Testing of ISO-compliant extreme water level calculations using raw wave data

Prepared by PhysE Limited
for the Health and Safety Executive 2009
Testing of ISO-compliant extreme water level calculations using raw wave data

Ian P Wade PhD CMarSci FIMarEST
PhysE Limited
The Harbour Offices
The Quay
Yarmouth
Isle of Wight
PO41 0NT

This report describes a study of raw (2 Hz) wave data with relevance to ISO-compliant derivations of extreme crest elevation and extreme water level.

This report and the work it describes were funded by the Health and Safety Executive (HSE). Its contents, including any opinions and/or conclusions expressed, are those of the author alone and do not necessarily reflect HSE policy.

HSE Books
CONTENTS

1. INTRODUCTION 1
   1.1 INTRODUCTION 1
   1.2 CONTEXT OF THE STUDY 1
   1.3 AIMS 1
   1.4 AVAILABILITY OF DATA / ACKNOWLEDGMENTS 2
2. GLOSSARY OF TERMS 3
3. DATA 5
   3.1 DATA SOURCES 5
   3.2 RAW DATA QC 5
   3.3 DATA PROCESSING 7
4. THE MAXIMUM CREST HEIGHT IN A SEA STATE 8
   4.1 SHORT TERM DISTRIBUTION OF WAVE CRESTS 8
   4.2 THEORETICAL SHORT-TERM DISTRIBUTIONS 8
   4.3 COMPARISON OF MEASURED AND THEORETICAL DISTRIBUTIONS 9
   4.4 COMPARISON OF MEASURED AND THEORETICAL MAXIMA 19
   4.5 THE MAXIMUM CREST AND THE HMAX WAVE 21
5. THE MAXIMUM CREST HEIGHT IN A STORM 23
   5.1 MOST PROBABLE MAXIMUM CREST IN A STORM 23
   5.2 COMPARISON OF THE MOST PROBABLE & MEASURED MAXIMUM CREST IN A STORM 24
6. IMPACTS ON THE ISO-COMPLIANT METHODOLOGIES 30
   6.1 THE PRINCIPAL ISO-COMPLIANT METHODS 30
   6.2 THE ISO-COMPLIANT RESULTS 31
7. MAXIMUM CREST EVENTS 37
8. INDIVIDUAL WAVE PERIODS 37
9. OVERVIEW 39

10. APPENDIX A: SHORT-TERM CREST EQUATIONS 40
11. APPENDIX B: WEIBULL PLOTS USED IN ISO ANALYSES 42
12. APPENDIX C: TVM PLOTS 46
13. APPENDIX D: TOP CREST EVENTS (CORMORANT A) 50
14. APPENDIX E: TOP CREST EVENTS (NTH. CORMORANT) 53
TABLES

Table 3-1  Study Locations ..................................................................................................................5
Table 4-1  Comparisons of Percentile and Distribution of Maxima Cmax (Cormorant A).................20
Table 4-2  Comparisons of Percentile and Distribution of Maxima Cmax (Nth. Cormorant) ..............20
Table 4-3  Comparisons of Measured and Theoretical Cmax (Cormorant A)........................................21
Table 4-4  Comparisons of Measured and Theoretical Cmax (North Cormorant) ...............................21
Table 6-1  Weibull Parameters Derived from Least Squares Fits ....................................................32
Table 6-2  Borgman / Krogstad Maximum Crest Heights Compared to Observed Maxima .............32
Table 6-3  Tromans and Vanderschuren Maximum Crest Heights Compared to Observed Maxima ....33
Table 6-4  Monte Carlo Maximum Crest Heights Compared to Observed Maxima ..........................34

FIGURES

Figure 4-1  Crest Height Distributions from 1-hour Sea States (CA and NC) .....................................9
Figure 4-2  Effect of the Successive Removal of Crestlets (CA only) ..................................................10
Figure 4-3  Effect of the Successive Removal of Crestlets on Exceedence (CA only) ............................11
Figure 4-4  Successive Removal of Crestlets on Distribution of Maxima (CA only) .............................12
Figure 4-5  Crest Height Exceedences from 1-hour Sea States (CA and NC) ......................................13
Figure 4-6  Sea State Duration and Probability of Exceedence (CA only) ...........................................14
Figure 4-7  Sea State Duration and Probability of Exceedence (NC only) ..........................................15
Figure 4-8  Effect of Sea State Duration on Distribution of Maxima with a Constant Nz Assumption (CA).16
Figure 4-9  Effect of Sea State Duration on Distribution of Maxima using a Realistic Nz (CA) ............16
Figure 4-10 Effect of Sea State Threshold on Individual Crest Distribution (CA) .................................17
Figure 4-11 Effect of Sea State Threshold on Individual Crest Distribution (F2D) ...............................18
Figure 4-12 Effect of Sea State Threshold on Distribution of Maxima with a Constant Nz Assumption ...18
Figure 4-13 Effect of Sea State Threshold on Distribution of Maxima with a Realistic Nz (F2D) .........19
Figure 4-14 Relationship Between Cmax and Cass_Hmax (Cormorant A) ...........................................22
Figure 4-15 Relationship Between Cmax and Cass_Hmax (North Cormorant) .....................................22
Figure 5-1  Distribution of Normalised Storm Cmax vs Cmp (Top 100 storms - CA and NC) .............24
Figure 5-2  Comparison of Cmax and Cmp for Top 50 Storms (Cormorant A) ........................................25
Figure 5-3  Comparison of Cmax and Cmp for Top 50 Storms (North Cormorant) .............................26
Figure 5-4(a,b)  Cmax vs Cmp for Top 50 Cormorant A Storms at Hs > 5 m (left) and 6 m (right) .......27
Figure 5-5(a,b)  Cmax vs Cmp for Top 50 North Cormorant Storms at Hs > 5 m (left) and 6 m (right) ....28
Figure 6-1  Comparison of Long-term Weibull Fits to Data (1-hr and 3-hr Sea States) .......................31
Figure 6-2  Comparison of Extreme Wave Crest Heights .................................................................36
Figure 8-1  Top Individual Waves Compared to Design Criteria (Cormorant A) .................................38
Figure 8-2  Top Individual Waves Compared to Design Criteria (North Cormorant) ...........................38
EXECUTIVE SUMMARY

KEY FINDINGS

Analysis of Sea States

1. The normalised theoretical distributions of individual wave crests do not take account of ‘crestlets’ that significantly alter the distribution at low $H_c/H_s$ values. Neither, do the theoretical distributions of individual wave crests locate true position of the mode. However, neither of these limitations significantly affects the distribution of the maxima.

   (Section 4.3, Figure 4-1 to 4-4)

2. The Forristall-2D wave crest distribution appears to outperform the alternatives in the extreme tails of the distributions (based on 1-hour sea state analyses).

   (Section 4.3, Figure 4-5)

3. The duration of the sea state under analysis affects the plotting position of normalised distributions (as $H_s$ changes but $H_c$ does not). The sea state duration should be taken into account when testing the suitability of the various theoretical short-term distributions.

   (Section 4.3, Figures 4-6 and 4-7)

4. Increasing the assumed sea state duration obviously increases the magnitudes of the distribution of maxima (as the number of waves increases). The effect of increasing sea state duration of the distribution of maxima was tested by setting (unrealistically) a fixed number of waves. This demonstrated that the distribution of maxima were more affected in the tail than in the bulk of the distribution as the sea state duration increased.

   (Section 4.3, Figure 4-8 and 4-9)

5. As we analyse successively higher sea states, the magnitudes of the distribution of maxima increase (again tested with an unrealistic fixed number of waves). However, in reality, this effect offset by the higher sea states containing fewer individual waves ($T_z$ tends to increase as $H_s$ increases) and the effect on the distribution of maxima is very small.

   (Section 4.3, Figures 4-10 to 4-13)

6. The highest individual wave in a theoretical distribution (taken as the $1-1/N^{th}$ percentile) compared favourably with the mode of the distribution of maxima (63% exceedence), verifying the validity of the method of generating a theoretical distribution of maxima.

   (Section 4.4.1, Tables 4-1 and 4-2)

7. However, the highest measured crest height in a given sea state appears to be ‘tail-side’ of the mode in the majority of cases (depending on the theoretical distribution used). The Forristall-2D distribution provides the closest matches with the measured values. However, even the F2D ‘most probable’ values are typically low by 4% to 5%.

   (Section 4.4.1, Tables 4-3 and 4-4)
8. The highest recorded crest (Cmax) was only associated the Hmax wave in approximately 1/3 of all sea states.  

(Section 4.5, Figures 4-14 and 4-15)

Analysis of Storms

9. The mode of the distribution of measured Cmax waves (for the top 100 storm events) agrees favourably with the most probable maxima crests in the storms (Cmp). Cmp’s therefore appear to be being calculated in an appropriate way.  

(Section 5.2, Figure 5-1)

10. However, as expected, the individual Cmax’s regularly exceed the Cmp’s during storm events due to short-term variability.  

