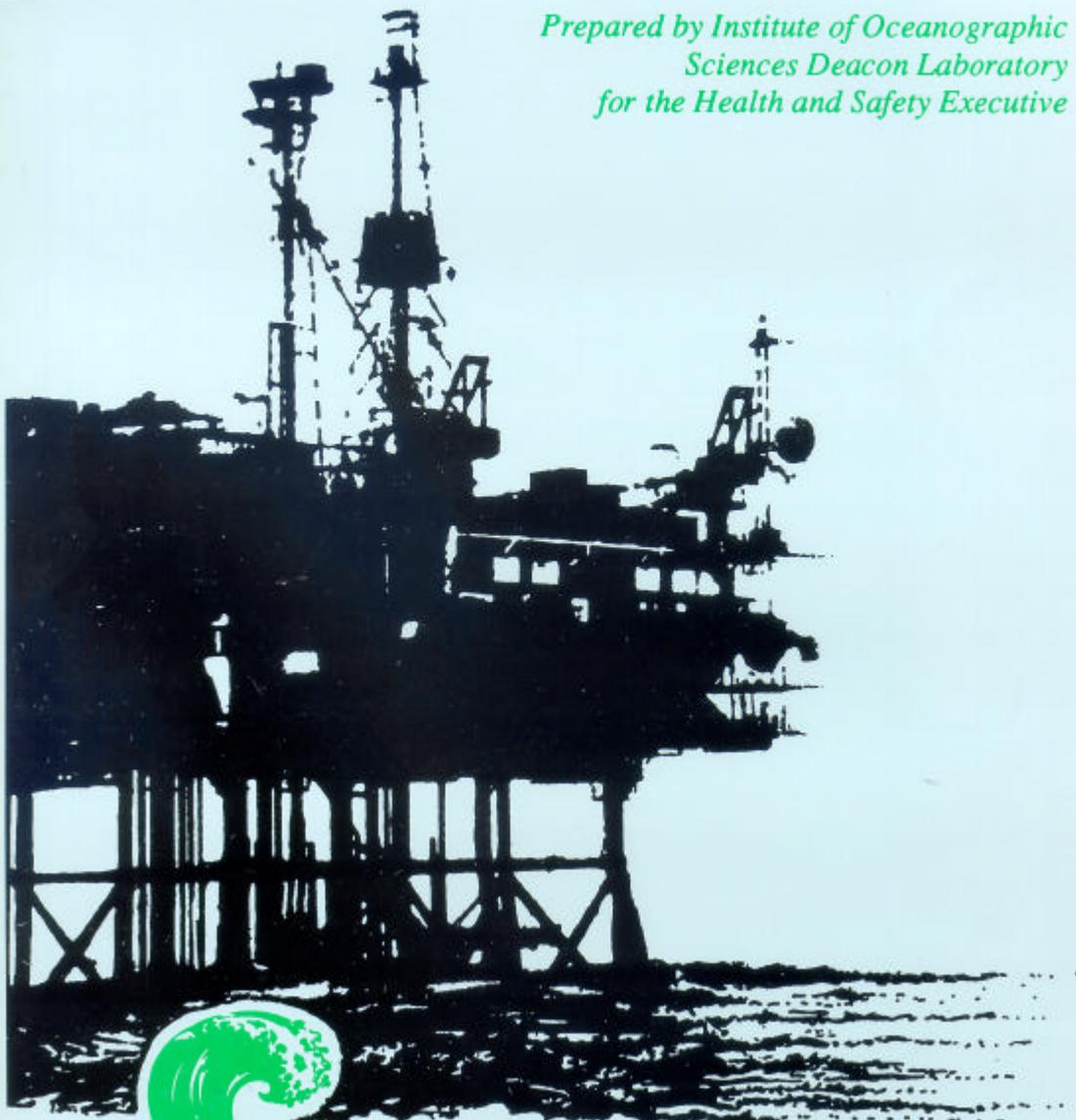


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# ESTIMATING EXTREME WAVE HEIGHTS IN THE NE ATLANTIC FROM GEOSAT DATA

*Prepared by Institute of Oceanographic  
Sciences Deacon Laboratory  
for the Health and Safety Executive*



*Offshore Technology Report*

**Health and Safety Executive**

**ESTIMATING EXTREME  
WAVE HEIGHTS IN THE NE  
ATLANTIC FROM GEOSAT DATA**

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## SUMMARY

Estimates of significant wave height ( $H_s$ ) can be obtained from a satellite as it orbits the globe using a downward looking radar, the primary purpose of which is to measure the satellite's altitude. This report explains briefly how the radar altimeter measures  $H_s$  and describes some of its limitations.

The US Navy's satellite 'Geosat' was launched in 1985, and its radar altimeter continued to function satisfactorily until 1989; it failed completely in January 1990. Geosat was thus the first satellite to obtain some years of global  $H_s$  data. After a discussion on the accuracy of the measurements of  $H_s$  from Geosat, this report investigates whether these data can be used to derive extreme wave heights - in particular the 50-year return value of  $H_s$ .

The method of analysis is essentially that used in the Department of Energy's *Offshore Installations: Guidance on design, construction and certification*; 4th edition of 1990. The statistical distribution of  $H_s$  throughout the year is assumed to be Fisher-Tippett Type 1; and the two parameters of the distribution are estimated from the available data - from long time series of measurements at a few sites for the "Guidance Notes" and from three years of satellite measurements in  $2^\circ$  latitude x  $2^\circ$  longitude bins for the Geosat analysis. The latter analysis was carried out over the area  $48^\circ\text{N} - 66^\circ\text{N}$  and  $20^\circ\text{W} - 10^\circ\text{E}$ .

The results are encouraging, except that Geosat's radar had difficulty in obtaining data close to the shore. Elsewhere the map of the 50-year return value of  $H_s$  from the Geosat data appears to be in good agreement with that in the "Guidance Notes". Analysis at specific sites indicates that the Geosat values were on average about 0.4m lower than the "Guidance Notes"; some possible reasons for this are suggested.





## 1. INTRODUCTION

Radar altimeters mounted on satellites give estimates of significant wave height ( $H_s$ ) as the satellite orbits the globe. (Annex A gives the definition of significant wave height.) Satellites usually report one value of  $H_s$  every second which corresponds to a distance along track of 7 km, and is an estimate from a footprint on the sea surface of about 5 km diameter - depending upon wave height as explained in Section 3.1.

The spatial coverage depends upon the orbit repeat duration which determines the spacing between tracks. For most of its life, the US satellite Geosat was in a 17-day repeat orbit with a distance between tracks of about 160 km at the equator and 80 km at a latitude of 60 degrees. So there are relatively few transects through an area of say  $2^\circ$  latitude by  $2^\circ$  longitude during any one month. However, data from these transects from Geosat have been found, for example by Challenor et al (1990) and by Carter et al (1991) to give useful indications of monthly mean values of  $H_s$ . This report explores the possibility of estimating the 50-year return value of wave height from these data in the NE Atlantic including the North Sea. The report first discusses briefly the history of altimetry and how the altimeter measures wave height, and the validation and accuracy of  $H_s$  estimates from Geosat. It then describes the method of analysis used to determine 50-year return values, and presents the results together with a discussion of them, including comparisons with results given in the UK "Guidance Notes" (Department of Energy, 1990).

