Review of probabilistic inspection analysis methods

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SUMMARY

Methods currently used in probabilistic inspection analysis for predicting fatigue crack growth in tubular joints in jacket structures are assessed especially based on recent research and development. Both SN and fracture mechanics approaches to represent the resistance are considered. Alternative procedures to determine the hot spot stress due to wave loading are examined, considering the following issues: hydrodynamic loading, joint flexibility, the pile-soil flexibility and stress concentration factor. Probabilistic measures of the uncertainty, associated with the models and parameters are assessed. Recent information about the probability of crack detection is examined. Also the reliability methodology used to determine the component and systems failure probability is evaluated. Recommendations for improved mechanics and probabilistic modelling are given.
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1. INTRODUCTION

The theory manual of PIA was completed on 15 January 1990, primarily based on information available up to May 1989.

The purpose of the present report is to review the PIA theory in view of recent developments, but also on a general basis.
2. RELIABILITY METHODOLOGY

Reliability measures may be expressed as component or system failure probabilities. The system failure is represented by sequences of component failures which may involve overload (collapse/fracture) and fatigue failures. While most design codes today only refer to component failure modes, it is increasingly recognised that system failure is a more rational feature to measure the quality vs. cost of a structure. For instance, a risk measure such as the likelihood of fatalities, depends more upon platform system failure than individual component failures.

However, system reliability analysis is very demanding in that it requires extensive structural and probabilistic modelling of the failure sequences in a structure. In special cases, however, such as for jacket structures, reasonable system reliability models have been established.

In this chapter component and system reliability analysis methodologies are briefly reviewed.

2.1 COMPONENT RELIABILITY

PIA addresses primarily component reliability analysis including updating of failure probability by means of FORM/SORM, using the PROBAN-2 version of 1989. In principle the state-of-the art within the mentioned area is still the same as it was in 1989-90.

In general this theory still provides acceptable estimates. However, it is noted that FORM/SORM represents an approximate method, and the probability estimates of intersections needed to calculate updated failure probabilities depends upon how the linearization of the failure function is carried out. Crude FORM is based on individual linearization of the failure and inspection events, while a more refined approach would be based on the “intersecting region” between the various limit states. In general, FORM/SORM needs to be validated by e.g. simulation methods that converge to the correct estimate by using large enough samples.

An interesting issue relating to updating is the possible effect of inspection of one joint on the reliability of other joints with correlated properties. This kind of reliability estimates does not formally require any other software than available in PIA, but would require handling of a large number of random variables. This issue is illustrated by Moan and Song (1998).

It is noted that failure probabilities may be referred to annual or service life values. If annual $P_s$'s are considered the contribution from fatigue (crack growth) will be the annual failure rate, which will vary (increase) with time.

Updating a component reliability is typically based on inspection results from non-destructive examinations (NDE) such as: no damage detection or some partial damage detection (with measurement of the size of the damage). Updating of the reliability based on a component failure, is straightforward.

In principle, a sequence of inspections (I) of joint no i, each with an outcome indicated by index k is envisaged. The total set of inspections of joint no i may be denoted by $I_i$:

$$I_i = \bigcap_{k,l} (I^l_{i,k})$$

where the indicator $I^l_{i,k}$ is a mathematical expression of the actual inspection result. It is noted that the actual event $I^l_{i,k}$ would generally depend upon previous events of the type $I^{l'}_{i,k'}$, for $l' > l$. 

2.2 SYSTEM RELIABILITY

2.2.1 General

Ultimate and fatigue design criteria in current codes are based on component failure modes (limit states) and commonly a linear global model of the structure to determine the load effects in the components. However, an approach, which is based on global (system) failure modes of the structure is desirable because significant consequences, e.g. fatalities, will primarily be caused by global failure. A suitable systems approach is also necessary to obtain the optimal balance between design and inspection plan since it is normally based on a certain damage tolerance, especially when the inspections rely on detecting flooded or failed members.

The prediction of ultimate behaviour of framework structures such as jackets is complex. However, recent developments of efficient methods which account for large deflections and plasticity and even premature fracture (e.g. Moan and Amdahl, 1989, Ueda and Rashed, 1991 and Stewart et al. 1993) facilitate systems reliability approaches. Significant progress has been made in reliability analysis of jackets over the last 5 years.

In the systems approach the relevant structure is assumed to be composed of different physical components (members, joints, piles, ..) which may each have different failure modes, e.g. different collapse, fracture or fatigue modes.

System failure is then expressed mathematically by load- and resistance parameters relating to all failure modes for all components, and the systems failure probability is calculated by the probabilistic properties of these parameters. Broadly speaking, this may be achieved by a failure mode (or survival mode) analysis, or direct-simulation methods (Karamchandani 1990, and Moses and Liu, 1992).

The failure mode analysis consists in

- identifying the sequences of component failures (system failure modes), $E_i$, considering members, joints and other components
- establishing a mathematical expression for the events of each sequence, $E_i$, based on structural mechanics. The event sequence (ES) No. 1 may involve failure of $n_i$ components such that

$$ES_1 : E_i^{(-1)} \cap E_i^{(-2)} \cap ... \cap E_i^{(-n_i-1)}$$

(1)

where $E_i^{(-)}$ is the event that component no. $j$ fails given that $j-1$ components already have failed. The numbers of these components are listed in the superindex ($\ldots$)
- establishing probabilistic measures for the random variables involved
- calculating the failure probability of the system.

In the following section, general system reliability analyses based on the failure mode approach is briefly reviewed to establish a basis for the simplified methods currently applied, e.g. in PIA (1990).

2.2.2 System failure modes

A statically determinate structure will fail when any one component fails. In this special case, the system failure probability is the probability of the union of failure of any one component and is equal to or greater than the probability of failure of a component. In particular it should be noted that the system failure probability will be much larger than the maximum component failure probability if there are many components with uncorrelated failure modes and failure probability which is equal to the maximum component failure probability.
Reviews of failure mode approaches for sequences of overload failures are given e.g. by Moses and Liu (1992) and Baker and Vrouwenvelder (1992) and Moan (1994).

Failure of redundant offshore structures may also be initiated by a fatigue failure or a fatigue induced fracture. If repair is not accomplished, a second fatigue or overload failure may occur. Even if the damage is detected, it may not be repaired until some time later, depending upon weather conditions. The increased stress in the remaining members will contribute to this second failure. While progressive overload failures are assumed to take place instantly (during the 18-20 sec period of a storm wave), the fatigue failures, occur at different points in time.

For each sequence (i) of fatigue failures, the failure functions for successive failures, in each mode may be established (Shetty, 1992 and Dalane 1993). This task is even more complex than for overload failure, because the successive failures depend upon the (random) time between different failure events (memory effect).