(Section 5.2, Figures 5-2 to 5-5)

11. Care should be taken when selecting the storm threshold when the said threshold (as Hs) is close to the maximum Hs in the storm. When the peak Hs in a storm is well in excess of the storm threshold, the Cmp is unaffected by the threshold selection (within reasonable limits). However, as the peak Hs approaches the threshold Hs the Cmp values are affected.  

(Section 5.2, Figures 5-2 and 5-3)

ISO Analyses

12. Care should be taken when considering the application of ISO-compliant techniques to such short datasets. However, in these analyses the Monte Carlo, TVM and direct extrapolation of measured Cmax resulted in very similar extreme crest heights. The Borgman / Krogstad methodology results in higher extremes. The Shell EP Europe Ratio Method (believed to be relatively conservative) concurred with the Borgman results.  

The different methodologies are not directly comparable (dealing with either all individual waves or just maximum individual waves within selected samples (e.g. sea states or storms)). One should not be overly concerned that the results do not converge.  

(see Section 6.2, Figure 6-2)

Additional Analyses

13. The highest recorded individual wave periods appear to agree well with current estimations of Tass (the wave period associated with Hmax) currently employed by Shell EP Europe for the two sites analysed.  

(see Section 8, Figures 8-1 and 8-2)
Concluding Remarks

14. The observed short-comings in the various short-term theoretical distributions appear to have a relatively small effect on the theoretical distribution of maxima. However, on the basis of these relatively limited datasets, these analyses do suggest that the modal values of the maxima underestimated the actual (measured) maxima. In the majority of analysed storms the measured maximum was closer to the mean (or median if we wish to discuss it in terms of exceedences) of the theoretical distribution of maxima.

15. A lot of previous effort has gone into trying to either get the various ISO-compliant methodologies to converge or to explain the differences. These analyses show the Borgman / Krogstad approach to produce relatively high extreme values. This is, in hindsight, expected due the method processing all individual waves in any given sea state (as discussed by Tucker and Pitt (2001) and Forristall (2006)). It therefore takes account of the 2nd and 3rd etc. highest individual waves which in high sea states may be of importance, depending in the engineering requirements.

16. The usefulness of comparing observed maxima against the various theoretical estimates is somewhat limited due to the natural short-term variability. This limit is particularly evident with such short datasets.
1. INTRODUCTION

1.1 INTRODUCTION

This report has been prepared by PhysE Limited, working within the BOMEL Framework Agreement, for the Health and Safety Executive as research contract C12530R-1, and describes a study of raw (2 Hz) wave data with relevance to ISO-compliant derivations of extreme crest elevation and extreme water level.

1.2 CONTEXT OF THE STUDY

BS EN ISO 19901-1:2005 is entitled “Petroleum and natural gas industries – Specific requirements for offshore structures Part 1: Metocean design and operating considerations”. The standard addresses in some detail acceptable mathematical approaches to the estimation of extreme wave crest elevation.

Previous work\(^1\) has shown a significant degree of variability in crest heights at extremely low levels of probability. This is not overly surprising considering the length of suitable workable datasets (a few decades at best) and the length of the required return periods (up to 10,000 years).

1.3 AIMS

Variability in the extreme results arise from either (1) fundamental problems with the IOS-compliant techniques and the various assumptions therein and/or (2) from natural variation within the extrapolation process.

The various ISO-compliant methodologies for determining extreme wave crests at very low probability levels are based on certain fundamental ideas. The primary object of this study is to examine the validity of these ‘building blocks’ in an attempt to either:

- make comment on best practice
- provide notes of caution as to the application of the various techniques/assumptions
- highlight where differences may arise in the various techniques

The ISO-complaint techniques will be applied to processed sea state parameters derived directly from the raw 2 Hz wave data. The ISO results will then be compared to the raw data, bearing in mind the relative paucity of data and the effect of short-term variability.

In addition to the above, this study aims to provide:

- ideas for further useful research concerning the derivation of extreme wave crests
- notes of interest regarding individual wave data with application to the derivation of design criteria

---

\(^1\) Extreme Total Surface Elevation and the Implications of ISO-19901-01.
1.4 AVAILABILITY OF DATA / ACKNOWLEDGMENTS

It was initially envisaged that 10 years worth of fully QC’d raw wave data would be available for this study. Unfortunately, this was not to be the case and alternative datasets had to be located.

Shell EP Europe kindly made available raw wave data from their METNET-3G North Sea measurement campaign for use in this study. The author’s of this report would like to acknowledge and thank Mr. Ian M Leggett, Discipline Head of Metocean Engineering, Shell EP Europe, for the use of their raw wave data and for his, and his team’s, valuable comments regarding this study.
# 2. GLOSSARY OF TERMS

<table>
<thead>
<tr>
<th>Wave Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero-crossings</td>
</tr>
<tr>
<td>A zero-crossing is the point in time at which the water level moves across an arbitrary datum (usually a mean water level). The use of zero-crossings to define individual waves minimises problems with crest to crest definitions of individual waves, which may be corrupted by small relatively insignificant wavelets atop the main waves.</td>
</tr>
<tr>
<td>Zero-crossing Individual Wave Height, $h_z$</td>
</tr>
<tr>
<td>The vertical excursion between two successive zero-crossings. The individual wave is a real wave that can be physically observed and recorded (as a time series of water elevation) as it passes through the measurement site. Several fundamental criteria are determined from this series of individual wave data.</td>
</tr>
<tr>
<td>Significant Wave Height, $H_s$</td>
</tr>
<tr>
<td>Is a purely statistical term, like the average or mean, that describes a sea state (or roughness of the sea) associated with a given sample duration. Unlike the mean it has a distinct positive bias and may be approximated to the mean height of the highest one third of the individual, $h_z$, waves. Several methods of deriving the significant wave height now exist.</td>
</tr>
<tr>
<td>Crest Height, $H_c$</td>
</tr>
<tr>
<td>The amplitude of a wave crest above the still water level. Care needs to be taken when defining the 'local' still water levels for the wave in question.</td>
</tr>
<tr>
<td>Maximum Crest Height (Measured), $C_{max}$ and Maximum Crest Height (Most Probable), $C_{mp}$</td>
</tr>
<tr>
<td>Care should be taken in defining the maximum crest height. Significant differences may exist between the two definitions due to short-term variability. $C_{max}$ is the maximum measured crest height recorded in a given sample. $C_{mp}$ is a theoretically derived maxima corresponding to the mode of the distribution of maxima (a most probable value with a 63% chance of exceedence).</td>
</tr>
<tr>
<td>Maximum Wave Height (Measured), $H_{max}$</td>
</tr>
<tr>
<td>In common with $C_{max}$ and $C_{mp}$, the maximum wave height may also be defined in a number of ways. However, in this report we only deal with the measured maximum wave height, $H_{max}$. Please note that elsewhere, through common usage, the term $H_{max}$ often applies to the modal value of the theoretical distribution of maxima. This $H_{max}$ is a most probable maximum wave height. If it were used here, the most probable maximum wave height would be defined $H_{mp}$ for consistency with the crest definitions.</td>
</tr>
<tr>
<td>Associated Crest Height, $C_{ass_H_{max}}$</td>
</tr>
<tr>
<td>The crest height associated with the $H_{max}$ wave.</td>
</tr>
<tr>
<td>Associated Wave Height, $H_{ass_C_{max}}$</td>
</tr>
<tr>
<td>The wave height associated with the $C_{max}$ wave.</td>
</tr>
<tr>
<td>Zero-crossing period, $t_z$</td>
</tr>
<tr>
<td>The wave period associated with an individual wave, $h_z$.</td>
</tr>
<tr>
<td>Mean Zero-crossing Period, $T_z$</td>
</tr>
<tr>
<td>The mean of all of the zero-crossing periods, $t_z$, within a given sea state.</td>
</tr>
<tr>
<td>Mean Wave Period, $T_1$</td>
</tr>
<tr>
<td>Despite the name, this is actually defined as the reciprocal of the mean frequency of the wave spectrum. It can be derived from the spectral moments ($m_0 / m_1$) or by applying a scaling factor, based on an assumed spectral model, to $T_z$. $T_1$ is always longer than $T_z$ and theory suggests that $T_1$ tends to better represent the periods of the highest individual waves. In this study $T_1$ is used in the derivation Forristall crest coefficients.</td>
</tr>
</tbody>
</table>
Wave Terms (Continued)

Wave Steepness, S or S_s

The wave steepness is rather arbitrarily defined as the ratio of the wave height to the wave length \((S = H/L)\). For convenience we generally classify wave steepness in terms of the more easily and commonly measured wave period \((S = 2 \pi H / g T^2)\) using the deep-water relationship between wavelength and wave period. In practice, this definition is used for all water depths but one should be aware of the shallow water implications.

The wave steepness is commonly derived in terms of the sea state parameters in which case it is referred to as the significant steepness \((S_s = 2 \pi H_s / g T_z^2)\). The significant steepness has no precise physical meaning but is used as a measure of the steepness of a sea state.

Steepness if often presented as the reciprocal steepness for convenience.