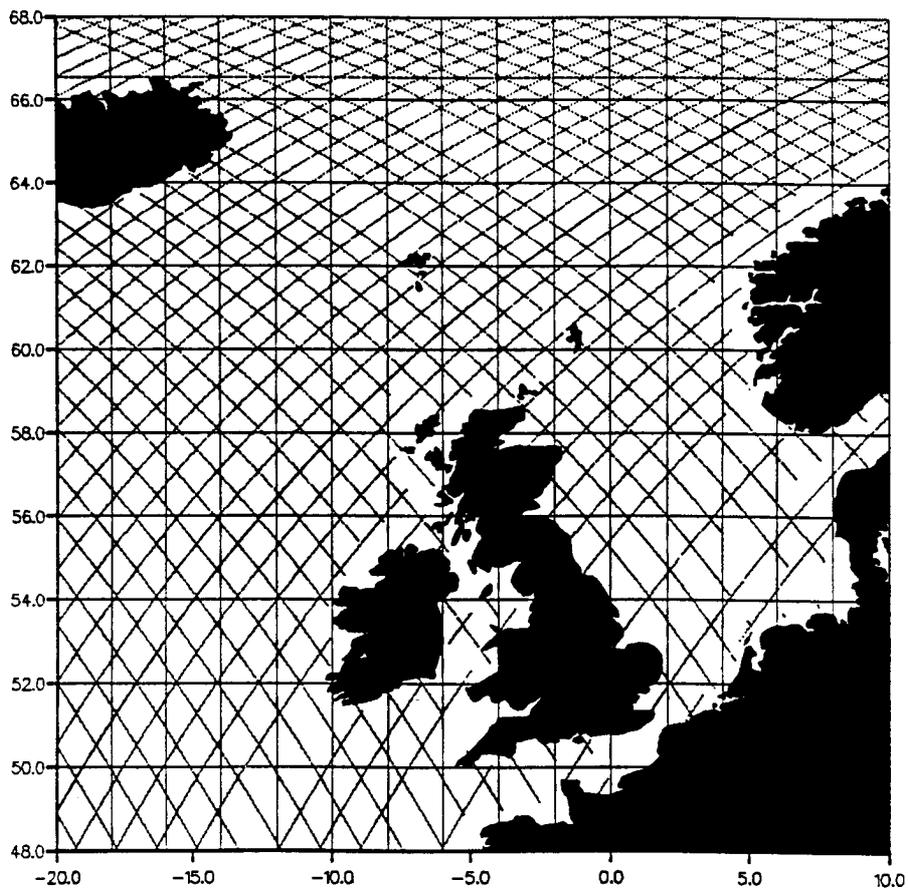
## 2. SATELLITE ALTIMETRY

the 'proof of concept' of a satellite radar altimeter was established by an instrument carried on SKYLAB in 1973. Seasat, which was only operational for three months in 1978, was the first satellite with an altimeter to give global coverage, from  $72^\circ\text{S}$  to  $72^\circ\text{N}$ . An earlier satellite, GEOS-3, carried an altimeter but it could not store the data on board so did not provide global coverage.

It was not until March 1985 that another satellite carrying an altimeter was launched: the US Navy's Geosat - with the same inclination as Seasat. As indicated by its name (GEOdetic SATellite), the satellite's primary purpose was to measure the marine gravity field with high precision. Because of the value of this information to the military, the first 18 months of observations were classified - although some data have recently been released. The Geosat exact repeat mission started on 8 November 1986 and continued until the satellite failed in January 1990; but it began to malfunction early in 1989, and there was a significant decline in global coverage from about March 1989.

Fig 1 shows the tracks of Geosat during a 17-day period. The missing portions illustrate gaps in the data coverage. Geosat had a particular problem in locking on to the sea surface as it came off the land, so ERS-1 and other satellites should give more data and better results in these regions, but the altimeter will not give measurements within 2 - 3 kilometre of the coast, where the footprint partly covers land which gives a spurious return.

Nevertheless, Geosat has provided, for the first time, several years of near-global coverage of wave data. This unique data set has given us the opportunity to carry out long term validation of the altimeter data and provides a useful foundation for a global climatology and for investigations of climate variability.



**Figure 1**

**Geosat tracks with successful observations of wave height  
over a 17 day period**

### 3. ESTIMATING $H_s$ FROM THE RADAR ALTIMETER

#### 3.1 HOW $H_s$ IS OBTAINED FROM THE ALTIMETER

The slope of the leading edge of the return pulse from the downward-looking satellite's altimeter provides an estimate of the sea surface variance,  $\sigma^2$ , and hence of significant wave height  $H_s$  ( $4\sigma$ ). Essentially, the higher the waves over the footprint of the radar pulse, the more spread-out the time of arrival of the return pulse; see Figure 2.

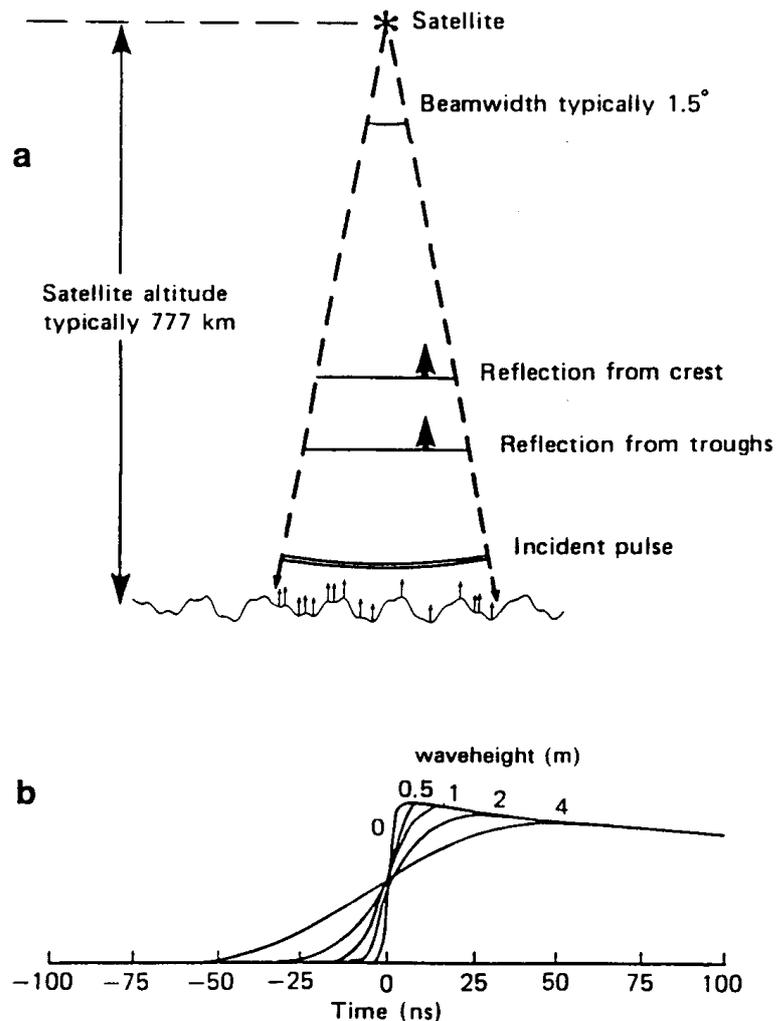


Figure 2

Estimating  $H_s$  from the shape of the return pulse from a vertical radar (adapted from Tucker, 1991)

- The rise time of the returned pulse is smeared by the different ranges to the reflecting facets.
- The average shape of the leading edge of the returned pulse is shown for various values of  $H_s$ .

An altimeter beam width of  $1.5^\circ$  at a height of 777 km, as in Fig. 2, would have a footprint on the sea surface of about 20 km diameter. However,  $H_s$  is estimated only from the portion of the sea surface close to the point vertically below the satellite which contributes to the leading edge of the returned pulse; returns from further away from nadir, which form the trailing edge of the returned pulse, are not taken into account. So the effective footprint is less than 20 km. The higher the waves, the more spread out is the leading edge and hence the larger the effective footprint. Further details are given by Chelton, Walsh and MacArthur (1989). They include a table for the effective footprint diameter for various values of  $H_s$ . The results depend upon the height of the satellite, and for 800 km the table shows 2.9 km for  $H_s=1\text{m}$ , and 7.7 km for  $H_s=10\text{m}$ . Diameters would be 3% less for a satellite at 777 km.