In reality the structural system may fail in different failure sequences involving overload and fatigue failure of components.

2.2.3 Uncertainty modelling

Modelling the uncertainties in loads and resistances in the components of the system is a crucial task. The system approach in addition requires an estimate of the uncertainty of the system model as well as the correlation between variables in the different failure functions that represent the system.

Correlation in strength variables is provided if joints belong to the same “batch”, since the between - batch variability is predominant. Correlation in stress due to common hydrodynamic factors depends upon location in the same vertical truss plane, and closeness in space. Correlation in stress concentration factors depends upon geometric similarity. Correlation will be 1.0 for joints with identical geometry. While the failure probability of a series system with n components may vary by a factor of n depending upon the correlation, the failure probability of a parallel system may vary even more - depending upon correlation and component characteristics.

Typically load uncertainties predominate in the calculation of the probability of wave overload of jackets, while load and resistance uncertainties are of the same order of magnitude in fatigue problems. Also, the correlation between component failure modes is less in fatigue. For fatigue failure modes the probability of first failure to the system failure probability will, hence, be large. Similarly, the small correlation between fatigue and overload failure events implies large systems effects for failure modes comprising fatigue events and a single overload failure.

2.2.4 Calculation of failure probability

Having established the limit state $g_{ij}(\cdot)$ and uncertainty measures for all random variables the failure probability may be calculated, using:

$$p_{SYS} = P_{\left[\bigcup_{i=1}^{N} \bigcap_{j=1}^{m} \{g_{ij}(\cdot) \leq 0\} \right]} \tag{2}$$
by FORM/SORM, bounding techniques or simulation methods. Due to the effort involved, it is important to apply some kind of technique to limit the number (N) of failure modes.

2.2.5 Simplified systems analysis

Fortunately, accurate estimates of the systems failure probability for jackets under extreme sea loading can be achieved with a very simple model, corresponding to a single systems failure mode, i.e. by referring both the load and resistance to a given load pattern and using the (overall) base shear as variable. This model has been validated for cases where the load uncertainties are dominant and the component forces are highly correlated (Wu and Moan 1989, and De et al, 1989).

This approach may be extended to include fatigue failure modes, using the basic overload case as the reference case. A variety of failure sequences should then in principle be considered. A first approximation to \( p_{FSYS} \), considering both overload and fatigue failure modes, may be achieved by,

\[
p_{FSYS} = P[FSYS] \approx P[FSYS(U)] + \sum_{j=1}^{n} P[F_j] P[FSYS(U)|F_j] + \sum_{j=1}^{n} \sum_{k=1}^{n} P[F_j \cap F_k] P[FSYS(U)|F_j \cap F_k] + \ldots
\]

where \( FSYS(U) \) is the overload system failure; \( F_j \) the fatigue failure of component \( j \) and \( F_j \cap F_k \), the fatigue failure of joint \( j \) and \( k \) before system overload failure. It is noted that the first term then accurately covers all pure overload failure sequences, i.e. all kinds of failure modes like \( FSYS[U_1, U_2, U_3, \ldots] \). The main approximation implied by Eq. (3) is to neglect sequences initiated by component overload, followed by component fatigue and overload failures or sequences initiated by two fatigue failures and followed by more fatigue or overload failures. It should be noted that the relative importance of system failure modes involving progressive fatigue failures increases with the service life of the structure for a given fatigue design life (Moan, 1994). Consideration of only a few of many alternative component failure sequences is obviously non-conservative. On the other hand neglecting the correlation between various sequences is conservative.

If the individual products in the sums in Eq (3) are approximately equal, the contribution from the last two sums in Eq (3) may be significant. In most cases, however, a few contributions will be dominant.

The first two terms of Eq (3) are easily interpreted and calculated. The failure probability, \( p_{FSYS} \) (Eq (3)) may most conveniently be referred to the service life or a period of a year. \( P[FSYS(U)|F_j] \) for the service life is estimated by the probability of a union of annual failure events. \( P[F_j] \) is then conservatively, computed as the probability of fatigue failure in the service life. \( P[FSYS(U)|F_j] \) in principle should be calculated as the probability of overload failure in the remaining time of the service life after fatigue failure. However, if it is assumed that inspection is carried out and that fatigue failures (of the complete member) can be reliably detected, e.g. by an annual visual inspection, the latter failure probability may be calculated as an annual overload probability.
To estimate $p_{FSYS}$ as an annual probability, the fatigue failure event $F_j$ could be split into mutually exclusive events $F_{jk}$, which denote fatigue failure of component $j$ in year $k$, given survival up to that time. The conditional probability of $FSYS(U)$ given $F_{jk}$ is then calculated as the probability of overload failure in the period from year $k$ to the end of the service life. Alternatively, if inspections with reliable detection of member failure is assumed, the probability of overload failure could be referred to all annual value, as mentioned above.

The third term of Eq (3) is relevant only for structures with redundancy beyond one member failure. The corresponding failure probability is more cumbersome to calculate. In reality fatigue failure $F_j$ and $F_k$ may occur at any time. The occurrence of one fatigue failure (or single component overload failure for that matter) increases the load level in other components and, hence, the likelihood of more component failure. The joint probability of two fatigue failures $F_j$ and $F_k$ over the service-life may, be approximately calculated by,

$$P[F_j \cap F_k] = P[F_j] \cdot P[F_k | F_j]$$  \hspace{1cm} (4)

The latter term may be conservatively calculated by assuming $F_j$ to occur in year one and hence increase the stress for joint $k$ correspondingly. A more realistic estimate is obtained by using a weighted average value of $P[F_k | F_j]$ to account for the fact that the likelihood of the event $F_k$ increases with time. In addition, the correlation between the events $F_k$ and $F_j$ should be accounted for. Based on knowledge about $P[F_j]$ of individual joints, and possibly $P[FSYS(U) | F_j \cap F_k]$ dominant failure modes may be selected.

Obviously, the correlation between failure of "identical" joints in the same vertical plane of the bay, is fairly high. This makes $P[F_k | F_j]$ relatively large for two such joints. Moreover, failure of two joints in the same bay leads to a large strength reduction, which increases the term $P[FSYS(U) | F_j \cap F_k]$. Such failure events may, therefore, contribute significantly to $p_{FSYS}$.

Eq. (3) provides a basis for comparing the contribution to the failure probability by systems overload failure and a fatigue-induced systems failure.