Spectral Moments

One of the key concepts in the analysis of waves is that of wave spectra (describing the distribution of wave ‘energy’ at various frequencies). The first few moments of the wave spectrum are of special importance in the description of ocean waves. The zeroth moment, \(m_0\), equals the total variance (‘energy’) of the wave system and is directly related to \(H_s\). Subsequent moments describe other wave parameters (e.g. \(T_1 = m_0 / m_1\)).

Water Level Terms

Level

The magnitude of the water surface relative to some fixed datum such as LAT (Lowest Astronomical Tide).

Elevation

A deviation in a water level due to some additional forcing such as a storm surge. Elevations have no fixed datum as they are added (or subtracted) from water levels (see above).

Amplitude

The tide can be represented by the sum of a series of sine waves of determined frequency. The parameters of each sine wave are called "harmonic constants". These are the amplitude (half the height) of the wave and its phase. The sum of the all the component tidal amplitudes results in the tidal level (once phases are taken into account).

Still Water Level, SWL

The instantaneous water level in the absence of waves. It is made up from the sum of the Tide and the Surge.

Total Water Level, TWL

The maximum elevation of "green water" due to the combined effects of Tide, Surge and Waves.

Tidal Level

The water level relative to a defined datum (usually LAT) due to astronomical forcing (mostly lunar and solar). Tidal elevations exclude all meteorologically induced forcing.

Surge (Residual) Elevation

The difference between the harmonically predicted tidal levels and the actual water levels. In most cases this Residual is made up from the storm surge component (due to meteorological forcing), however, it also includes other relatively minor effects such as the inverse barometer effect etc. Due to the dominance of the storm surge component to the Residual, in these analyses the Residual and Surge Levels are assumed to be the same.

Miscellaneous Terms

Short-term Distribution

The distribution of individual waves (heights or crests) within a single sea state of a given duration in the order of hours.

Long-term Distribution

The distribution of multiple sea states over a number of years.
3. DATA

3.1 DATA SOURCES

Raw wave (level) data collected from a number of sites in the North Sea, as part of Shell EP Europe’s ongoing measurement campaign, were kindly made available with permission of Mr. Ian M Leggett, Discipline Head of Metocean Engineering.

This study has concentrated on analyzing the raw 2 Hz data collected in the northern North Sea. The following data were available:

- **Cormorant A**
  
  Data collected under the Fugro GEOS measurement regime between 07/11/2002 and 01/05/2003 and the Muir Matheson regime between 20/01/2004 and 01/04/2008.
  
  = 918 days of non-continuous data at a sampling rate of 2 Hz.
  
  The mean individual wave period, Tz, of the data was approximately 6.4 s resulting in some 12.3 million individual waves for analysis.

- **North Cormorant**
  
  Data collected under the Fugro GEOS measurement regime between 05/10/2001 and 19/01/2004 and the Muir Matheson regime between 04/05/2004 and 30/04/2008.
  
  = 944 days of non-continuous data at a sampling rate of 2 Hz.
  
  The mean individual wave period, tz, of the data was approximately 6.5 s resulting in some 12.5 million individual waves for analysis.

<table>
<thead>
<tr>
<th>Name</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Region</th>
<th>Depth</th>
<th>UK Licence Block</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cormorant Alpha</td>
<td>61° 06' 09&quot; N</td>
<td>001° 04' 22&quot; E</td>
<td>Northern North Sea</td>
<td>~149 m</td>
<td>211/26</td>
</tr>
<tr>
<td>North Cormorant</td>
<td>61° 14' 26&quot; N</td>
<td>001° 08' 58&quot; E</td>
<td>Northern North Sea</td>
<td>~160 m</td>
<td>211/21</td>
</tr>
</tbody>
</table>

Table 3-1 Study Locations

3.2 RAW DATA QC

3.2.1 Automatic QC Procedures

The initial data QC of the 2 Hz raw data is automatic and consists of a number of relatively broad tests to remove grossly erroneous data from the series. The following tests\(^2\) are performed on successive 20 minute segments of the data.

---

For each successive 20 minute sample:

- **Absolute Min. / Max. Value = 10 / 50**: Any values less than / greater than this value are removed. No interpolation of removed data is performed.

- **Gross Error Limit = 6**: Values > Gross Error Limit × the standard deviation of the 20 minute sample are removed. Single removed values are linearly interpolated.

- **Steepness Threshold = 10**: if the rate of change between points is too large then the second point is removed. The equation relating the rate of change of two points is as follows:

\[
\text{Maximum allowable rate of change} = \Delta t \sqrt{\frac{\pi g \sigma}{S_t}} \ln \left(\frac{T \sqrt{g/S_t}}{8 \pi \sigma}\right)
\]

where \(S_t\) is the steepness threshold, \(T\) is the record length (2,400 data) and \(\sigma\) is the standard deviation of the sample. Single removed values are linearly interpolated.

- **Constants Threshold = 0.001**: this determines how close two values can be before they are deemed to be different – if two adjacent values exceed this threshold then they are deemed to be different, otherwise they are said to be constant.

- **Constants Limit = 10**: if the number of equal (see previous bullet point) sequential values exceeds this values then they are all removed. No interpolation of removed data is performed.

### 3.2.2 Manual QC Procedures

The automated QC procedures described above are only capable of removing the more extreme erroneous data. The procedures need to be set ‘wide enough’ to avoid false positives. Therefore it is not expected that raw data can be QC’d purely with automatic procedures.

Due to the sheer quantity of data, neither is it practical to review each and every individual wave manually. The datasets used in this study each contain in excess of 12 million individual waves. A practical solution is to process the post auto QC’d data to 3 hour sea state parameters (Hs, Tz, Hmax, Cmax etc.) and to review these more limited datasets to highlight where potential faults lie in the underlying raw datasets. Individual spot values (Hmax, Cmax etc.) are compared as ratios against statistical parameters that are less prone to outliers (Hs etc.).

As a final check the post auto and partially manually QC’d data were processed to extract the top individual crest heights. The raw data time series corresponding to each of these peak (key) events were then reviewed to ensure the veracity of these low probability events.

---

3 Radar to Sea surface distance in metres.
3.3 DATA PROCESSING

After QC the raw wave data were processed deterministically (i.e. from the time domain as opposed to the spectral domain) to provide the following measurement based parameters (at sea state durations of 1 and 3 hours):

- Significant Wave Height, $H_s$
- Mean zero-crossing Wave Period, $T_z$
- The measured maximum wave height in the given sea state, $H_{\text{max}}$
- The measured maximum crest height in the given sea state, $C_{\text{max}}$
- The measured crest height associated with the $H_{\text{max}}$ wave, $C_{\text{ass}_{-}H_{\text{max}}}$

Various theoretical criteria, such as $C_{\text{MP}}$, were derived from these measurement-based criteria.
4. THE MAXIMUM CREST HEIGHT IN A SEA STATE

4.1 SHORT TERM DISTRIBUTION OF WAVE CRESTS

Short-term wave statistics are the statistics of the sea surface assuming stationarity. That is, that the variance of the sea surface elevation (expressed as a statistical term called the significant wave height) remains constant. The significant wave height, Hs, may usually only be regarded as being constant for just a few hours – hence ‘short-term’.

Generally the short-term statistical constancy is assumed to persist for anything up to three hours and this is a commonly used duration in the derivation of design wave criteria.

Whatever its duration a single sea state (described by a constant Hs) comprises a number of individual (real observable) waves. The distribution of individual waves within a single sea state may be described by a variety of theoretical distributions.

4.2 THEORETICAL SHORT-TERM DISTRIBUTIONS

4.2.1 Rayleigh (Linear) Crest Distribution

Individual waves in deep water are assumed to follow the Rayleigh distribution. This distribution is asymptotically correct as the wave spectrum tends to a narrow band of frequencies. A consequence of this is that the waves are assumed to be linear (i.e. the sea surface is Gaussian and the waves are symmetric about the mean; troughs equal crests).

The Rayleigh distribution is known to be limited by the assumptions of linearity and therefore several modifications of this distribution have been proposed. See Appendix A for relevant equations.

4.2.2 Forristall 2D / 3D Crest Distributions

The Forristall 2D / 3D crest distributions\(^4\) are essentially non-linear transformations of the Rayleigh distribution (Forristall, 2000)\(^5\). Forristall’s crest distributions are actually based on a modified Weibull\(^6\) distribution with the coefficients expressed as functions of the wave steepness and the Ursell number which provide estimates of the degree of non-linearity. The Forristall models converge by default to the Rayleigh distribution in very deep water as the coefficient fits are forced to match Rayleigh at zero steepness and Ursell number (i.e. as depth \(\rightarrow \infty\)). See Appendix A for relevant equations.

---

\(^4\) Forristall’s 2D distribution describes a long crested sea, the 3D a short crested situation.


\(^6\) The Rayleigh distribution is a form of the Weibull distribution (see Appendix A for more details)
4.2.3 Jahns & Wheeler Crest Distribution

The Jahns and Wheeler crest distribution is also a non-linear transformation of the Rayleigh distribution. The transformation is dependent on the crest height normalized by water depth (Jahns and Wheeler, 1973)\(^7\). In this study we have used the coefficients determined by Haring and Heideman (1978)\(^8\). The Jahns and Wheeler model also converges to the Rayleigh distribution in deep water. See Appendix A for relevant equations.