The shape of the return pulse from a radar which is directed vertically downwards - or nearly so - is determined by specular reflection and depends upon the statistics of the reflecting surface. The physics of the process are well understood. The satellite processor assumes that the sea surface statistics are linear and Gaussian, but even if assumed to be slightly non-linear then it makes very little difference to the estimate of  $H_s$ , Srokosz 1986 and 1990. (The theory does not account for highly non-linear behaviour, such as breaking waves.) So the derivation of  $H_s$  is based upon sound physical principle, no empirical factor is involved, and estimates of  $H_s$  from satellite altimeters are widely considered to be satisfactory. However, there is some evidence that errors can be introduced by the data processing, and Geosat appears to have under-estimated  $H_s$  by about 13%. See Section 3.3.1 below.

The only problem expected from the theory is that individual returns from the sea surface will be severely contaminated by noise. To reduce the effect of this, the pulses, which are transmitted at 1000 or 2000 Hz, are averaged to give one estimate of  $H_s$  each 0.1 s, and ten of these 0.1 s values are averaged again to give the 1 s values which are distributed as the Geophysical Data Records.

The standard deviation of the ten 0.1 Hz values comprising each 1 s value is also given in the Geophysical Data Record, and provides a useful check on the validity of the  $H_s$  value.

### **3.2 PRECISION OF ESTIMATES OF $H_s$**

The standard deviation (sd) of the ten 0.1 Hz values also provides a check on the precision of the estimates of  $H_s$  from the altimeter. An inspection of the Geosat data from the open North Atlantic indicates values of sd from less than 0.1 m for low  $H_s$ , rising to about 0.1 when  $H_s$  is 2.5 m and to about 0.2 when it is around 6 - 7 m. The standard error in the estimate of the mean of  $N$  independent values is given by  $sd/\sqrt{N}$ ; but the processing on board the satellite introduces a degree of smoothing into successive estimates of  $H_s$ , so they are not independent. The precise effect of this smoothing on the correlation structure of the  $H_s$  values has not been determined; but a rough measure of the standard error of the 1 s means from ten 0.1 s values is probably around  $sd/\sqrt{5}$  - rather than  $sd/\sqrt{10}$  assuming independence. This would suggest a standard error in the Geosat altimeter 1 s estimates of roughly 1% - 2% of  $H_s$ .

The estimates of  $H_s$  are also affected by sampling variability; two measures of  $H_s$  from a statistically stationary sea surface at different times or locations would not be identical. The size of this variability is determined by the wave spectrum and the area of duration over which the measurement is made. For an estimate of  $H_s$  from a 17 minute record, Carter and Tucker (1986) give the standard error from sampling variability as 4% - they assume a generalised Pierson-Moskowitz spectrum with a peak period of 7 s. For a peak period of 9 s, it is 4.5%.

Challenor (1983) obtains an approximate relationship between spatial estimates and those from time series using the group velocity of the wave and the dispersion relationship between wave period and wave length. For a wave period of 9 s, he shows that 7 km = 15 minutes. (In fact, it is more precisely 7 km = 17 minutes.)

Therefore the sampling variability of estimates of  $H_s$  from the satellite altimeter 1 s values (7 km apart) - and significantly greater than the instrument precision of the altimeter of 1% - 2%.

However, precision is only a measure of repeatability of the instrument (limited by noise and by sampling variability) and is not the same as absolute accuracy.

### **3.3 THE ACCURACY OF $H_s$ FROM GEOSAT**

#### **3.3.1 Comparisons of individual altimeter values with buoy data**

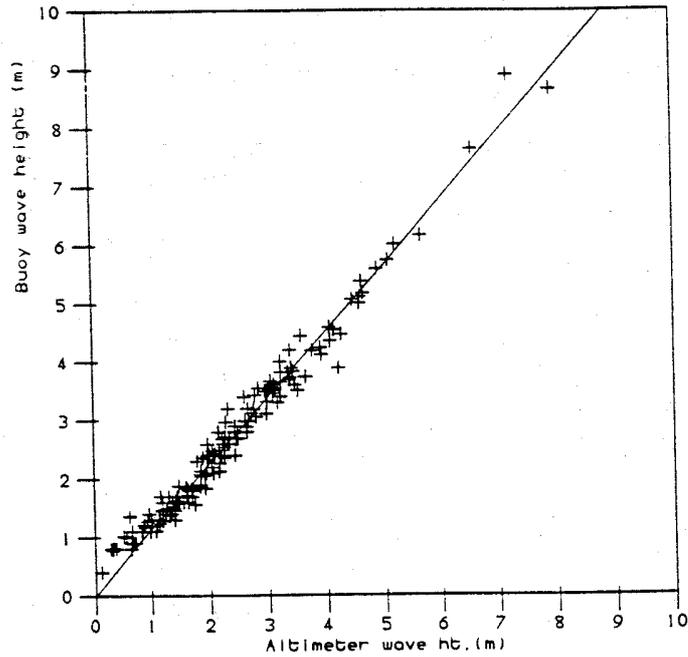
The accuracy originally specified for Geosat's estimates of significant wave height,  $H_s$ , was 0.5 m rms or 10% of  $H_s$ , whichever is higher (the same as that for ERS-1). Dobson, et al., (1987) validated Geosat's estimates of  $H_s$  against values from buoys in NOAA's National Data Buoy Center network and concluded that the altimeter was performing within its measurement goal, but that on average the geosat estimate was lower than the buoy's value by 0.4m. Glazman and Pilorz (1990) report a similar bias of 0.4 m from a comparison of a larger set of buoy data and Geosat data from the exact repeat mission.

Glazman and Pilorz derived a sub-set of these data by averaging the Geosat values obtained within  $1/4^\circ$  of the buoy, and a detailed analysis of this subset which contains data from the vicinity of 13 buoys is reported by Carter, Challenor & Srokosz (1992). A linear regression through the origin of the buoy  $H_s$  against the Geosat altimeter  $H_s$ , shown in Fig. 3 taken from Carter et al. (1992), gives

$$H_s \text{ buoy} = 1.130 H_s \text{ alt} \quad (1)$$

where the standard error of the slope is 0.007; indicating that the buoy  $H_s$  values are on average about 13% greater than those estimated from the Geosat altimeter. (The reduction in deviance from including an intercept in the regression was insignificant.)

Some of this discrepancy appears to come from the procedure used on board the satellite to fit the return waveform. Hayne and Hancock (1990) compare the on-board values of  $H_s$  with those obtained by them using an improved waveform fitting procedure, and derive an empirical correction factor. Applying this factor to the Glazman and Pilorz Geosat  $H_s$  values, and comparing the corrected values with the buoy data, Carter et al (1992) find that the excess of the buoy data over the Geosat values is reduced by about half, from 13% to 6%.



**Figure 3**

**Comparison of NOAA buoy and Geosat altimeter estimates of significant wave height, from Carter et al (1992)**

*The regression through zero has a slope of 1.13*

### **3.3.2 Comparison with monthly means from buoy data**

Estimates of the global climate of wave height for each month of the year can be obtained from the altimeter Geophysical Data Records, such as the Geosat data, by averaging values in  $2^\circ$  latitude x  $2^\circ$  longitude bins. These values can then be compared with monthly mean values from buoy data.

There is no obvious choice of bin size. However, here the data are analysed in the same bin size -  $2^\circ$  x  $2^\circ$  - as used by Challenor, et al. (1990) and by Carter, et al., (1991) for their analyses of the Geosat data. Tournadre & Ezraty (1990) give some support for a minimum of  $2^\circ$ . They analyse  $H_s$  values from Geosat in the vicinity of the Frigg Field in the open North Sea (about  $60^\circ\text{N}$   $2^\circ\text{E}$ ) and decide that an area of 50 km radius is too small to determine statistics of the distribution of  $H_s$  because of under-sampling, but that a radius of 100 km ( $1^\circ$ ) is satisfactory. The area of the bin reduces with increasing latitude, but this is more than compensated for by the corresponding convergence of the satellite tracks. The smallest number of transects of a  $2^\circ$  x  $2^\circ$  bin during a month is at the equator, where - for the 17-day repeat mission - the tracks are separated by 160 km, or 1.4 degrees, and give a minimum of 4 transects a month (up-crossings and down-crossings in 34 days). So, at least in the tropics, some smoothing would probably be desirable, using for example the Gaussian filter applied by Challenor, et al (1990). (This smooths over  $3 \times 3$  ins with weights 1:2:1, 2:4:2, 1:2:1.)