Moreover it is noted that PIA (1990) ranks inspection of the different joints ($j$) according to the magnitude of

$$p_j = P[F_j] \cdot P[FSYS(U) | F_j]$$  \hspace{1cm} (5)

2.2.6 Effect of inspection on the systems failure probability

Estimates of the effect of a general set of inspections of a structural system:

$I: \bigcap_{i,k,l} \{I_{ik_l}\}$

is very complex.
In general the systems failure probability, Eq. (2) may be updated by

$$p_{SYS} = P(F_{SYS} | I) = P\left[ \bigcup_{i=1}^{N} \bigcap_{j=1}^{n_i} \left( g_{i,j}^{(k)} \leq 0 \right) \right]$$

(6)

For a multi-component system and an inspection event tree with multiple branches, the computational efforts to calculate Eq. (6) are significant.

Simplifications may be introduced by considering different inspection methods/outcomes, e.g.
- use of visual inspection (VI) to detect significant member damage or failure
- use of NDE to detect cracks

VI to detect significant member damage would only affect ULS failure modes associated with damage induced by impacts or other accidental loads. There will be no effect on the reliability associated with wave-induced overloads, because subsequent overload failures in this case are highly correlated with the initial damage.

The effect of using NDE methods to detect (fairly small) crack damage (on $p_{SYS}$) could be assessed by assuming that the method used affects failure modes associated with crack growth.

The system failure probability may then be approximately updated based on Eq. (3) as follows:

$$p_{SYS,up} = P(F_{SYS} | I) \approx P(F_{SYS}) + \sum_{j=1}^{n} P(F_{j} | I) P(F_{SYS} | F_{j}) + \sum_{j=1}^{n} \sum_{k=1}^{n_{j}} P(F_{j} \cap F_{k} | I) \cdot P(F_{SYS} | F_{j} \cap F_{k}) + \ldots .$$

(7)

A further simplification may be to update the failure probability of each joint based on the inspection of that joint. This is conservative if no cracks are detected, but non-conservative otherwise. In this case updating of component failure probabilities $P(F_{j} \cap I_{j})$ is necessary and may be accomplished by well-established methods as described, e.g. in the PIA Theory Manual (1990). Such an approach was used by Moan et al. (1993) to demonstrate how fatigue design criteria could be calibrated to yield the same component failure probability using a given inspection/repair strategy and a relaxed design criterion, with no inspection and a given design criterion.

2.3 INSPECTION QUALITY

PIA is based on probability of detection curves which refer to crack depth, a, and are based on a fixed ratio of depth to length (a/2c) of 0.075. The data were obtained by the NDE Centre at University College on braces in a jacket. It would, of course, be an advantage to use both crack depth and length as parameters. However, such data are not always available for the relevant application (underwater inspection of welds in tubular joints). By referring to a crack depth with a fixed aspect ratio of the crack, uncertainty is introduced.

In a recent study (Moan et al., 1997) inferred the mean detectable crack depth to be 1.95 mm for a combined underwater MPI/EC inspection. This value corresponds fairly well to the average of MPI-UCL and Hocking-EC-UCL data given in PIA Theory Manual (1990). However, it should be noted that the results in Moan et al. (1997) refer to non-propagating cracks, with significantly varying aspect ratios. This would introduce extra scatter, since most inspection methods rely upon the crack length as the primary parameter.
Kam (1990) discusses NDE reliability based on results from 200 inspections and emphasized the difference in the quality of detection and sizing processes.

In the PIA Theory Manual (PIA, 1990) crack sizing with ultrasonic and ACPD(AC) potential drop methods is discussed. The error is estimated to be ± 10% of the crack depth or ± 2 mm. On this basis, the default value of sizing uncertainty is estimated to correspond to a COV of 20%. This means that the absolute measurement error is proportional to the mean depth. According to the discussion in PIA (1990) the error is partly determined by the total depth. However, for small crack depths it is the margin ± 2 mm that will be governing the sizing error. Hence, for small cracks, say less than 10 mm deep, a constant standard deviation is a better measure of the error than the COV.

Alternatively, the size of the crack may be estimated while remedying the crack by grinding in increments of 0.5 mm and checking after each grinding whether the crack is removed or not. In this case the sizing error will be influenced partly by the total crack depth (length) and partly an error, which is independent of the total depth, but rather the size of the increment (0.5 mm).
3. FATIGUE RESISTANCE

3.1 SN-APPROACH

SN-data for tubular and other welded joints have been revised since 1990.

Based on a significant offshore industry review in the period 1987 to 1990 the HSE revised their 1990 guidance in 1995 (HSE, 1995, HSE 1992). A summary of the revised HSE documents is given by Stacey and Sharp (1995). Also, the API RP2A (1993) recommendations are noted. Recently the ISO TC67/SC7/WG3 panel on joints/Fatigue Technical Core Group has also worked on harmonised guidance on fatigue of tubular joints (Bærheim et al., 1996).

Recent (HSE, 1995) fatigue guidance contains SN-curves and revision on the effect of
- thickness
- environment
- treatment of low and high stress ranges

Guidance on cast and ring-stiffened joints and fracture mechanics is included. The fracture mechanics approach refers to the BSI:PD6493 document in conjunction with the HSE (1995) document. Also the PD6493 document has undergone a revision recently (Stacey et al., 1996), and appears now as BS7910:1998.


In a recent study Lotsberg (1997) reassessed the available SN data for tubular joints and proposed curves for use in the new NORSOK standard. The resulting curves deviate somewhat from those given by HSE (1995).

3.2 FRACTURE MECHANICS APPROACH

3.2.1 Model

PIA is based on the Paris’ crack propagation law

\[
\frac{da}{dN} = \begin{cases} 
C (\Delta K)^m & \text{for } \Delta K > \Delta K_{th} \\
0 & \text{for } \Delta K \leq \Delta K_{th} 
\end{cases}
\]  

and a stress intensity factor range, \( \Delta K \)

\[
\Delta K = Y \cdot S \sqrt{\pi a} = (Y_m S_m + Y_b S_b) \sqrt{\pi a}
\]  

(8a)

with a one dimensional compliance function \( Y(a) \) based on constant aspect ratio \( a/c \). Sub-indices \( m \) and \( b \) refer to membrane and bending, respectively.

Propagation Law

The fatigue crack growth equations are still written in the form of Paris law, although many modifications have been suggested. These modifications are aimed to take into account the effects of stress ratio, load sequence, crack retardation and others, which were not considered in the original Paris-Erdogan formula. Probably the most important effect on the fatigue crack growth rate is that of crack closure taking place during part of the fatigue load cycle even if this cycle is entirely tensile. Regardless of the particular form of the fatigue crack propagation equation, it is always imperative to first determine the stress intensity factors (SIF) for the particular crack geometry under investigation.
The character of the stress in tubular joints is such that the plastic zone will be small compared to the crack dimension, hence, making linear elastic fracture mechanics applicable.