4.3 COMPARISON OF MEASURED AND THEORETICAL DISTRIBUTIONS

4.3.1 The Effect of ‘Crestlets’

In this study all the measured individual wave crests have been recorded as positive vertical deviations from a local 5 minute still water level. Each measured crest height, \(H_c\), was then normalised against the corresponding \(H_s\) for the sea state within which it occurred in order to produce a single distribution of measured individual wave crests.

Figure 4-1 shows the distribution of measured individual crest heights (normalised against the 1 hour \(H_s\)) from an analyses of almost 25 million individual waves (~12.3 million at Cormorant A and ~12.5 million at North Cormorant). The measured distributions are compared against the four theoretical distributions discussed in Section 4.2.

---

\(^7\) Jahns, H.O. and Wheeler, J.D. (1973)
‘Long-term wave probabilities based on hindcasting of severe storms’

\(^8\) Haring, R.E. and Heideman, J.C. (1978)
‘Gulf of Mexico raw wave return periods’
The four theoretical distributions are broadly similar. This is expected as all these distributions
tend to converge to the Rayleigh distribution in deep water (Cormorant A and North Cormorant
are situated in a relatively deep 149 m and 160 m of water respectively).

The most notable differences between the theoretical and measured distributions are twofold:

- A sharp increase in the probability of occurrence at low Hc/Hs values
- A shift in the modal position from ~ 0.25 (theoretical) to ~ 0.30 (measured)

**Note 1:** All the theoretical distributions, with the exception of Rayleigh, are functions
of individual sea state statistics other than just the normalized crest height (Hc/Hs). Therefore the
Forristall 2D / 3D and the Jahns and Wheeler distributions will alter slightly depending on the
datasets used. With Rayleigh distribution the probability density is purely a function of Hc/Hs
(our abscissa) and is therefore independent of the dataset used. See Appendix A for further
details.

### 4.3.2 Pre-normalising Removal of ‘Crestlets’

The sharp increase in the probability of occurrence at low Hc/Hs values is due to the presence of
small crests in relatively large sea states (‘crestlets’). This is demonstrated, with the Cormorant
A data, in Figure 4-2 by examining the normalised wave crest distributions after the removal of
successively higher (pre-normalised) wave crests.

![Figure 4-2 Effect of the Successive Removal of Crestlets (CA only)](image-url)
Figure 4-2 shows that the removal of crestlets does have a noticeable effect on the bulk of the distribution of individual wave crests. However, from a design point of view the most interesting part of these various distributions is towards the tail and in this region the differences are small. Figure 4-3 (on a log scale to emphasise the tail of distribution) demonstrates that in this key area the distributions are virtually identical.

![Figure 4-3](image)

**Figure 4-3 Effect of the Successive Removal of Crestlets on Exceedence (CA only)**

### 4.3.3 Effect of ‘Crestlet’ Removal on the Distribution of Maxima

Irrespective of the distribution used, the most probable maximum wave crest in any given sea state (Cmax) can be determined in the same way. For any given stationary sea state, defined by Hs, we assume that the individual waves making up that sea state follow a given distribution, \( P(H_c < c \mid H_s) \). Now the probability of any individual wave crest, Hc, being less than a threshold, c, is given by the cumulative formulae of the preferred distribution (see Appendix A).

An elementary law of probability states that the probability of all of a number of events occurring is the product of the probabilities of each of them occurring individually. Therefore, the probability of all the wave crests in the given sea state being less than the threshold, c, is given by the product of the probabilities of each individual wave in turn being less than, c. If there are Nz waves then

\[
P(C_{\text{max}} < c \mid H_s) = P(H_c < c)^{Nz}
\]

---

For example, the probability of throwing a six with a die is \( \frac{1}{6} = (\frac{1}{6})^1 = \frac{1}{6} \)

likewise, the probability of throwing two successive sixes is \( \frac{1}{6} \times \frac{1}{6} = (\frac{1}{6})^2 = \frac{1}{36} \)

or more generically, the chance of throwing N successive sixes = \( (\frac{1}{6})^N \)
Nz can be determined from the duration of the sea state (1 hour in these analyses) and the mean zero up-crossing period (Tz). \( Nz = \frac{3,600}{Tz} \) (for a 1 hour sea state).

The cumulative distributions from Figure 4-3 were converted to distributions of maxima using the following values of Nz. The value of Nz varies due to the removal of crestlets but in fact the differences caused by the variation in Nz are negligible.

- CDF of maxima (no crestlet removal) \( Nz_{(1 \text{ hour})} = 530 \) Mean Tz of 6.8 s from all sea states
- CDF of maxima (crestlets > 0.05 m removed) \( Nz_{(1 \text{ hour})} = 500 \) 94\% of above (from Figure 4-2)
- CDF of maxima (crestlets > 0.10 m removed) \( Nz_{(1 \text{ hour})} = 470 \) 89\% of above (from Figure 4-2)
- CDF of maxima (crestlets > 0.15 m removed) \( Nz_{(1 \text{ hour})} = 450 \) 85\% of above (from Figure 4-2)

Figure 4-4 demonstrates that the removal of crestlets has no impact on the distribution of maxima. We therefore conclude that the inability of the various theoretical distributions to adequately model the ‘crestlet effect’ has no effect on our ultimate goal of modelling likely maxima.

![Figure 4-4 Successive Removal of Crestlets on Distribution of Maxima (CA only)](image)

4.3.4 Position of the Mode of the Normalised Crest Distributions

The reason for the shift in the positions of the theoretical and measured modal values remains uncertain. The removal of the pre-normalised crestlets does not alter the position of the measured modes (just their probabilities of occurrence).

The exact modal position of the theoretical Rayleigh (and approximate modal position of the alternatives) distribution of individual crest heights is \( 0.25 \times Hs \) (from \( \{1/16\}^{0.5} \)). The modal position of the measured data is somewhat higher and corresponds more closely to the mean.
value of \( \approx 0.31 \) (from \( \{\pi/32\}^{0.5} \)). The reasoning for this is uncertain. However, our review of the effects on the distributions of maxima (see Section 4.3.3) demonstrates that the shortcomings in the theoretical distributions (including this modal position) have no significant effect on the position of the more important (from a design point of view) distribution of maxima.

### 4.3.5 Comparison of the Tails of the Normalised Crest Distributions

In Figure 4-5 the measured distributions at Cormorant A and North Cormorant are compared against the four theoretical distributions discussed in Section 4.2 (all normalised against the 1 hour sea state). Rayleigh clearly under-estimates as expected\(^{10}\). The Jahns & Wheeler and Forristall 3D partially correct this under-estimation. However, from these analyses, the Forristall 2D model clearly out-performs the alternatives at the extreme end of the distributions. At alternative sea state durations the effects on the distributions should also be taken into account (see Section 4.3.6).

![Figure 4-5 Crest Height Exceedences from 1-hour Sea States (CA and NC)](image)

\(^{10}\) The Rayleigh distribution assumes a Gaussian surface (wave troughs are equal in magnitude to crests). Forristall (2000) describes the deviation that real waves experience from this Gaussian assumption, "Real waves show a small but easily noticed departure from a Gaussian surface. The crests are higher and sharper than expected from a summation of sinusoidal waves with random phase, and the troughs are shallower and flatter. It is easy to tell by inspection whether a wave record is the right side up."

As Rayleigh is Gaussian it will, therefore, always under-estimate the wave crest height.
4.3.6 Effects of the Sea State Duration on the Normalised Distributions

The assumed duration of the sea state has an impact on the measured normalized crest height distributions. Although the measured crest heights extracted during the analyses are independent of the sea state duration, the plotting positions are affected because we are normalizing against \( H_s \) which will vary with sea state duration.

The theoretical distributions that are also functions of the individual sea state statistics (see Note 1, Section 4.3.1 above) should also, in principle, be affected by the assumed sea state duration. However, even small deviations in \( H_c/H_s \) due to sea state duration are not observed in the theoretical distributions until we reach exceeding low probability levels \(<10^{-12}\). As we may only expect approximately \( 5 \times 10^{10} \) individual waves in 10,000 years (based on an individual wave period of 6.5 s) this need not concern us here.

![Figure 4-6 Sea State Duration and Probability of Exceedence (CA only)](image)

Figures 4-6 and 4-7 confirm (Cormorant A and North Cormorant respectively) this at all probability levels that we are likely to deal with. The minimum / maximum ranges of the analysed sea state durations, 10 minutes and 3 hours, result in Forristall 2D exceedence distributions (our best model for 1 hour sea states – see Section 4.3.5) that are virtually identical.

The other theoretical distributions show the same, but offset, effect – omitted from Figures 4-6 and 4-7 for clarity.
The effect of the duration of the sea state is evident in the normalization of the measured crest heights. In the tails of the distributions the shorter sea states tend to higher $H_s$ values (i.e. higher sea states may persist for shorter periods) and hence lower $H_c/H_s$ values.