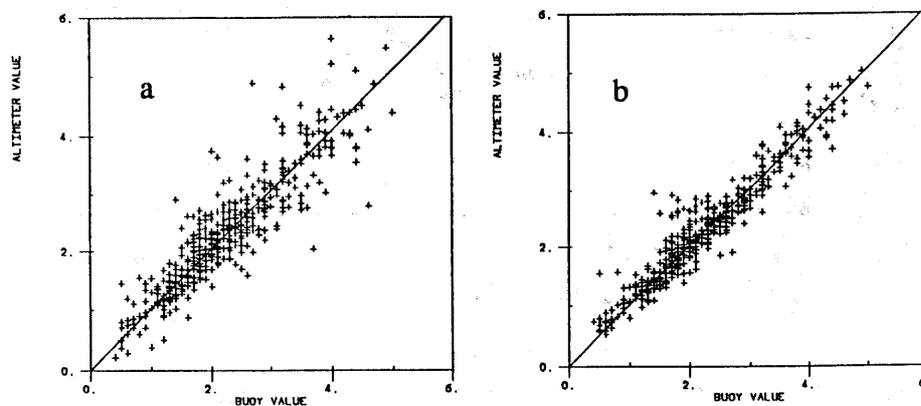
The average value of  $H_s$  in a  $2^\circ$  x  $2^\circ$  bin can be obtained by taking the median of the 1 Hz value of  $H_s$  along each transect of the bin and then calculating the average of these transect medians.

The median value is recommended by Carter (1990) because it is more robust against outliers than the mean. He finds no evidence of skewness in the distribution of  $H_s$  values across a  $2^\circ \times 2^\circ$  bin, with an average for the difference between the median and the mean which is not significantly different from zero.

The US Department of Commerce publish quarterly the *Mariners Weather Log*. This contains monthly mean values of wave height obtained from (generally) hourly measurement from the NOAA's National Data Buoy Center's network. Carter (1992) extracted data for 20 buoys, from October 1985 to December 1988, and compared these with the corresponding estimates of monthly means from the Geosat data.

Fig. 4 a & b show the regressions of about 400 monthly means of  $H_s$  from Geosat multiplied by 1.13 (from equation 1) against the values from the buoys. In 'a' the unsmoothed Geosat values are used and in 'b' those from applying the Gaussian smoothing function once. The regression slopes are not significantly different from 1.0. The correlation improves considerably if the smoothed Geosat values are used, with the correlation coefficient increasing from 0.90 to 0.96. This is not a completely independent check of equation 1, since data from some of the buoys were used to establish that equation. However, a comparison, reported by Carter (in press), of monthly means from Geosat and from six buoys not used in its derivation does support equation 1.

The differences between the estimates of monthly mean values from altimeter data and from buoy data could be due to several reasons. Spatial variation across the  $2^\circ \times 2^\circ$  bin could occur, especially around the buoys closer to the shore, such as in the NW Atlantic. However, probably the largest source of the differences is the few transects used to calculate the altimeter values. This raises the question of how many observations are required during a month to give a useful estimate of the monthly mean.



**Figure 4**

**Monthly mean  $H_s$  from Geosat compared with buoy data**

**a: Unsmoothed Geosat values**

**b: Spatially smoothed**

### 3.3.3 How many observations are required to estimate a monthly mean?

Fig. 5 (from Carter, 1992) shows the residual rms and correlation coefficient from regressing altimeter monthly unsmoothed  $2^\circ \times 2^\circ$  bin means on monthly buoy values as a function of the number of Geosat transects of the bin (each transect giving one value towards the altimeter monthly mean). Ignoring the result for 9 transects calculated from only 6 events, the correlation coefficient is seen to rise with the number of transects  $N_t$  until  $N_t = 5$ , it then remains nearly constant; the residual rms is around 0.5 m for  $N_t < 5$  and roughly constant at 0.35 for higher  $N_t$ .

This suggests that 5 transects of a  $2^\circ \times 2^\circ$  bin give about as much information as can be obtained from an altimeter, and that further transects do not appear to improve the estimate of the monthly mean; but to obtain a good estimate of the climatic monthly mean would require more data from other years to allow for inter-annual variability. Moreover, these results are from an analysis of data from open ocean buoys and should not be applied to coastal situations where considerable spatial variability is to be expected within a  $2^\circ \times 2^\circ$  bin. Variances of the buoy data are not reported, but we can expect that estimates of the standard deviation of wave height would require more observations to obtain the same percentage accuracy.

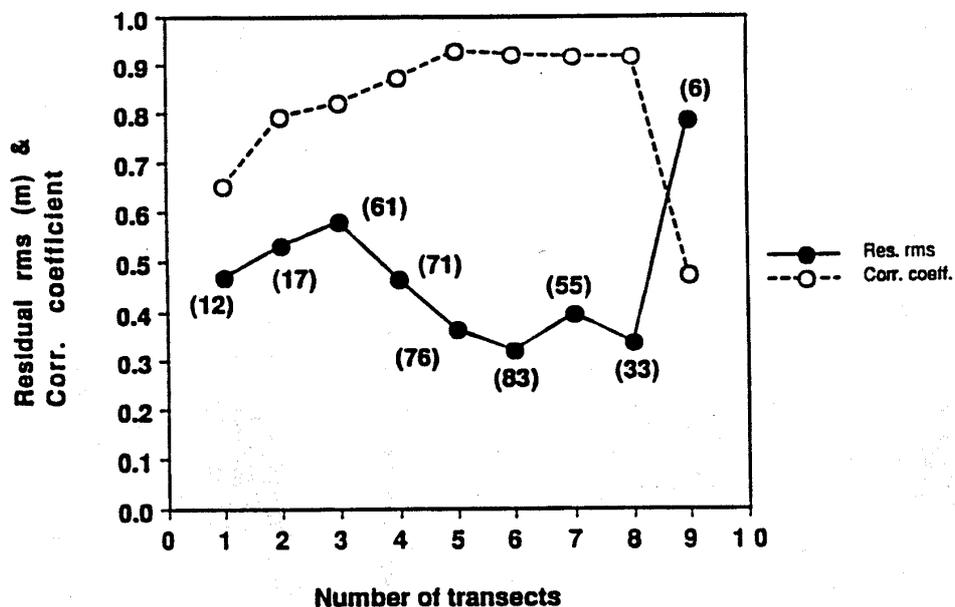


Figure 5

Results of regression of monthly mean Geosat  $H_s$  on buoy data as a function of the number of transects used to calculate the Geosat values

*The number of pairs in each regression is shown in brackets*

#### 4. ESTIMATION OF 50-YEAR RETURN VALUES OF $H_s$

The 50-year return value is defined as that value which is exceeded on average once in 50 years. So estimating the 50-year return value of  $H_s$  ( $H_{s50}$ ) requires some assumptions on the statistical distribution of  $H_s$  and on the time interval between measurements. Various methods have been used over the years and are summarised in Carter et al (1986). Those which involve the analysis of maxima, such as annual maxima, are to be preferred in that they require the fewest assumptions. However, this approach is clearly not possible with the infrequent measurements within a small area from the satellite altimeter. The method used here is essentially that employed to derive the extreme design wave height in the UK "Guidance Notes":  $H_s$  throughout the year is assumed to have a Fisher-Tippett Type 1 (FT-1) distribution; and the 50-year value is computed assuming measurements are at 3-hourly intervals. In the "Guidance Notes", the distribution was fitted using the method of moments applied to the histogram of wave heights with 0.5 m bins -because of the large amount of data available, some of them only in this form. But there are only a maximum of a few hundred observations available in a  $2^\circ \times 2^\circ$  bin from Geosat, so the methods of moments and of maximum likelihood using individual transect values have been used.