**Stress Intensity Factor**

The stress intensity factors can be evaluated by various fracture mechanics models. Several modelling approaches have been attempted in the past with varying degree of success. The main approaches include:

- finite element and other numerical models
- plate models, modified for shells
- weight function approach
- experimental methods

BS7910:1998 briefly describes relevant approaches.

While direct methods based on three-dimensional finite element methods are feasible, such methods are prohibitively expensive.

More commonly, stress intensity factors available for idealised components and crack geometries are used together with correction factors. PIA is based on a solution proposed by Newman and Raju (1981) for semi-elliptical surface cracks in flat plates. A conservative correction factor is introduced to account for the weld toe geometry.

The model uncertainty of this approach is represented by a bias of 1.0 and COV of 5%. The constant a/c ratio is randomised with a mean value of 0.2 and a typical COV of 15%.

This model uncertainty probably is on the low side. It is also noted that the uncertainty will vary with (a/T).

The use of a two-dimensional approach would provide more realistic information about the development of the crack length, which is a feature that influences the probability of crack detection.

The two-dimensional character of the crack could be pursued by using:

- a one-dimensional crack propagation law in conjunction with a crack length (i.e. aspect ratio) as a function of depth.
- a two-dimensional crack propagation law using a set of simultaneous differential equations for the depth as well as in-plate direction.

Measured data are given by Tweed and Freeman (1987). A parametric numerical study is published by Sigurdsson and Torhaug (1993). Numerical and experimental results are presented by Kam et al. (1995). The development of a/c versus a/t depends is sensitive to the initial a/c-ratio. Alternative models are indicated in Figure 1.
A large a/c is conservative.

While modified plate solutions provide a relevant basis for stress intensity factors in tubular joints for shallow cracks, the particular load shedding that occur in a shell structure needs to be considered for deeper cracks (Aaghaakouchak et al., 1989, Du and Hancock, 1989). The assumption is that as the crack grows, the cracked section will progressively lose the local bending stiffness and rotational constraints, and gradually becomes a hinge. Therefore, the tensile stress (direct force) component does not change while the bending stress (moment) component decreases as the crack grows (thus called “moment release” or “moment redistribution”). The excess loading would be transmitted through the uncracked part of the joint.

Two moment release models have been reported so far (Du and Hancock, 1989). The most popular one is a linear release proposed as a “limiting case”.

\[
\sigma_{b, \text{ for crack ends}} = \sigma_{b, \text{original}} \left(1 - \frac{a}{T}\right). \tag{9a}
\]

The moment release model needs to be used in conjunction with one of the plate or numerical solutions mentioned above. Studies combining the linear moment release with the Newman-Raju solution and other solutions have given promising results.

A modified model was proposed by Kam et al. (1995) for in-plane bending:

\[
\sigma_{b, \text{ for crack ends}} = \sigma_{b, \text{original}} \left(1 - \left(\frac{a}{T}\right)^{0.25}\right). \tag{9b}
\]

It is advised that the load shedding correction should be applied to both crack tip and ends. See also discussion by Maddox (1997).

**Aspect Ratio**

There is a significant uncertainty in the initial aspect ratio a/c and how the microcracks may develop. One issue is possible coalescence of microcracks to form a long crack (Vosikovsky et al., 1985, Snijder et al., 1988). This phenomenon may give rise to jumps in aspect ratio at the
early crack growth stage. But this effect is expected to be less than for plates due to the more significant non-uniform variation of stress in a tubular joint than in a flat plate.

According to Maddox (1997), there is no need to account for interaction between adjacent fatigue cracks until they actually coalesce.

The measure of \( a/c \) used in PIA for small \( a/T \) (say \( \leq 0.05 \)) implies probably a too small mean value and COV. Conolly and Dover (1987) indicate a mean aspect ratio of 0.3 and a COV of 0.5. Based on the data for observed initial cracks in the Hutton TLP, Kountouris and Baker (1989) suggest modelling the initial \( a/c \)-ratio as a log-normal variable with a mean value of 0.62 and a COV of 0.4. Since Kountouris and Baker analysed plated structures the data should be applied to tubular joints with some caution. For larger values of \( a/T \), the mean value is probably underestimated by using a constant \( a/c = 0.2 \), especially for \( a/T > 0.4 \) when the neglected effect of load-shedding becomes important. The \( a/c \) for through-thickness cracks, however, seem to be quite close, independent of their initial shape (Kam et al., 1995).

3.2.2 Crack propagation parameters

Fatigue crack propagation is seen to be an inherently random phenomenon and experimental results for Paris law constants often exhibit very high scatter. Virkler et al. (1979) have carried out carefully controlled tests on 68 replicable centre cracked specimens under constant amplitude loading. While various types of Markov and other random process models have been proposed to model the stochastic behaviour, it seems that a simple random model of the parameters \( C \) (and \( m \)) of the Paris’ equation yields a good representation. The nature of Paris law implies a high negative correlation (\( \approx -0.95 \)) between \( C \) and \( m \) and it is therefore satisfactory to fix \( m \) and introduce the uncertainty through \( C \). Some of the data collected for the crack growth parameters \( C \) and \( m \) are summarized in Table 1.

All data in Table 1 are based on data collected from several investigations. Several other sources, e.g. Johnston (1983), Besuner (1987) and Snijder (1987) exist. In reliability applications of fracture mechanics analysis in recent years the uncertainty in \( C \) is modelled with a COV of typically 0.25 (Shetty and Baker, 1990) to 0.55 (PIA). As the uncertainty measure chosen in reliability analysis seems to be made by non-specialists, limited support can be obtained on the uncertainty modelling of \( C \) from published applications.

King et al. (1996) recently reviewed fatigue crack growth rates for offshore steels in air and seawater for the forthcoming revision to PD6463: 1991, BS7910: 1998. By grouping the data according to the stress ratio \( R \) the scatter is reduced. The two-segmented approach used for corrosive environment further reduces the uncertainty for such conditions, compared to that used in PIA. While a single relation is applied for \( \Delta K \) above \( \Delta K_{th} \) for steels in air, a two-stage model is used for corrosive environment.
The small uncertainty in the Cortie and Garrett (1988) data is noted. Even if their data are from different sources, the uncertainty is much smaller than given in other sources. This information indicates that there may be a small in-batch variability, in relation to the total uncertainty. If the uncertainty in $C$ is modelled by

$$C = C_{wb} \cdot C_{bb},$$

(10)

where $C_{wb}$ and $C_{bb}$ represent the within - batch and between batch - variability. While $C_{wb}$ is independent within the batch, $C_{bb}$ is dependent for a given batch. If each factor is assumed to be log-normal, the correlation of $C$ between components in the same batch will be
Obviously, by using specific information obtained for the relevant steel in a given structure, the uncertainty will be significantly reduced.