In Figures 4-6 and 4-7 the Forristall 2D distribution matches the measured distribution based on 1 hour sea states very well. This is entirely expected as Forristall based his analyses on hourly data (see Footnote #1 for reference). However, this does emphasise that we should be aware of such factors when selecting suitable distributions and dealing with normalized data.

4.3.7 Effect of Sea State Duration on the Distribution of Maxima

Figure 4-8 shows the distribution of individual waves and their corresponding distribution of maxima for a range of sea state durations. For comparison purposes only, Figure 4-8 assumes a constant number of waves (530) irrespective of the sea state duration used and we observe the divergence of the distributions of maxima at low probability level (towards the top of the non-exceedance plot). This corresponds to the divergences seen in the tails of Figures 4-6 and 4-7.

In reality a 3 hour sea state will include more waves than a 1 hour sea state etc. Figure 4-9 shows the same data as Figure 4-8 but with the distribution of maxima derived from a more representative number of waves based on a mean $T_z$ of 6.8 s (3 hr = 1590 waves, 1 hr = 530 waves, 20 min = 177 waves, 10 min = 88 waves).
Figure 4-8  Effect of Sea State Duration on Distribution of Maxima with a Constant Nz Assumption (CA)

Figure 4-9  Effect of Sea State Duration on Distribution of Maxima using a Realistic Nz (CA)
4.3.8 Effect of Higher Sea State Sub-setting on Distribution of Maxima

The ISO-compliant methodologies are storm based and as such, the effects of fitting to successively smaller upper sea state subsets (storms) have been tested. Figure 4-10 demonstrates that as we successively subset the 1 hour data at higher sea state thresholds the tail end of the distribution of individual waves has higher $H_c/H_s$ values. This effect eventually breaks down at high thresholds ($H_s > 8$ m) due to the relative paucity of data.

The effect that this has on the individual wave height distribution is more clearly seen if we examine the theoretical distributions. Figure 4-11 shows the equivalent Forristall 2D distribution of individual waves (1 hour sea states) for the same $H_s$ thresholds.

What effect does this have on the distribution of 1 hour sea state maxima? Figure 4-12 shows the distribution of maxima based on a constant $N_z$ (530 waves in 1 hour). The effect is as expected with the distribution being pushed to higher values as we subset higher sea states. However, in reality, as we subset higher sea states we experience less waves in any given sea state due to the fact that as $H_s$ increases we have, on average, a corresponding increase in $T_z$. Figure 4-13 takes this into account and demonstrates that although fitting to higher sea state subsets tends to higher maxima the effect is countered by the longer wave periods ultimately resulting in very similar distribution of maxima.
Figure 4-11  Effect of Sea State Threshold on Individual Crest Distribution (F2D)

Figure 4-12  Effect of Sea State Threshold on Distribution of Maxima with a Constant Nz Assumption
Figure 4-13  Effect of Sea State Threshold on Distribution of Maxima with a Realistic Nz (F2D)

4.4    COMPARISON OF MEASURED AND THEORETICAL MAXIMA

4.4.1   Comparison of Maximum Crest Heights in Given Sea States

The maximum wave crests in a given sea state may be obtained as either:

- The \((1-(1/N))^n\) percentile of a distribution, where \(N\) = number of individual waves
- The most probable (modal) maximum value from the distribution of maxima
- Alternative (non-modal) maximum values from the distribution of maxima
- The maximum recorded measured value

The first two of the maxima listed above should be directly comparable and this is confirmed in Tables 4-1 and 4-2 (for Cormorant A and North Cormorant data respectively). Tables 4-1 and 4-2 list regression statistics between the percentile based maximum and the maxima taken from various positions within the distribution of maxima below highlights the comparison. This is not that surprising as we are effectively comparing the theoretical distributions against themselves.

What is more useful is that the tables confirm that the method of taking a distribution of individual waves within a sea state and generating a distribution of the likely values of the maximum wave in that sea state (of which the modal 63% exceedence value is the most likely)\(^{11}\), via raising the cumulative distribution to the power of the number of waves, is entirely valid.

\(^{11}\) The 63% comes from the limit \((N \to \infty)\) of the term \(1-(1-(1/N))^N\), which is the form of our distribution of maxima. At the limit, \(1-(1-(1/N))^N = 1-(1/e) \approx 0.632\).
What we now need to test is how well the actual measured maximum crest height within a sea state matches the various theoretical values. Tables 4-3 and 4-4 summarise these findings for the Cormorant A and North Cormorant data respectively.

Tables 4-1 and 4-2 demonstrate to us that if we model the distribution of individual waves appropriately then taking the modal value of the distribution of maxima is a good estimate of our maximum crest height. However, Tables 4-3 and 4-4 show significant deviations between the observed maximum crest heights and the modal value of the distribution of maxima. The best short term model (Forristall 2D) is still providing most probable maximum crest heights that are approximately 4% to 5% low relative to our measurements. The limitation is thus in the short-term distributions used.

### Table 4-1 Comparisons of Percentile and Distribution of Maxima Cmax (Cormorant A)

<table>
<thead>
<tr>
<th>Y</th>
<th>X</th>
<th>Relationship</th>
<th>r²</th>
<th>Y/X difference as %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rayleigh</td>
<td>1-(1/N)N %ile</td>
<td>y = 1.0006 x</td>
<td>1.0000</td>
<td>+0.1%</td>
</tr>
<tr>
<td>63% exceedence</td>
<td>1-(1/N)N %ile</td>
<td>y = 1.0006 x</td>
<td>1.0000</td>
<td>+0.1%</td>
</tr>
<tr>
<td>50% exceedence</td>
<td>1-(1/N)N %ile</td>
<td>y = 1.0006 x</td>
<td>1.0000</td>
<td>+0.1%</td>
</tr>
<tr>
<td>10% exceedence</td>
<td>1-(1/N)N %ile</td>
<td>y = 1.0006 x</td>
<td>1.0000</td>
<td>+0.1%</td>
</tr>
</tbody>
</table>

### Table 4-2 Comparisons of Percentile and Distribution of Maxima Cmax (Nth. Cormorant)

<table>
<thead>
<tr>
<th>Y</th>
<th>X</th>
<th>Relationship</th>
<th>r²</th>
<th>Y/X difference as %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rayleigh</td>
<td>1-(1/N)N %ile</td>
<td>y = 1.0006 x</td>
<td>1.0000</td>
<td>+0.1%</td>
</tr>
<tr>
<td>63% exceedence</td>
<td>1-(1/N)N %ile</td>
<td>y = 1.0006 x</td>
<td>1.0000</td>
<td>+0.1%</td>
</tr>
<tr>
<td>50% exceedence</td>
<td>1-(1/N)N %ile</td>
<td>y = 1.0006 x</td>
<td>1.0000</td>
<td>+0.1%</td>
</tr>
<tr>
<td>10% exceedence</td>
<td>1-(1/N)N %ile</td>
<td>y = 1.0006 x</td>
<td>1.0000</td>
<td>+0.1%</td>
</tr>
</tbody>
</table>

Based on 17,722 100% complete 1-hour sea states

Based on 22,272 100% complete 1-hour sea states
Based on 17,722 100% complete 1-hour sea states

\[ y = mx \] (fixed at origin)

<table>
<thead>
<tr>
<th>Relationship</th>
<th>Theory compared to Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rayleigh 63% exceedence</td>
<td>Measured Cmax</td>
</tr>
<tr>
<td>50% exceedence</td>
<td>Measured Cmax</td>
</tr>
<tr>
<td>10% exceedence</td>
<td>Measured Cmax</td>
</tr>
<tr>
<td>Jahns &amp; Wheeler 63% exceedence</td>
<td>Measured Cmax</td>
</tr>
<tr>
<td>50% exceedence</td>
<td>Measured Cmax</td>
</tr>
<tr>
<td>10% exceedence</td>
<td>Measured Cmax</td>
</tr>
<tr>
<td>Forristall 2D 63% exceedence</td>
<td>Measured Cmax</td>
</tr>
<tr>
<td>50% exceedence</td>
<td>Measured Cmax</td>
</tr>
<tr>
<td>10% exceedence</td>
<td>Measured Cmax</td>
</tr>
<tr>
<td>Forristall 3D 63% exceedence</td>
<td>Measured Cmax</td>
</tr>
<tr>
<td>50% exceedence</td>
<td>Measured Cmax</td>
</tr>
<tr>
<td>10% exceedence</td>
<td>Measured Cmax</td>
</tr>
</tbody>
</table>

Table 4-3 Comparisons of Measured and Theoretical Cmax (Cormorant A)

Based on 22,272 100% complete 1-hour sea states

<table>
<thead>
<tr>
<th>Relationship</th>
<th>Theory compared to Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rayleigh 63% exceedence</td>
<td>Measured Cmax</td>
</tr>
<tr>
<td>50% exceedence</td>
<td>Measured Cmax</td>
</tr>
<tr>
<td>10% exceedence</td>
<td>Measured Cmax</td>
</tr>
<tr>
<td>Jahns &amp; Wheeler 63% exceedence</td>
<td>Measured Cmax</td>
</tr>
<tr>
<td>50% exceedence</td>
<td>Measured Cmax</td>
</tr>
<tr>
<td>10% exceedence</td>
<td>Measured Cmax</td>
</tr>
<tr>
<td>Forristall 2D 63% exceedence</td>
<td>Measured Cmax</td>
</tr>
<tr>
<td>50% exceedence</td>
<td>Measured Cmax</td>
</tr>
<tr>
<td>10% exceedence</td>
<td>Measured Cmax</td>
</tr>
<tr>
<td>Forristall 3D 63% exceedence</td>
<td>Measured Cmax</td>
</tr>
<tr>
<td>50% exceedence</td>
<td>Measured Cmax</td>
</tr>
<tr>
<td>10% exceedence</td>
<td>Measured Cmax</td>
</tr>
</tbody>
</table>

Table 4-4 Comparisons of Measured and Theoretical Cmax (North Cormorant)

4.5 THE MAXIMUM CREST AND THE HMAX WAVE

4.5.1 The Maximum Wave Height, \( H_{\text{max}} \)

As well as recording the maximum crest height in a given sea state we have recorded / generated a number of other wave parameters (see Section 3.3). One of the key wave height parameters is the maximum recorded wave height, \( H_{\text{max}} \). The recorded maximum should be, in theory, be equivalent to the modal value of the distribution of maximum wave heights.