The FT-1 distribution is given by

$$P = \text{Prob}(X < x) = \exp \{-\exp[-(x-a)/\beta]\} \quad (2)$$

so

$$x = a + \beta \{-\log_e[-\log_e P]\} \quad (3)$$

Assuming 3-hourly intervals between observations, so there are  $50 \times 2922$  observations in 50 years, then  $H_{s50}$  is given from equation 3 with  $P = 1 - 1/(50 \times 2922)$ . That is:

$$H_{s50} = a + 11.89 \beta \quad (4)$$

The mean and variance of the FT-1 are given by

$$\left. \begin{aligned} \text{mean} = m &= a + \gamma\beta \\ \text{variance} = s^2 &= \beta^2 \pi^2/6 \end{aligned} \right\} \quad (5)$$

where  $\gamma$  is Euler's constant, equal to 0.5772...

Given estimates of  $m$  and  $s^2$  from the data, moments estimates of  $a$  and  $\beta$  are obtained from (5). The maximum likelihood estimates cannot be derived algebraically but are readily obtained for a few hundred values using numerical iteration. See Johnson and Kotz (1970) for details. If  $N$ , the numbers of observations, is large then these maximum likelihood estimates are unbiased and normally distributed, and Johnson and Kotz give their variances and their covariance. From these we get that the standard error of the estimate of  $H_{s50}$  from 3-hourly values using the maximum likelihood estimates of  $a$  and  $\beta$  is given by:

$$\text{SE}(H_{s50\text{MLE}}) = 9.654 \beta/\sqrt{N} \quad (6)$$

The corresponding value using the moments estimates are (from Tiago de Oliveira, 1963):

$$\text{SE}(H_{s50\text{MOM}}) = 12.64 \beta/\sqrt{N} \quad (7)$$

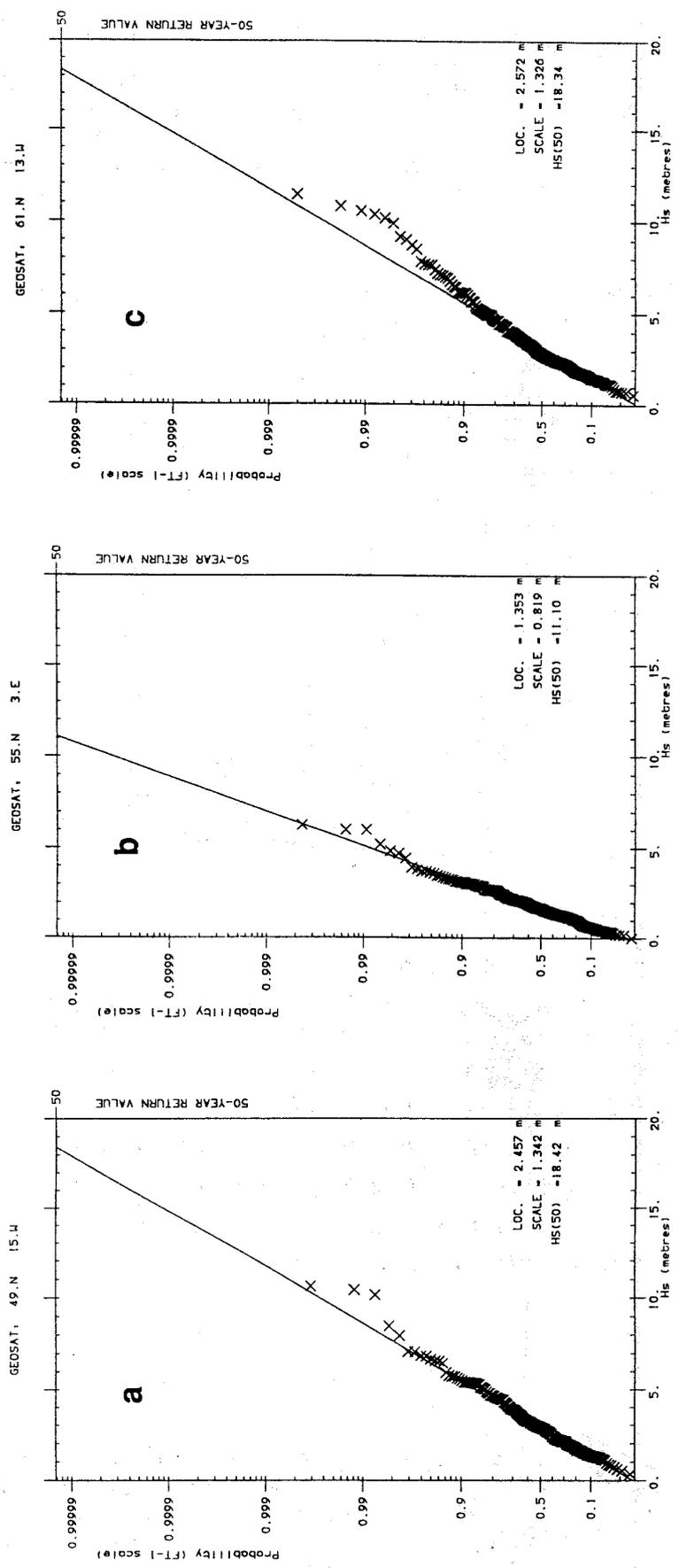
A method for testing the goodness-of-fit for the FT-1 with maximum likelihood estimates, which is used below, is described by Stephens (1977). He gives three statistics which can be calculated and tested against their specified distribution. The null hypothesis is that the data come from the FT-1 with the calculated values of  $a$  and  $\beta$ . If the value of one of the three statistics is then such that it has a probability of less than say 10% of being from its distribution, this indicates that the hypothesis is probably incorrect. It is assumed in this report that if 2 out of the 3 statistics 'pass' at 90% or less, then there is insufficient evidence for doubting the hypothesis.

## 5. APPLICATION TO GEOSAT DATA

For each transect of a  $2^\circ \times 2^\circ$  bin, an estimate of  $H_S$  was obtained by calculating the median of all the 1 second (7 km) values if there was a minimum of 5 such values which passed some simple checks - see Carter (1990) for a discussion of these checks. This procedure was carried out for all Geosat data from November 1986 to October 1989 and for all bins from  $48^\circ\text{N}$  to  $66^\circ\text{N}$ ,  $20^\circ\text{W}$  to  $10^\circ\text{E}$ . About 200 such transect values were obtained in each open ocean bin in the South of the area and about 300 values in the North, where the satellite tracks come closer together. Fig. 6 a-c shows plots of the data and the FT-1 fit obtained by maximum likelihood at three locations. (The axes are scaled so that a cumulative FT-1 distribution is a straight line. The formula recommended for plotting position by Gringorten (1963) has been used.).

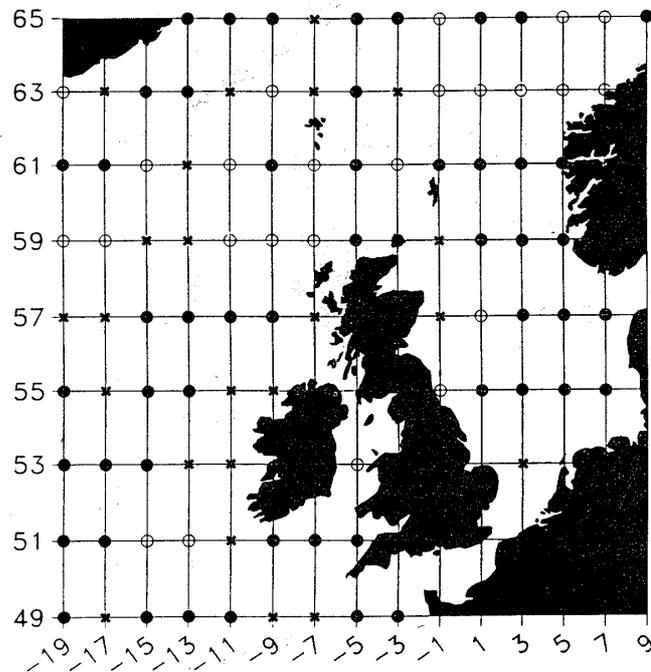
Fig. 6a for the bin centred at  $49^\circ\text{N}$   $15^\circ\text{W}$  and Fig. 6b at  $55^\circ\text{N}$   $3^\circ\text{E}$  both pass the Stephens goodness-of-fit test. But Fig. 6c at  $61^\circ\text{N}$   $13^\circ\text{W}$  fails (at the 99% level for all three statistics).