In case of small defects and loading typical for marine structures, the threshold values are important. But on the other hand the use of threshold for small grades may be questionable. The value of $\Delta K_{th}$ is strongly dependent on microstructure, mean stress and environment (Beevers and Carlson, 1985). According to PD6493: 1991, no threshold value should be applied for ferritic steel in a corrosive environment. Fatigue threshold is relevant for carbon and carbon manganese steels in air and seawater environments. The considerations in PIA reflect this fact. Based on UEG data (1985), PIA proposes to represent $\Delta K_{th}$ with a mean value of 100 Nmm$^{-3/2}$ and a COV of 20% for large stress ratios, $R$ ($R > 0.8$). The dependence of $\Delta K_{th}$ on stress ratio $R$ can be described by a deterministic relationship, e.g. $\Delta K_{th} = 240 - 173 \cdot R$ (Nmm$^{-3/2}$) also mentioned in PIA. It may be that a slightly smaller COV (COV≈0.15) could be applied to represent the uncertainty in the whole range of $R$. The data reviewed by King et al. (1996) indicate a $\Delta K_{th} \sim 300 \cdot (1-R)$ and the same COV of about 0.15.

Thresholds for “long cracks” are given by Hobbacher (1997) and PD6493

$$\Delta K_{th} = 190 - 144 \cdot R \geq 62 \text{ Nmm}^{-3/2}$$
$$\Delta K_{th} = 170 - 214 \cdot R \geq 63 \text{ Nmm}^{-3/2}$$

It is known that “short” cracks behave differently. From a practical point of view, the actual initiation stage, or the microstructural short crack propagation stage, can be neglected. However, the mechanically short crack propagation stage should be considered from the very first cycle of loading (Verreman, 1997). The divergences between short and long crack growth rates are associated with initial transients of the crack opening level.

The LEFM theory can be used to predict the “propagation life” of welded joints beyond a certain “initiation” crack depth, a. For example, a Canadian Offshore Corrosion Fatigue Research Programme has demonstrated the ability of a multiple crack model (using a forcing function for $a/2c$ to account for coalescence) to predict the propagation lives of manually welded joints from $a_i = 0.5$ mm (Verreman, 1997). In general, $\Delta K_{th}$ of a long crack is greater than that of a short crack. Hertzberg et al. (1992) point out that one major reason for this discrepancy may be attributed to the different crack-closure levels at the same $\Delta K$ value found for long and short cracks. They show that if during a test the maximum $K$-level is maintained constant while the applied $\Delta K$ range is decreased, the upper limit $\Delta K_{th}$ for a short crack can be obtained. They also proposed that the $\Delta K_{th}$ value obtained by such a procedure be used for an assessment of fatigue crack growth.

Recently Hartt (1997) conducted experiments to characterize crack growth including thresholds, for several high strength steels in natural seawater for different levels of cathodic protection. Short and long cracks were considered.

### 3.2.3 Initial defect size

Crack-like defects are invariably present in welded joints and fatigue cracks often initiate from such defects. The weld defects are of several types and size and occurrence rates are found to be highly random being influenced by such factors as fabrication year, welding procedure, welding position, type of joint, etc. as well as control procedures. Very limited data have been
reported on weld defect size distribution typical of offshore construction and tubular joints. Only surface breaking defects are of concern here.

Based on a review of the limited data available, PIA recommends the use of Bokalrud and Karlsen (1982) data for the depth of weld toe undercuts in butt welded plates. An exponential distribution with a mean value equal to 0.11 mm and COV of 1.0 is fitted to these data. PIA does not mention that the frequency of occurrence rate of these defects is 16 cracks per m. The occurrence rate of cracks may be modelled as a Poisson process with the given occurrence rate.

A statistical analysis of a large amount of weld defect data obtained from the Conoco Hutton TLP structure was carried out by Kountouris and Baker (1989). Based on these studies, the initial defect depth has been modelled as a log-normal variable with a mean of 0.73 mm and a standard deviation of 0.78 mm. The defect rate, defined as the surface length of crack to the total length, was found to be between 0.08 and 1.36% for cruciform and toe butt welds and between 0.01 % and 0.27% for plate butt and fillet welds for surface-breaking cracks. It should be noted, however, that tubular joints may have a different distribution of defect sizes than those found in a TLP. This may be due to the difference in quality required and quality control implemented depending upon the importance of the joint.

Moan et al. (1997) analysed non-propagating cracks detected in about 30 North Sea jackets and found that initial crack size was well fitted by an exponential distribution with a mean value of 0.94 mm. It is noted that these cracks were observed in-situ using an inspection method, which is characterized by a mean detectable crack size of 1.95 mm. For this reason the mentioned initial crack size distribution corresponds to a mean occurrence rate of one per 3 tubular joints. By transforming these data to an occurrence of one crack per hot spot area in each joint, it was found that the mean crack size is 0.38 mm.

3.2.4 Final crack size

The final crack size, \( a_f \), needs to be modelled if the probability of reaching a truly final fracture is to be determined. This may for instance be the case for inspection planning based on the flooded member principle. The crack depth \( a_f \) depends upon the imposed stress level, which is a stochastic variable in itself; as well as the residual stress. Usually is it very difficult to measure the magnitude of residual stresses and their distribution at tubular joints and it is common practice to assume residual stresses of yield magnitude in tension. However, limited measurements, for example Porter-Goff et al. (1988), indicate that the magnitude of the residual stress can vary considerably, often reaching yield value at the surface but falling off rapidly in the through-thickness direction. Based on these findings, the residual stress at the surface is modelled at a log-normal variable with a mean value of 300 MPa (tensile) and a COV 0.25.

Fracture strength depends e.g. upon the linear elastic fracture toughness, \( K_{mat} \) and the yield strength. Data on fracture toughness is obtained from laboratory tests on simple specimens and usually, large scatter is observed in test results. In addition, the actual toughness of a structure is affected by many factors such as plate thickness, stress state, mode of loading, service temperature, etc. Johnston (1983) proposes a log-normal model for \( K_{mat} \), while others also propose a Weibull distribution. The COV applied varies between 8 and 25% (Johnston. 1983; Shetty and Baker, 1990; Urabe, 1987).

Usually adequate data are available from the steel mills and on-site quality control tests to develop a suitable distribution for the chosen material. Yield strength of the material has a considerable influence on the plastic collapse load of the joint and hence on the fracture behaviour. Here yield strength has been modelled as a log-normal variable with appropriate mean value and COV of 0.08.

The application of the PD6493: 1991 CTOD level fracture mechanics approach to cracked tubular joints is examined in view of test results by e.g. Cheaitani et al. (1995, 1996) and Klasén
and Wästberg (1997). It is demonstrated that the failure assessment approach gives a lower
bound on test results. The scatter in the method is, however, significant.