4.5.2 The Crest Height Associated with the Maximum Wave, \( C_{\text{Hmax}} \)

Intuitively one may feel that the highest individual wave in a given sea state (the \( H_{\text{max}} \) wave) would also provide the highest crest. In fact this is only so in a 39.4% and 39.8% of the 1 hour sea states (Cormorant A and North Cormorant respectively). Figures 4-15 and 4-16 show the \( C_{\text{max}} / C_{\text{Hmax}} \) relationships for the two sites.
It would therefore be wholly inappropriate to make assumptions about the maximum crest height in a sea state based solely on the Hmax wave. In approximately 14% of the 1 hour sea states the crest height associated with the Hmax wave was less than 80% of the actual maximum crest. In ~ 0.7% of cases it was less than 60% of Cmax.

**Figure 4-14** Relationship Between Cmax and Cass_Hmax (Cormorant A)

**Figure 4-15** Relationship Between Cmax and Cass_Hmax (North Cormorant)
5. THE MAXIMUM CREST HEIGHT IN A STORM

5.1 MOST PROBABLE MAXIMUM CREST IN A STORM

A storm event may comprise several sea states. The most probable maximum crest height, \( C_{MP} \), in any given storm is derived from a combination of the crest distributions within each sea state making up the storm.

The principle of determining a distribution of maximum crest heights from a storm comprising several sea states is essentially the same as that of determining a distribution of maxima from a single sea state of a known number of individual waves.

The distribution of maxima within a storm is derived thus:

From each sea state (identified by the subscript \( i \)) contributing to the storm…..

\[
P(H_c > c | H_{s_i}) = 1 - Q_i
\]

Probability of \( H_c \) exceeding a threshold \( c \) for sea state, \( i \).

where \( Q_i \) is the preferred short term distribution.

\[
e.g. \text{for Rayleigh distributed crests}, \quad Q = \exp \left( -8 \left( \frac{H_c}{H_s} \right)^2 \right)
\]

If there are \( N_i \) individual waves in the given sea state, \( i \), then…..

\[
P(C_{\text{max}} > c | H_{s_i}) = (1 - Q_i)^{N_i} = \exp (-N_i Q_i)
\]

Probability of \( C_{\text{max}} \) exceeding a threshold \( c \) for sea state, \( i \).

Finally if all the sea states comprising the storm are numbered \(-I < i < J\), where \( i = 0 \) at the storm peak, then…..

\[
P(C_{\text{max}} > c | \text{Storm}) = \exp \sum_{i=1}^{J} (-N_i Q_i)
\]

Probability of \( C_{\text{max}} \) exceeding a threshold \( c \) in a given storm

The most probable maximum crest height in a given storm, \( C_{MP} \), is then defined as \( c \) when

\[
P(C_{\text{max}} > c \mid \text{Storm}) = 1 - 1/e = 0.63
\]

or the mode of the first derivative of \( P(C_{\text{max}} > c \mid \text{Storm}) \)

These \( C_{MP} \)'s are the basis of the Tromans & Vanderschuren ISO-complaint methodology.
5.2 COMPARISON OF THE MOST PROBABLE & MEASURED MAXIMUM CREST IN A STORM

As a test of this most fundamental ISO-compliant parameter, $C_{MP}$, the top fifty storms were extracted from each of the Cormorant A and North Cormorant datasets and derived $C_{MP}$’s for each using the methodology described in Section 5.1. These $C_{MP}$’s were then directly compared with measured maximum crest heights from each storm event.

Figure 5-1 shows the distributions of normalised $C_{max}$ and $C_{MP}$ from each of the top 100 storms (top 50 storms from each site based on a storm threshold of $H_s > 5$ m with a 48 hour peak separation). The analyses were based on the 1 hour datasets with $C_{MP}$ values being generated using the Forristall 2D short-term distributions (see Section 4.3.5 for justification).

Figure 5-1 demonstrates that the theoretical $C_{MP}$ values derived for each storm do indeed match (at least in majority of cases from our arbitrarily selected 0.05 m bins) the modal value of the real distribution of measured $C_{max}$. Of course, this does not mean that $C_{MP}$ is necessarily the best crest height to use; just that it is approximately what it says. The difference between the top $C_{MP}/H_s$ and top $C_{max}/H_s$ is approximately 0.35 which, in a top sea state, say $H_s = 11$ m, would correspond to a crest difference of just under 4 m.

Figures 5-2 and 5-3 (overleaf) show the detail of the $C_{max}$ and $C_{MP}$ values for Cormorant A and North Cormorant respectively. These figures contain analyses for two storm thresholds ($H_s > 5$ m and 6 m) for comparison.

---

12 Normalised against the maximum value of $H_s$ in that particular storm.
Figure 5-2 Comparison of Cmax and Cmp for Top 50 Storms (Cormorant A)
Figure 5-3 Comparison of Cmax and Cmp for Top 50 Storms (North Cormorant)
Figure 5-4(a,b)  Cmax vs Cmp for Top 50 Cormorant A Storms at Hs > 5 m (left) and 6 m (right)

\begin{align*}
\text{y} &= 0.9812x + 0.5619 \\
R^2 &= 0.8143
\end{align*}

\begin{align*}
\text{y} &= 0.9825x + 0.5552 \\
R^2 &= 0.8241
\end{align*}
Figure 5-5(a,b)  Cmax vs Cmp for Top 50 North Cormorant Storms at Hs > 5 m (left) and 6 m (right)
In Figures 5-2 and 5-3 the $C_{MP}$ values for the top storms are obviously the same. Deviations from the two thresholds appears as our storm $C_{MP}$’s and Cmax’s approach our selected threshold and we effectively look at different storms (samples of time).

Figures 5-4 (a,b) and 5-5 (a,b) show the corresponding least squares regressions between Cmax and $C_{MP}$ for the two sites and for the two storm thresholds. These figures show Cmax consistently exceeds $C_{MP}$ (by up to 3.2 m, North Cormorant 01/02/2008 08:00 hrs). This is the expected short-term variability and shows that even in very short datasets we should expect to find several Cmax values from the extreme tails of the distribution of maxima.
6. IMPACTS ON THE ISO-COMPLIANT METHODOLOGIES

6.1 THE PRINCIPAL ISO-COMPLIANT METHODS

6.1.1 Borgman Integral

The Borgman integral is an elegant mathematical technique for convolving the short and long-term probability distributions. A fuller description of the method is provided in an HSE Research Report\textsuperscript{13} and in the original papers of Borgman (1973)\textsuperscript{14} and Krogstad (2004)\textsuperscript{15}.

6.1.2 Tromans and Vanderschuren

This is a storm-based approach which, according to ISO 19901-1, is to be preferred. However, the method includes a number of user-defined variables such that, unless very tightly specified, two experienced analysts are unlikely to produce the same result. A fuller description of the method is provided in an HSE Research Report\textsuperscript{13} and in the original paper of Tromans and Vanderschuren (1995)\textsuperscript{16}.

6.1.3 Monte Carlo (Crest Only Mode)

In these analyses, we are ignoring the advantages of the Monte Carlo technique to incorporate other parameters, such as tide and surge, and concentrate solely on generating crest heights. In it’s crest only mode the Monte Carlo technique does the following:

- Generate a number (the number required to fill our required return period) random sea states (defined by Hs) from a given long-term distribution.
- For each sea state and an assumed short-term distribution, pick N random individual waves and record the maximum value.
- Then record the maximum of all the individual sea state maxima within the required return period. This provides a single extreme maxima.
- Run all of the above with a large number of iterations and thus generate a distribution of the extreme maxima.
- Record the ‘most probable’ (or otherwise) extreme maxima.