Fig. 7 shows the results of this test over the area. One would expect about 10% of the tests to fail; but the figure shows a rather higher percentage have done so. There are several reasons which might explain this (besides the possibility that the data do not come from the FT-1).



**Figure 6**  
**Cumulative distributions of Geosat  $H_s$  values in  $20^\circ \times 20^\circ$  bins**  
*The straight line is the FT-1 fitted by maximum likelihood.*

One reason could be that the test assumes the values are independent and identically distributed. The shortest time between successive transects is about 15 hours so the assumption of independence is not strictly true, but the correlation between the data must be small. Because of the large differences in wave heights over the NE Atlantic between summer and winter, the data are clearly not identically distributed, but the magnitude of the effect of this upon the test is not known.

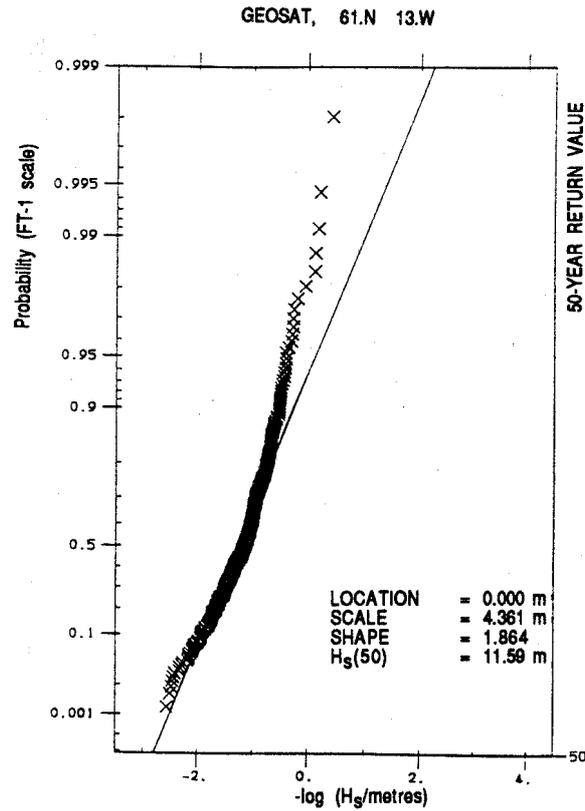


**Figure 7**

**Results of the test for goodness of fit of an FT-1 distribution to Geosat  $H_s$  values (fitted using maximum likelihood).**

- Key**
- *No evidence that the data are not form an FTR-1.*
  - *All data unlikely to be from an FT-1, but no evidence that selected data, with equal number of observations for each of the 12 months are not from an FT-1.*
  - × *Unlikely to be from an FT-1.*

Another reason is that the measurement of low wave heights (less than about 1 m) by Geosat appears questionable. Fig. 8 shows a plot of  $-(\log_e H_s)$  on FT-1 scales of data at  $61^\circ\text{N } 13^\circ\text{W}$ . The discontinuity around  $H_s = 1 \text{ m}$  ( $\log_e H_s - 0$ ) is evident. (Note that if the data were from a two-parameter Weibull, then this plot would be a straight line.)



**Figure 8**

**Cumulative distributions of log(H<sub>s</sub>) values in 2° x 2° bin centred on 61°N 13°W**

A third reason is that the data are not uniformly spread throughout the year. In particular, there are often significantly fewer data from the autumn (September, October and November) than from the other seasons. (This arises partly because Geosat was being manoeuvred into its 3-day repeat in early November 1986 so no data were obtained until 8 November, and partly because the altimeter was malfunctioning intermittently during September and October 1989, towards the end of its life.) Examples, from the sites in Fig. 6 are given in Table 1.

**Table 1**

**Total number of observations in 2° x 2° bins by season**

Position	Spring	Summer	Autumn	Winter
49°N 15°W	54	48	33	55
55°N 3°E	60	54	54	72
61°N 13°W	94	73	43	75

*(Spring = March-May, Summer = June-August, Autumn = Sept-October, Winter = December-February)*

This third reason was checked by reworking the analysis, but if NI was the minimum number of observations in any one calendar month, then only the first NI values from each other month were used. The significance of the fit of these results are indicated in Fig. 7 and show an increase in the number of 'passes' but this must be partly due to the fewer observations being fitted.

Because of the wide range in the number of observations in different seasons, it was decided to use the method of moments but to use weighted means and variances so that each month contributed equally. If  $p_i$  and  $q_i$ ,  $i = 1, 12$  are the 12 monthly sums and sum of squares of  $n_i$  values respectively, then the weighted mean and variance are given by:

$$\begin{aligned} \text{mean} &= (\sum p_i/n_i)/12 \\ \text{and} & \\ \text{variance} &= SS - (\text{mean})^2 \end{aligned} \quad (8)$$

where  $SS = (\sum q_i/n_i)/12$

Note that if there are the same number of observations in each month then the above expression for variance gives a value which is  $N/(N-1)$  times that from the usual unweighted definition, where  $N$  is the total number of observations throughout the years; but with  $N$  around 100, the difference is small.

A drawback to this method is that the analysis cannot be carried out if there are no observations in any calendar month. This was rarely the case but it did occur at some near-coastal sites, such as in the English Channel. Also, the confidence limit on  $H_{S50}$  given in equation 7 is no longer strictly applicable. However, the estimates obtained were found generally to be very close to those obtained from the unweighted method of moments, so equation 7 has been used, and is thought to give a good indication of the accuracy of  $H_{S50}$  and of the variation in accuracy over the NE Atlantic.

## 6. RESULTS

Estimates of  $H_{S50}$  from this analysis are shown in Fig. 9, which should be compared with Fig. 10 from the "Guidance Notes". Figs. 11-13 show the standard error of the 50-year return value and the values of the location parameter  $\alpha$ , and the scale parameter  $\beta$ . These plots have not been smoothed in any way and the contours are from linear interpolation between grid points which are the centres of the  $2^\circ \times 2^\circ$  bins; but the 'correction factor' of 1.13 has been applied (see Section 3.3.1 above). The values for the standard error of the return value reflect the number of observations used in the analysis, and show in particular the poor quality of the results in coastal regions, where Geosat experienced particular difficulties.

The results illustrated in Fig. 4 a & b suggest that estimates of  $\alpha$ ,  $\beta$  and  $H_{S50}$  in the open ocean could be improved by smoothing.

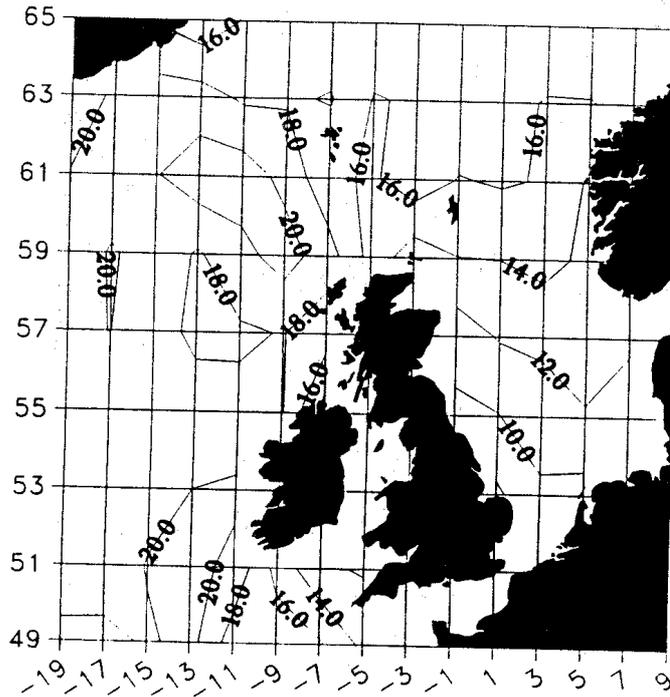


Figure 9

Estimate of the 50-year return value of  $H_s$  (m) from Geosat data from November 1986 to October 1989.