In fatigue analysis, the critical crack size is often taken to be the wall thickness.

### 3.2.5 Fracture mechanics (FM) vs. SN-approach

Both the SN and FM method can be used to predict time to failure (implicit in SN data and
explicitly defined in the FM approach).

Fatigue lives predicted by the fracture mechanics (FM) approach are sensitive to some
parameters which are difficult to control, but which are implicit in SN data. For this reason it
would be an advantage to ensure consistency between the FM and SN approach both in a
deterministic and probabilistic sense.

The main use of the FM approach is for the assessment of the growth of through-thickness
cracks.
4. FATIGUE LOAD EFFECTS

4.1 GENERAL

Fatigue load effects are described by the long-term distribution of stress ranges, \( S \). In PIA the following two-parameter Weibull distribution is applied:

\[
F_S(s) = 1 - \exp\left(-\frac{s}{A}\right)^B \tag{12}
\]

where \( A \) and \( B \) are shape and scale parameters, respectively. The corresponding uncertainty is modelled by the parameters \( A \) and \( B \), which then depend upon uncertainties associated with

- environmental conditions
- wave load model
- structural - soil model

Various methods for calculating load effects, \( S \) are envisaged. Default uncertainty measures in PIA are given without reference to specific assumptions regarding the method used to determine the distribution of \( S \). This fact may result in inconsistent uncertainty measures.

In the PIA correlation/calibration studies by Vårdal and Moan (1997) the expected values of \( A \) and \( B \) are based on the fatigue design calculations of load effects, \( S \), while using default values for the uncertainty measures.

The following issues are major factors that affect the uncertainty of the Weibull distribution:

- response analysis procedure
- environmental conditions
- hydrodynamic loading model
- structure-pile-soil analysis model to determine member forces and moments
- stress concentration factors in tubular joints

These factors are discussed in the following sections.

4.2 RESPONSE ANALYSIS PROCEDURE

Two alternative response analysis procedures are discussed in PIA Theory Manual (1990):

- deterministic single wave approach, based on stress response obtained by a set of regular long-crested waves with height \( H \) and period \( T \) from representative directions, and a long-term distribution of the wave heights.
- linearized stochastic frequency analysis. Response in short-term sea states are determined by linearized frequency domain approach, possibly considering short-crestedness. The long-term response is obtained by combining the short-term responses according to the long-term probability of various sea states and mean direction.

The simplest deterministic approach is to use a relation between stress range, \( S \) and wave height, \( H \) and a long-term distribution of wave heights. The \( S-H \) relationship may be conservatively established by considering waves with a period (length) corresponding to the peaks of the transfer function and an appropriate height according to a defined wave steepness.

However, the first method does not represent the character of the waves properly. This deficiency increases if there are dynamic response effects. This fact results in increased model uncertainty compared to a stochastic analysis method.

The latter method is preferable especially if stochastic linearization (of drag forces) is carried out. However, the latter method cannot capture surface elevation effects of importance to
splash-zone loading. In this case the short-term response should be obtained by a non-linear
time domain approach.

Linearized short-term response may be assumed to be Rayleigh distributed. The error in fatigue
damage due to possible wide-bandedness is commonly less than 15%.

Non-linear response should in principle be determined by rainflow cycle counting, or simplified
methods, which are verified for the relevant application. A conservative approach would be to
assume that the stress range is twice the amplitude and use the normalized distribution of
positive amplitudes as the amplitude distribution.

Studies by Olufsen (1989) and Karunakaran (1993) demonstrate that fairly good accuracy of the
long-term response in submerged members relevant for fatigue, can be obtained by a linear
approach. Karunakaran (1993) showed that the non-linear responses could be very well
represented by determining a equivalent "transfer function" using non-linear time domain
response for a single equivalent sea state.

Application of a long-crested wave model rather than the more realistic short-crested wave
model, implies a conservative bias. See e.g. Karunakaran et al. (1991, 1993).

For given response quantities the assumption that all sea states have the same mean direction
rather than the actual long-term directionality, over-estimates the stress relevant for fatigue,
typically by 20-40%.

4.3 ENVIRONMENTAL CONDITIONS

Frequently occurring sea states contribute most to stress cycles, which are important for fatigue.
Several observations of the relevant sea states of importance to fatigue will be available, and the
statistical uncertainty will be small compared to the situation for extreme values.

4.4 HYDRODYNAMIC LOAD

Hydrodynamic loading on submerged members are calculated by Morison’s equation. Wave
kinematics is obtained by Airy or some modified Airy theory. The choice of appropriate
hydrodynamic coefficients is a crucial issue.

Fatigue loads are mainly contributed by waves with significant wave height of about 3-4 m. If the
typical member diameter is about 1 m, inertia forces in Morison’s equation will predominate. In
this case Airy theory provides a relatively good estimate of the kinematics, and the uncertainty will
be governed by that in \( C_M \). The default values often used in fatigue (as well as extreme) load
analysis of North Sea jackets up to now, have been \( C_D = 0.7 \) and \( C_M = 2.0 \). It is noted that the
true \( C_M \) varies with the Keulegan-Carpenter number, \( K_C \), which is defined by \( K_C = \frac{U_{\text{max}} \cdot T}{D} \),
where \( U_{\text{max}} \), \( T \) and \( D \) are the maximum particle velocity, wave period and diameter of the
structural member, respectively. According to API RP2A (1993) \( C_M = 1.2 \) for \( K_C > 18 \) and
increases linearly with decreasing \( K_C \) to be \( C_M = 2.0 \) for \( K_C \approx 3 \) (for members with rough surface).
For small wave lengths the MacCamy and Fuchs correction on \( C_M \), should be introduced. Using
\( C_M = 2.0 \) would imply a conservative bias under such circumstances.

Similarly the drag coefficient, \( C_D \) depends upon the Reynolds number and \( K_C \), as well as surface
roughness. The \( C_D \) for members with a rough surface varies for instance between 1.0 and 1.4
for a \( K_C \) which varies from 30 to 10 and less, and a Reynolds number above \( 5 \times 10^5 \).

Hydrodynamic loads on members in the splash zone, that successively are above and below the
wave surface are more complex to model, since local loading governed by slamming and
varying buoyancy predominate. The wave kinematics in the splash-zone is particularly uncertain.
Moreover, dynamic response may be excited by such transient loading. Commonly the loading is
modelled in a conservative, simplified manner by e.g. assuming long-crested waves with crest
parallel to the relevant member. In general time-domain simulation is necessary to determine the response in this case.

Slamming loading may be modelled by a momentum formulation, which appears as a drag-type loading with equivalent $C_D$ equal to at least 3.0 (DNV, 1991). Ridley (1982) explored models for determining the slamming response of members in the splash zone.