---


6.2 THE ISO-COMPLIANT RESULTS

6.2.1 Borgman / Krogstad Results

As the Borgman / Krogstad methods are based on all sea states, via the specified long-term significant wave height, via the Weibull scale, shape and location parameters, it is important to assess the sensitivity and consistency of these to the fitted data.

The Weibull function was fitted to the upper 95% and 30% of the 1-hr and 3-hr data. Due to the relative paucity of the measured data used in these analyses (< 3 years), fitting further into the tail proved problematic. The results of the Weibull long-term fitting are summarised below:

![Weibull Fits to Data (1 hour Sea States)](image1)

![Weibull Fits to Data (3 hour Sea States)](image2)

Figure 6-1 Comparison of Long-term Weibull Fits to Data (1-hr and 3-hr Sea States)
In Figure 6-1 the 1-hr analyses show more consistent results than the 3-hr. The 3-hr Cormorant A data show significant deviations (particularly the 30% tail fit) from the alternative fits and, therefore, these should be treated with some caution. The differences in the Weibull parameters emphasise this.

Corresponding EVA plots given in Appendix B

<table>
<thead>
<tr>
<th>Site</th>
<th>Duration</th>
<th>Tail</th>
<th>Shape</th>
<th>Scale</th>
<th>Location</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cormorant A</td>
<td>1-hr</td>
<td>95%</td>
<td>1.4325</td>
<td>2.3263</td>
<td>0.6884</td>
<td>0.9988</td>
</tr>
<tr>
<td>Cormorant A</td>
<td>1-hr</td>
<td>30%</td>
<td>1.5646</td>
<td>2.7619</td>
<td>0.1468</td>
<td>0.9928</td>
</tr>
<tr>
<td>Cormorant A</td>
<td>3-hr</td>
<td>95%</td>
<td>1.3778</td>
<td>2.2491</td>
<td>0.6645</td>
<td>0.9991</td>
</tr>
<tr>
<td>Cormorant A</td>
<td>3-hr</td>
<td>30%</td>
<td>1.2095</td>
<td>1.7767</td>
<td>1.1734</td>
<td>0.9955</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Site</th>
<th>Duration</th>
<th>Tail</th>
<th>Shape</th>
<th>Scale</th>
<th>Location</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>North Cormorant</td>
<td>1-hr</td>
<td>95%</td>
<td>1.5153</td>
<td>2.5186</td>
<td>0.5744</td>
<td>0.9992</td>
</tr>
<tr>
<td>North Cormorant</td>
<td>1-hr</td>
<td>30%</td>
<td>1.5548</td>
<td>2.6071</td>
<td>0.5091</td>
<td>0.9977</td>
</tr>
<tr>
<td>North Cormorant</td>
<td>3-hr</td>
<td>95%</td>
<td>1.5153</td>
<td>2.5115</td>
<td>0.5809</td>
<td>0.9988</td>
</tr>
<tr>
<td>North Cormorant</td>
<td>3-hr</td>
<td>30%</td>
<td>1.5480</td>
<td>2.5712</td>
<td>0.5538</td>
<td>0.9957</td>
</tr>
</tbody>
</table>

Table 6-1 Weibull Parameters Derived from Least Squares Fits

The effect of these parameter variations on the Borgman / Krogstad estimation of the extreme maximum crest height are presented below (based on a Forristall-2D short-term crest distribution):

<table>
<thead>
<tr>
<th>Site</th>
<th>Duration</th>
<th>Tail</th>
<th>Hc_2.0-yrs</th>
<th>Hc_100-yrs</th>
<th>Hc_10K-yrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cormorant A</td>
<td>1-hr</td>
<td>95%</td>
<td>13.09</td>
<td>16.75</td>
<td>21.37</td>
</tr>
<tr>
<td>Cormorant A</td>
<td>1-hr</td>
<td>30%</td>
<td>13.00</td>
<td>16.60</td>
<td>20.88</td>
</tr>
<tr>
<td>Cormorant A</td>
<td>3-hr</td>
<td>95%</td>
<td>13.44</td>
<td>17.30</td>
<td>22.06</td>
</tr>
<tr>
<td>Cormorant A</td>
<td>3-hr</td>
<td>30%</td>
<td>13.83</td>
<td>18.17</td>
<td>23.56</td>
</tr>
<tr>
<td>Measured Cmax</td>
<td></td>
<td></td>
<td>12.90</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Site</th>
<th>Duration</th>
<th>Tail</th>
<th>Hc_2.6-yrs</th>
<th>Hc_100-yrs</th>
<th>Hc_10K-yrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>North Cormorant</td>
<td>1-hr</td>
<td>95%</td>
<td>13.13</td>
<td>16.45</td>
<td>20.75</td>
</tr>
<tr>
<td>North Cormorant</td>
<td>1-hr</td>
<td>30%</td>
<td>13.03</td>
<td>16.29</td>
<td>20.46</td>
</tr>
<tr>
<td>North Cormorant</td>
<td>3-hr</td>
<td>95%</td>
<td>13.10</td>
<td>16.42</td>
<td>20.70</td>
</tr>
<tr>
<td>North Cormorant</td>
<td>3-hr</td>
<td>30%</td>
<td>12.98</td>
<td>16.23</td>
<td>20.36</td>
</tr>
<tr>
<td>Measured Cmax</td>
<td></td>
<td></td>
<td>13.15</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 6-2 Borgman / Krogstad Maximum Crest Heights Compared to Observed Maxima

Comparisons of the predicted maxima crest heights against the observed values for such short datasets is slightly questionable. However, the results are reassuringly good considering the data limitations. The 10,000 year results from North Cormorant are very consistent (as expected considering the similarity of the Weibull parameters). The Cormorant A results reflect the uncertainty in the Weibull fitting.
6.2.2 Tromans and Vanderschuren Results

The Tromans and Vanderschuren analyses are highly sensitive to the storm picking routines employed. The best procedures to use are still an area on ongoing research and the methods used here have been based on best practice (with some compromises – see below) at the time of writing.

The procedure used in these analyses was a compromise (due to the brevity of our datasets) between obtaining enough ‘most probable’ maximum crest heights, \( C_{MP} \)'s, but not too many storms in a single year that would potentially violate the independent nature of each event.

The following settings were used for each of the analyses:

- Storm selection: \( H_s > 6 \) m with a 48 hour peak separation
- Short-term crest distribution: Forristall-2D
- Site specific water depths: \( CA = 140 \) m; \( NC = 160 \) m
- Target Probability of \( C_{MP} \): 36.79% non-exceedence (63.21% exceedence)
- Selected \( C_{MP} \) Distribution: 3 parameter Weibull (100% of data)
- Fitting Type: NonLeastSquaresLin
- Plotting Position: Weibull
- Probability Types: POT (Peaks Over Threshold)
- Number of waves in a random storm: 5000; \( [\mu = \ln(5000) = 8.5] \)
  (Assuming a storm typically last ~ 9 hours, with an individual wave period of ~ 6.5 s)

Table 6-3 summarises our storm cut-off thresholds and the resulting number of storms per year. Appendix C contains the Weibull distributions of the \( C_{MP} \)'s and the Long/Short-term Convolved distribution of maxima.

Table 6-3 Tromans and Vanderschuren Maximum Crest Heights Compared to Observed Maxima

<table>
<thead>
<tr>
<th>Site</th>
<th>Duration</th>
<th>Storm Threshold</th>
<th>Storms per Year</th>
<th>( H_c )_2.0-yrs</th>
<th>( H_c )_100-yrs</th>
<th>( H_c )_10K-yrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cormorant A</td>
<td>1-hr</td>
<td>( H_s &gt; 6 ) m</td>
<td>23.5</td>
<td>12.15</td>
<td>15.73</td>
<td>19.43</td>
</tr>
<tr>
<td>Cormorant A</td>
<td>3-hr</td>
<td>( H_s &gt; 6 ) m</td>
<td>23.0</td>
<td>12.27</td>
<td>16.23</td>
<td>20.51</td>
</tr>
<tr>
<td>Measured Cmax</td>
<td></td>
<td></td>
<td></td>
<td>12.90</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Site</th>
<th>Duration</th>
<th>Storm Threshold</th>
<th>Storms per Year</th>
<th>( H_c )_2.6-yrs</th>
<th>( H_c )_100-yrs</th>
<th>( H_c )_10K-yrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>North Cormorant</td>
<td>1-hr</td>
<td>( H_s &gt; 6 ) m</td>
<td>25.0</td>
<td>12.27</td>
<td>15.50</td>
<td>19.50</td>
</tr>
<tr>
<td>North Cormorant</td>
<td>3-hr</td>
<td>( H_s &gt; 6 ) m</td>
<td>21.9</td>
<td>12.12</td>
<td>15.18</td>
<td>18.61</td>
</tr>
<tr>
<td>Measured Cmax</td>
<td></td>
<td></td>
<td></td>
<td>13.15</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 6-3 shows the TVM results are rather low relative to the observed maxima. However, considerable variability in the results may be introduced into this technique during storm selection. Reviewing a rather subjective method such as TVM using so few measured data may be testing the credibility of the analyses.