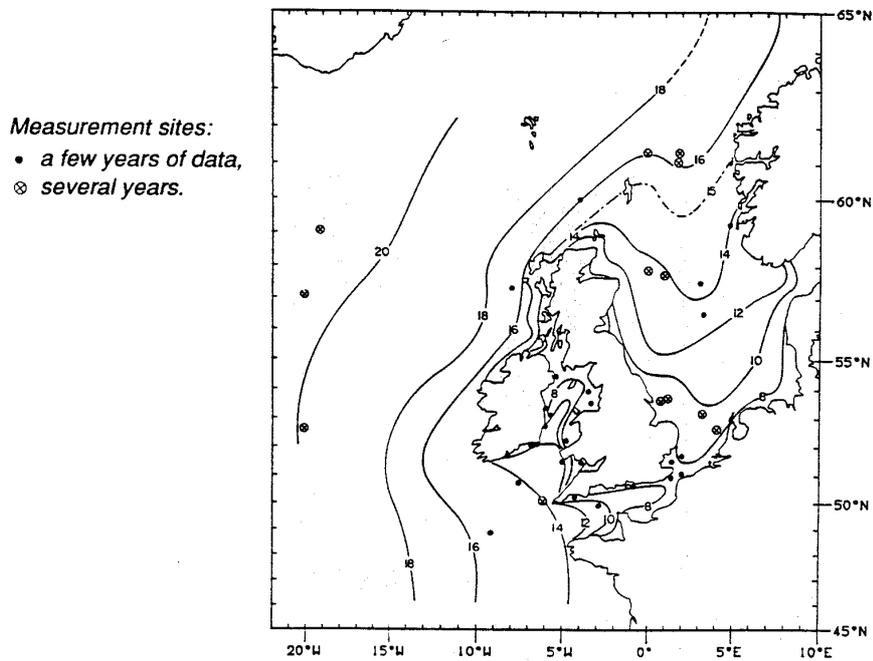


Figure 10

Estimate of the 50-year return value of  $H_s$  from the "Guidance Notes" (Dept. of Energy, 1990)

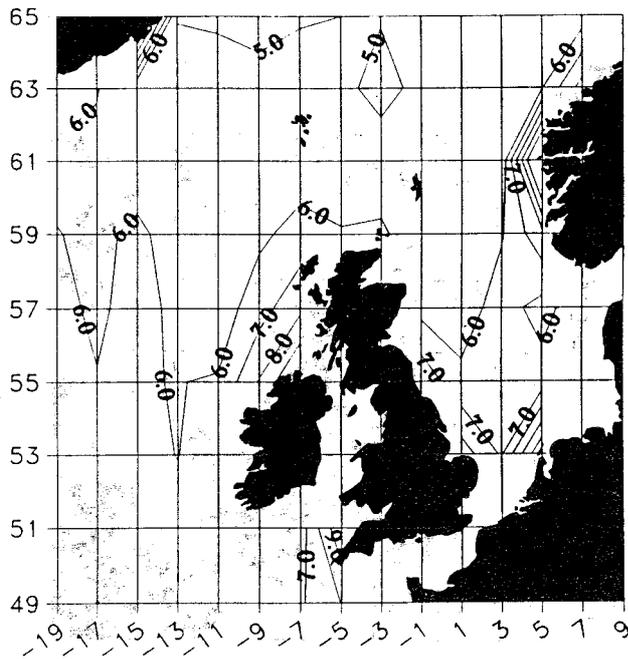


Figure 11

Estimate of standard error of  $H_{S50}$  (%) from Geosat data.

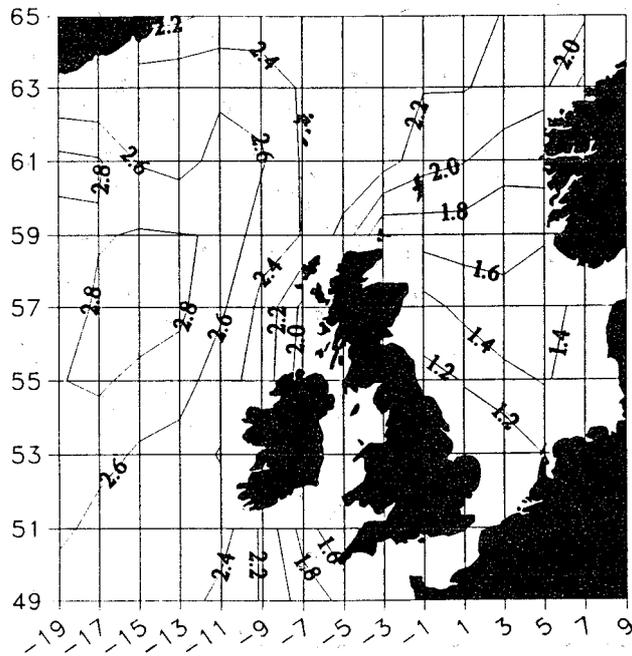
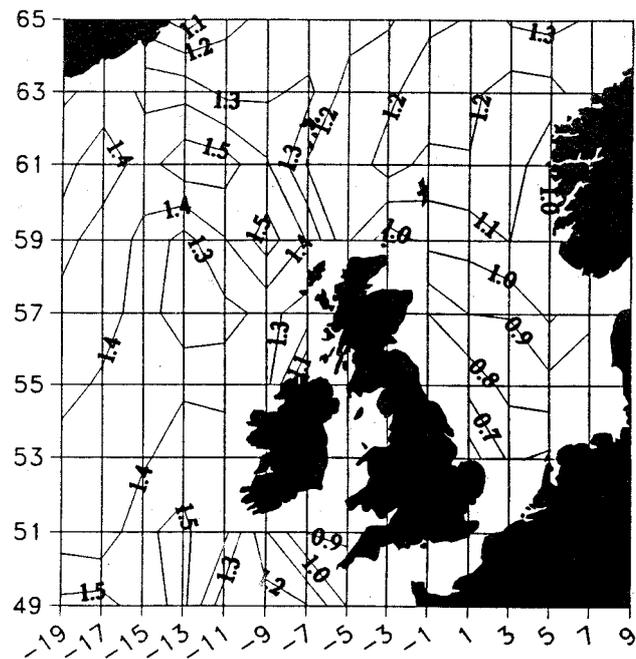


Figure 12

Estimate of the location parameter  $\alpha$  (m) of an FT-1 fitted to Geosat  $H_S$  data.



**Figure 13**

**Estimate of the scale parameter  $\beta$  (m) of an FT-1 fitted to Geosat  $H_s$  data.**

Value of  $a$ ,  $\beta$   $H_{S50}$  from some locations used in the construction of the "Guidance Notes" are shown in Table 2 together with the "Guidance Notes" values taken from Table 16 of Carter & Challenor (1990).