Finally, the loads acting on conductors, anodes and other equipment as well as the effect of marine growth (on diameter, and drag coefficient) should be noted. Fatigue analysis based on the maximum marine growth and full corrosion allowance is obviously conservatively biased.

It is also noted that a random model uncertainty associated with loads/response in individual waves will be cancelled when the contribution from several cycles at each load level are “added”. Alternatively, the rms value (or variance) of the response may be used to characterize fatigue. Since several short-term sea states of a given intensity contribute to fatigue damage, there will even be a cancellation effect in the model uncertainty associated with the response variance.

A possible constant current acting together with waves will usually have a small effect on (quasi-static) fatigue loading. This is because the current affects the drag forces, which are fairly small in the fatigue loading regime, and because the fatigue loading is governed by the stress range. See e.g. Olufsen (1989). Load effects, which are amplified by structural dynamics effects, however, will be more sensitive to a possible current because the current may affect the equivalent damping.

4.5 STRUCTURE - PILE - SOIL MODEL

Since the structural behaviour is well represented by a linearly elastic model in fatigue analysis, the main concern is the model of

- joint flexibility
- pile soil
- possible structural dynamic amplification as depending upon eigenfrequencies and damping properties

Joint flexibility may affect axial forces, but especially bending moments. Bending moments are of primary importance for members in the splash zone where direct lateral loads are largest, and axial forces which are due to the integrated loads on the structure above the relevant member, are relatively small.

The Norske Veritas rules (DNV 1977) were the first publication to report flexibility coefficients for tubular joints.

The UEG (1984) report presented a study on the effects of joint flexibility on the response of offshore jackets. Joint flexibility was examined with respect to deflections, axial force, buckling length and dynamic characteristics. A methodology was proposed in the UEG report in order to form a stiffness matrix which takes into account the effects of joint flexibility.

Fessler et al(1986a, 1986b) presented improved equations for flexibility coefficients in terms of the joint parameters, regarding single-braced joints as well as X and gap-K joints.

Efthymiou (1985) published parametric equations for the local rotational stiffness of T, Y and K joints. Stiffness coefficients for rotational degrees of freedom were proposed in terms of joint parameters. Other interesting work on joint flexibility was reported by Buitrago et al (1993) who presented new equations for local joint flexibility coefficients. The investigation was carried out using thick shell elements. The authors pointed out the difficulty of incorporating these coefficients in a structural analysis program. Instead, two models, referred to as the “spring model” and the “flex model”, were proposed.
Holmås (1987) developed efficient analytical approaches to determine the flexibility of tubular joints. The model of pile-structure connection may involve uncertainty due to eccentricities, imperfect grouting (e.g. due to cracking under an extreme storm during operation).

The uncertainty in the pile-soil model for fatigue analysis is due to uncertainty in the model as well as equivalent elastic soil material properties. In addition scour and horizontal soil displacement due to extreme lateral loads may change the effective span length of the piles at mudline and, hence, especially affect the bending moments in the lower joints of the jacket.

4.6 STRESS CONCENTRATION FACTORS

The PIA theory manual refers to SCF’s in tubular joints given in UEG (1985) as well API RP2A 1987-version. Since then significant work on SCF’s for tubular joints has been completed.

While SCF’s can be determined on a case-by-case basis by a 3D or 2D FEM or experimental methods, practical design is largely based on non-dimensional parametric formula, calibrated by the mentioned theories. The most recent parametric formulae have been proposed by Efthymiou (1988), Hellier et al (1990) as well as Smedley and Fisher (1991).

The Efthymiou equations were constructed to provide a mean fit through SCF values obtained by 3D finite element shell models. The new Lloyd's Register (Smedley and Fisher, 1991) equations have a similar form to Efthymiou’s equations, but were obtained as a mean fit to available tubular test data. This fit was then modified by the inclusion of design factors to adjust the level of conservatism of the estimate. These data may include experimental bias and are partly distributed over the design parameters of interest.

Although the Lloyd's Register data provide a better fit to existing experimental data than Efthymiou’s equations, the wider range of parameter coverage have made the latter data more widely accepted.

Hellier et al. (1990) only cover single braces attached to a chord. However, they go into additional detail describing the SCF all along the weld, and not just at a (few) hot spot locations.

Existing SCF equations were compared with measured data in HSE Guidance (1995). Equations were considered acceptable if the SCF was underpredicted by up to 25%. They were considered conservative if 50% or more joints gave SCF predictions which exceeded recorded value by 50% or more. Based on this comparison, formulae for SCF are recommended in HSE (1995). Neither the Efthymiou nor the Smedley & Fisher equations are recommended for all situations. It is noted that no recommendation is given for KT joints due to lack of experimental data.

The draft of the new ISO code for jackets is mainly based on the Efthymiou equations.

It follows from the above approaches that the parametric formulae yield conservatively biased SCF. Based on a range of the model error of 0.67 to 1.25, and the assumption that this range corresponds to 4 standard deviations (95% of the outcomes), the “mean value” and the standard deviation become 0.96 and 0.15, respectively. Larger uncertainties are given by Shetty (1992). This is a generic uncertainty estimate, the true model uncertainty will vary with type of joint and the scantlings, especially if the parameters exceed the range of validity of the parametric equations.

Moreover, the SCFs apply to simple, plane joints. In practice many types of complex and overlap joints are used and braces at a node can be in more than one plane. Moreover, the ratio of axial loads and moments in different braces at a node vary for different wave approach through the structure. Also, the stress needs to be calculated at different positions (crown, saddle) around the tube circumference since the worst location depends on the relative magnitude of axial, in-plane and out of plane bending. This means that joint type identification and SCF calculations have to be repeated for every wave position. This will increase the computational cost considerably. If simplified approaches are used, model uncertainty will increase.
Efthymiou (1988) proposed an “influence function” approach, which avoids repetitive calculation of SCFs. The approach is essentially that of linear superposition of hot-spot stress contributions from each of the braces meeting at a node. Influence function formulations for hot-spot stress in different types of joints under different loading conditions have been derived in Efthymiou (1988). The proposed formulations can be easily extended to multi-planar joints. A comparison of this approach with the conventional SCF approach has shown that multi-planar effects are usually small and the conventional SCF approach generally yields conservative estimates of hot-spot stresses.

4.7 COMPARISON BETWEEN PREDICTED AND MEASURED NOMINAL STRESS

Methods for predicting extreme wave response in jackets have been validated by comparison with measurements. Heideman and Weaver (1992) compared the new API procedure with full-scale measurements. The API procedure is based on kinematics according to Stokes’ fifth order theory, $C_D = 0.65$ and $C_M = 1.6$ (smooth members), and $C_D = 1.05$ and $C_M = 1.2$ (rough members). Since they compared the API design wave approach with measurements in real irregular seas, they had to approximate the irregular waves with a piecewise regular wave. These comparisons for the Tern jacket resulted in a bias of 1.06 and a COV of 0.25. It is shown by others (Atkins JIP, 1994, and Jonathan and Taylor, 1996) that the uncertainty may be reduced by comparing the predicted and measured most probable maximum load in a (random) sea state. The COV will then be approximately 12-14%. These results for extreme loads are not directly transferable to fatigue loads because different sources of uncertainty will dominate in the two situations, e.g. uncertainty in $C_D$ dominates in extreme seas while $C_M$ may be most important for sea states affecting fatigue. The full-scale measurements, however, show that the model uncertainty is reduced when a stochastic approach rather than the deterministic one is applied.

Nerzic and Lebas (1988) compared measured and predicted loads on a tubular member in a North Sea platform. A wave by wave analysis was done, using regular waves in the prediction. The wave short-crestedness and wave components with different length than the dominant wave length, which were more or less present in the real sea-states, were not modelled in the analysis. With $C_D = 0.7$ and $C_M = 1.7$ the predicted loading was smaller than the measured values. The scatter is very large, especially for small wave heights. However, for fatigue analysis many waves at each wave height will contribute. Hence, the expected model error is of main concern. The mean ratio $\bar{R}$ between measured and predicted loading was found to vary, as $\bar{R} = 2.1/H^{0.32}$, which represents a fairly large discrepancy.

An experimental study of (extreme!) wave-current loads on jacket is described by Chakrabarti and Giu (1996). They concluded that a drag coefficient 0.7 was relevant for steady state drag. In regular waves the $C_D$ was found to be in the range of 1.3 to 1.6, while the $C_D$ was reduced towards the steady state value for irregular waves.

4.8 LONG-TERM UNCERTAINTY MODEL

The long-term stress ranges are described by the Weibull distribution, Eq (12). The uncertainty model applied for the distribution of $S$ is to describe $(\ln A, 1/B)$ by a joint normal distribution with correlation coefficient, $\rho$.

The implication of this assumption on the uncertainty in the $S$ corresponding to a given probability, $Q(s)$ of exceedance level, given by

$$Q(s) = 1 - F_s(s) = \exp\left[-(s/A)^B\right]$$

(13)

can be determined from

$$S = \left[-\ln Q(s)\right]^{1/B} A$$

(14)
as follows

\[ \sigma_{\ln S}^2 = c^2 \sigma_{1/B}^2 + \sigma_{\ln A}^2 + 2 \rho c \sigma_{1/B} \sigma_{\ln A} \]  

(15)

where

\[ c = \ln(-\ln Q(s)) \]  

(15a)

The true uncertainty of \( S \) at different exceedance levels varies. For instance, drag forces become relatively more important compared to inertia forces with increasing wave heights \( H \) and hence large response levels (small \( Q(s) \)). Moreover, for large wave heights nonlinearities in wave elevation and kinematics are more pronounced than for small wave heights. Since large wave heights imply large loads, nonlinearities associated with pile-soil behaviour also would first affect large responses (at small \( Q(s) \)). Structural dynamics effects tend to increase \( S \) at small wave heights (periods or, large \( Q(s) \)) first if they are significant.

The main contribution to fatigue damage stems from stress ranges corresponding to \( Q(S) \) in the range \( 10^{-1} \) to \( 10^{-4} \). The uncertainty measures of \( \ln A \) and \( 1/B \) should therefore be chosen to yield reasonable uncertainty measures in the given stress range level.

It is interesting to see the implication of the probabilistic model for (\( \ln A, 1/B \)) on the uncertainty of \( S \), at a given probability of exceedance level.

If the default values specified by PIA: \( \sigma_{\ln A} = 0.2, \mu_{1/B} = 1.0, \sigma_{1/B} = 0.07 \) and \( \rho = -0.8 \), are used, the uncertainty \( \sigma_{\ln S} \) is found to be of the order \( 0.12 - 0.15 \) for values of \( S \) that are most relevant to fatigue. If only the uncertainty of \( \ln A \) is considered, \( \sigma_{\ln S} \) becomes 0.2 over the entire range of \( S \).

If the marginal uncertainties are considered as before, but \( \rho \) is taken to be -0.34 or zero, \( \sigma_{\ln S} \) becomes 0.19 - 0.20 or 0.21 - 0.24, respectively.

The uncertainty \( \sigma_{\ln S} \) is not very sensitive to the \( Q(s) \) level for values of \( S \) most important to fatigue. However, it is seen that the uncertainty is influenced by the uncertainty in \( 1/B \).

The most important issue is that the uncertainty of \( \ln A \) and \( 1/B \) are chosen so that the uncertainty for \( Q(s) \) in the range \( 10^{-1} \) to \( 10^{-4} \) is as accurate as possible.
5. CONCLUDING REMARKS

Recent work relating to uncertainties that affect predicted crack growth in tubular joints of jacket structures has been reviewed. Uncertainties in load effects and resistance have been considered.

It is noted that the PIA model could be improved with respect to

- fracture mechanics model, especially by using a two-dimensional model
- using more recent data for the parameters C and m, which are differentiated with respect to stage in crack growth (magnitude of da/dN), the stress ratio, R and corrosive environment
- stress analysis, including stress concentration factors.

The uncertainty associated with the hot spot stress is a significant source of the uncertainty in predicted crack growth. This uncertainty should be further assessed by comparing predicted nominal stresses with in-service measurements, and by comparing current methods to determine the transfer from nominal to hot spot stress by means of refined analysis and possibly laboratory experiments.

When probabilistic analyses are applied in PIA to correlate with observed behaviour of specific structures, the uncertainties may be reduced based on the specific knowledge available.

The first phase in such correlation studies is based on generic uncertainties. Some of the uncertainties may be reduced due to specific information available. This may include for instance the uncertainty in C and m if crack propagation data for the specific material under relevant environmental condition are available. Then only the small in-batch variability will remain.

The model uncertainty of the fracture mechanics model as well as SCF depends upon the geometry of the joint and could be reduced if available information is utilized.

The uncertainty of the nominal stresses (member forces) in the structure depends on the hydrodynamic model and joint flexibility. The uncertainty associated with the hydrodynamic loading in particular needs to be separated into continuous and slamming loading on members.

While commonly the uncertainty in extreme loading is found to dominate over that in the resistance for ultimate failure modes, the uncertainties in resistance against crack propagation and load histories are found to be of about the same magnitude in fatigue analysis. Work is especially needed to better quantify uncertainties in load effects (hot spot stresses) and inspection quality.
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