Comparison of Figs. 9 and 10 show generally very good agreement, except that the Geosat data give lower estimates of  $H_{S50}$  north of  $63^\circ\text{N}$  and higher estimates in an area centred on  $51^\circ\text{N}$   $13^\circ\text{W}$ . Analysis of data from the sites listed in Table 2 indicate that the Geosat data usually gives slightly lower values for  $H_{S50}$  than those in the "Guidance Notes". The average reduction is 0.4 m; but 0.2 m is accounted for by the large difference at O.W.S. India; and at four of the twelve sites analysed in Table 2, the "Guidance Notes" gives a lower estimate. The difference at most sites is well within 10% which is about 2 x standard error of the estimate. Estimates of  $a$  here are on average 0.17 m higher than in the "Guidance Notes", whilst estimates of  $\beta$  are on average 0.06 m lower. From equation 5, these figures give an increase in the mean value of  $H_s$  over that from the "Guidance Notes" of 0.14 m. This is probably not significant, but might be due to the increase in average wave heights in the Atlantic which has been observed in recent years. (Bacon & Carter, 1991; Barratt, 1991.) The slight decrease in  $\beta$ , which results in the decrease in  $H_{S50}$ , might be because with data from less than three years, the variability from inter-annual and decadal variations are not fully included in this estimate of variance.

Tournadre & Ezraty (1990) analyse Geosat wave data from around the Frigg Field in the northern North Sea at about  $59.9^\circ\text{N}$   $2.0^\circ\text{E}$ . They analyse all data from the first 2 years of Geosat in 17-day orbit which fell within a radius of 50, 100, 200 and 300 km of Frigg, averaging the Geosat data over 10 seconds (70 km). They fit an FT01 by least squares and estimate  $H_{S50}$  using equation 4 - but they do not increase the  $H_s$  values, as in this report, by 13%. Their results differ little for radii of 100 - 300 km. For radii of 100 and 200 km they obtain  $H_{S50} = 14.9$  m; increasing this values by 13% gives  $H_{S50} = 16.8$  m compared with 15.2 m from the present analysis. This difference could well be caused by the

different method of fitting and the slightly different data set analysed. From all data within 300 km Tournadre & Ezraty get 13.8 m, increasing this by 13% gives 15.6 m.

**Table 2**

**Comparison of estimates from Geosat with those in the "Guidance Notes"**

Site	Position		Guidance Notes			Geosat estimates			Diff in H <sub>SS0</sub>
			<i>a</i>	<i>β</i>	H <sub>SS0</sub>	<i>a</i>	<i>β</i>	H <sub>SS0</sub>	
Brent B	61.1N	1.7	2.05	1.20	16.3	2.05	1.18	16.1	0.2
Buchan	57.9N	0.0	1.47	1.03	13.7	1.55	0.94	12.7	1.0
Ekofisk	56.5N	3.2E	1.29	0.98	12.9	1.52	0.89	12.1	0.8
Fitzroy	60.0N	4.0W	1.94	1.21	16.3	2.11	1.11	15.3	1.0
Forties	57.8N	1.0E	1.57	1.01	13.6	1.56	0.96	13.0	0.6
Haltenbank	65.1N	7.6E	1.85	1.20	16.1	2.00	1.24	16.7	-0.6
Ijmuiden	52.6N	4.1E	0.88	0.61	8.1	1.13	0.69	9.3	-1.2
India	59.0N	19.0W	2.22	1.61	21.4	2.73	1.35	18.8	2.6
Juliett	52.5N	20.0W	2.45	1.56	19.9	2.80	1.36	19.0	0.9
Lima	57.0N	20.0W	2.58	1.50	20.4	2.65	1.44	19.8	0.6
S. Stones	50.1N	6.1W	1.37	0.98	13.0	1.68	1.00	13.6	-0.6
S. Uist Deep	57.3N	7.9W	2.24	1.26	17.2	2.21	1.30	17.7	-0.5

## 7. CONCLUSIONS

A method has been developed for estimating the 50-year return value of significant wave height,  $H_{S50}$ , from satellite radar altimeter measurements of  $H_s$ , by analysing the value in  $2^\circ$  latitude x  $2^\circ$  longitude bins, and fitting a Fisher Tippett Type 1 distribution to the data in each bin. The method has been applied to three years of Geosat data over the NE Atlantic and North Sea from  $48^\circ\text{N}$  to  $66^\circ\text{N}$ ,  $20^\circ\text{W}$  to  $10^\circ\text{E}$ . The method is essentially similar to that employed in the UK "Guidance Notes", but with this analysis there were at most about 300 values for analysis in any one bin, although most bins, except close to the coast, had sufficient data for analysis. The "Guidance Notes" was based upon relatively few sites each with some thousands of values recorded at 3-hourly intervals; but, these values are correlated so the increase in information is not as high as the ten-fold increase in data might suggest. Indeed, analysis from open ocean sites of Geosat and buoy data suggest that 5 observations per month might be sufficient to estimate that month's mean - although not to determine the climatic monthly mean or the variance of the population.

Using maximum likelihood to fit the Fisher-Tippett Type 1 distribution together with a test of goodness-of-fit due to Stephens (1977) shows that there was no reason to doubt the choice of this distribution over large areas of the NE Atlantic and North Sea, but there were some areas where its use was questionable. However, some of the assumptions required for the test are not met by these data - in particular they are not identically distributed or uniformly or evenly distributed throughout the year - so the inapplicability of the Fisher-Tippett Type 1 distribution in these parts is not resolved. However, the analysis does indicate how satellite data can be used to investigate the choice of statistical distribution for fitting  $H_s$  data.

A weighted method of moments has been used, to allow for the large discrepancies between the numbers of observations in the twelve calendar months. Surprisingly, using an unweighted method of moments gave very similar estimates for  $H_{S50}$ .

A map of  $H_{S50}$  over the NE Atlantic has been produced, (Fig. 9), in which values have been increased by 13% in accordance with reported comparisons of Geosat and US buoy data. These values of  $H_{S50}$  are estimated to have a standard error of about 5 - 6% in the open ocean, but the error increases markedly in coastal regions, because of the lack of data from the satellite. In many coastal regions there were insufficient data to carry out the weighted moments analysis. Geosat's altimeter had particular difficulty in measuring  $H_s$  close to land, so there should be an improvement in the data return from ERS-1 and other satellites, but there will always be a problem within about 2 km of the shore where the radar footprint picks up a return from the land.

These values of  $H_{S50}$  are generally in good agreement with those in the UK "Guidance Notes" (Fig. 10); but comparisons at some sites with wave measurements, analysed for the "Guidance Notes", suggest that estimates of  $H_{S50}$  from the Geosat data are on average slightly lower than those in the "Guidance Notes". This is possibly because the Geosat data covers a relatively short period, and so does not allow for trends and long-term variabilities in the wave climate. More years of data are required in order to investigate properly the inter-annual and decadal variability in wave height and the effect upon estimates of  $H_{S50}$ .

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## ANNEX A

### DEFINITION OF SIGNIFICANT WAVE HEIGHT

Significant wave height,  $H_s$ , is a measure of the general sea state, an 'average' value of a prevailing wave height. It was originally defined, about fifty years ago, as the mean height of the one-third highest waves, and was thought to give about the same value as an experienced seaman's estimate by eye of wave height. With the development of instruments giving time series of sea surface elevation,  $H_s$  was redefined in terms of the variance of the elevation as:

$$H_s = 4\sqrt{\sigma^2}$$

where  $\sigma^2$  is the surface elevation variance. The '4' was introduced so that, for a narrow-band sea, the old and new definitions have the same value. It is sometimes expressed in terms of the spectrum of the time series:

$$H_s = 4\sqrt{m_0}$$

where  $m_n$  is the  $n^{\text{th}}$  moment of the variance spectrum  $S(f)$  in terms of the frequency  $f$ , given by

$$m_n = \int f^n S(f) df$$

Note that  $m_0$  (or  $\sigma^2$ ) can be measured either over an area of the sea surface at any instant or over a period of time at a single position. Assuming stationarity in both space and time over the area and period of measurement, then the spatial and temporal definitions of  $m_0$  are numerically identical. So this definition of  $H_s$  is equally applicable to the radar altimeter which takes a 'snap-shot' of the sea surface over several  $\text{km}^2$ .



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