Corresponding EVA plots given in Appendix C
6.2.3 Monte Carlo Results

In common with the Borgman / Krogstad integration, the Monte Carlo simulations require the Weibull parameter pre-requisites.

Despite this, the Monte Carlo results are low relative to the Borgman runs. Tucker and Pitt\textsuperscript{17} provide a reason behind the differences, namely that the Monte Carlo runs only extract a single highest crest from each sea state whereas the Borgman / Krogstad technique effectively takes all the individual waves forward. In any particular run of, say, 100 years, significant wave heights in excess of 100 years occur rarely but when they do, there may be many very large individual waves. The Monte Carlo technique ignores all but the highest individual wave.

Whether we should take just the highest wave in a sea state or all of the individual waves forward depends entirely on the type of analyses we wish to perform.

<table>
<thead>
<tr>
<th>Site</th>
<th>Duration</th>
<th>Tail</th>
<th>Hc_2.0-yrs</th>
<th>Hc_100-yrs</th>
<th>Hc_10K-yrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cormorant A</td>
<td>1-hr</td>
<td>95%</td>
<td>12.56</td>
<td>16.44</td>
<td>19.98</td>
</tr>
<tr>
<td>Cormorant A</td>
<td>1-hr</td>
<td>30%</td>
<td>12.38</td>
<td>16.27</td>
<td>19.21</td>
</tr>
<tr>
<td>Cormorant A</td>
<td>3-hr</td>
<td>95%</td>
<td>12.56</td>
<td>16.76</td>
<td>21.09</td>
</tr>
<tr>
<td>Cormorant A</td>
<td>3-hr</td>
<td>30%</td>
<td>12.94</td>
<td>17.49</td>
<td>22.88</td>
</tr>
<tr>
<td>Measured Cmax</td>
<td></td>
<td></td>
<td>12.90</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Site</th>
<th>Duration</th>
<th>Tail</th>
<th>Hc_2.6-yrs</th>
<th>Hc_100-yrs</th>
<th>Hc_10K-yrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>North Cormorant</td>
<td>1-hr</td>
<td>95%</td>
<td>12.66</td>
<td>16.13</td>
<td>18.96</td>
</tr>
<tr>
<td>North Cormorant</td>
<td>1-hr</td>
<td>30%</td>
<td>12.56</td>
<td>15.92</td>
<td>18.33</td>
</tr>
<tr>
<td>North Cormorant</td>
<td>3-hr</td>
<td>95%</td>
<td>12.35</td>
<td>15.71</td>
<td>18.58</td>
</tr>
<tr>
<td>North Cormorant</td>
<td>3-hr</td>
<td>30%</td>
<td>12.31</td>
<td>15.60</td>
<td>18.12</td>
</tr>
<tr>
<td>Measured Cmax</td>
<td></td>
<td></td>
<td>13.15</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**Table 6-4 Monte Carlo Maximum Crest Heights Compared to Observed Maxima**

6.2.4 Comparison of ISO-compliant Results

Figure 6-2 compares the results of the ISO-compliant methodologies alongside alternate values, namely:

- **Measured Maxima**
  The actual measured maximum crest in the 2.0 year Cormorant A and the 2.6 year North Cormorant datasets. The crest heights are relative to their local 5-minute still water level.

• **Borgman / Krogstad**  
The average Borgman / Krogstad crest heights each derived from the long-term Cormorant A and North Cormorant derived Weibull Parameters (1 hour) and a short-term Forristall 2D crest distribution.

• **Monte Carlo**  
The average Monte Carlo crest heights each derived from the long-term Cormorant A and North Cormorant derived Weibull Parameters (1 hour) and a short-term Forristall 2D crest distribution.

• **Tromans and Vanderschuren**  
The average Tromans and Vanderschuren crest heights each derived from the 1 hour Cormorant A and North Cormorant datasets using the methodology described in Section 6.2.2.

• **Extrapolation of Measured Maxima**  
The average of the Weibull (95%) extrapolations of the measured 1 hour Cmax values.

• **Shell EP Europe Ratio Method**  
The Shell EP Europe ratios applied to the 100-year Hs. For comparison purposes, this is the 100-year Hs, derived from the 2.0 and 2.6-year study datasets, and not Shell’s official 100-year design criteria of 15.6 m.

  Average 100-year Hs from study datasets = 14.94 m

  - 100-year crest height = 100-year Hs × 1.10
  - 10K-year crest height = 100-year Hs × 1.42

Based on our rather limited datasets the results fall into two main groups. The Monte Carlo, Tromans & Vanderschuren and direct extrapolation of measured crest agree favourably, tending to converge at the 10,000 year return. The Borgman / Krogstad method results in relatively high extreme values. The Shell ratios match the Borgman / Krogstad results in these analyses even though they were calibrated from Tromans & Vanderschuren analyses.

The comparisons with measured maxima appear to suggest the Borgman / Krogstad methodology fits best. However, the variation at such short return periods is large and to conclude anything of statistical significance from this would be quite unsatisfactory.

Figure 6-2 demonstrates the results of fundamentally different techniques, in that we are dealing with single maxima extracted from each sample (sea state or storm) OR we are dealing with all individual waves. Forristall\(^\text{18}\) concludes that the “correct” methodology depends entirely on the design requirements. Our findings (Figure 6-2) would tend to concur with that.

Figure 6-2 Comparison of Extreme Wave Crest Heights
7. MAXIMUM CREST EVENTS

Appendices D and E contain 10 minute series of the raw (2 Hz) wave data that contain the top 12 crest events from Cormorant A and North Cormorant respectively. These images are inverted and represent radar to sea surface distances.

Please note that the ordinate in these plots is automatically scaled and therefore not constant. The abscissa is a fixed 10 minutes.

8. INDIVIDUAL WAVE PERIODS

This section, though not strictly part of the ISO methodology testing, is included here as an interesting note on the individual wave heights and periods extracted during these analyses. Significant uncertainties exist in determining design wave periods and the availability of raw data is vital in a better understanding of this area of research.

Figures 8-1 and 8-2 compare the top \( H > 10 \text{ m} \) individual wave height and periods (for Cormorant A and North Cormorant respectively) against their current design criteria\(^{19}\).

Mean zero-crossing period design ranges are based on sea state steepness of 1/12 and 1/18 for sea states in excess of 12.5 m\(^{20}\). The ranges of significant steepness are based on the 5\(^{th}\) and 95\(^{th}\) percentiles of \( T_z \) as a function of \( H_s \) for local long-term datasets.

Wave periods associated with the \( H_{\text{max}} \) waves, \( T_{\text{ass}} \) are derived from a median \( T_z \) (based on a significant steepness of 1/15) and established\(^{21}\) recommended \( T_{\text{ass}}/T_z(\text{median}) \) ratio ranges of 1.14 and 1.41.

The plots demonstrate that the upper individual waves are well modelled by the current derivations within our usual working ranges. However, current methods of deriving \( T_z \) (and ultimately \( T_{\text{ass}} \)) from wave steepness have inherent problems. As we move to higher sea states, the ranges broaden when intuitively they should be converging to (ultimately) a physically determined, maximum single wave with one height, period and steepness.

---

\(^{19}\) Extrapolated to shorter return periods to coincide with the relatively short observation periods of the measured individual waves.

\(^{20}\) Based on Shell EP Europe’s Northern North Sea Engineering Reference Document analyses (EN079) and summarised in a May 2007 update, DEP 37.00.10.10 EPE.

\(^{21}\) Metocean Parameters – Wave Parameters, Department of Energy Offshore Technology Report, OTH 89-300
Figure 8-1  Top Individual Waves Compared to Design Criteria (Cormorant A)

Figure 8-2  Top Individual Waves Compared to Design Criteria (North Cormorant)
9. OVERVIEW

The observed shortcomings in the various short-term theoretical distributions appear to have a relatively small effect on the theoretical distribution of maxima. However, on the basis of these relatively limited datasets, these analyses do suggest that the modal values of the maxima underestimated the actual (measured) maxima. In the majority of analysed storms the measured maximum was closer to the mean (or median if we wish to discuss it in terms of exceedences) of the theoretical distribution of maxima.

A lot of previous effort has gone into trying to either get the various ISO-compliant methodologies to converge or to explain the differences. These analyses show the Borgman / Krogstad approach to produce relatively high extreme values. This is, in hindsight, expected due the method processing all individual waves in any given sea state (as discussed by Tucker and Pitt (2001) and Forristall (2006)). It therefore takes account of the $2^{nd}$ and $3^{rd}$ etc. highest individual waves which in high sea states may be of importance, depending in the engineering requirements.

The usefulness of comparing observed maxima against the various theoretical estimates is somewhat limited due to the natural short-term variability. This limit is particularly evident with such short datasets.
10. APPENDIX A: SHORT-TERM CREST EQUATIONS

10.1 RAYLEIGH CREST DISTRIBUTION

Probability of a crest height, $H_c$, exceeding a threshold, $c$, for a given $H_s$ sea state….

$$P(H_c > c \mid H_s)_{\text{RAYLEIGH}} = \exp \left(-8 \left(\frac{H_c}{H_s}\right)^2\right)$$  \[A1\]

Probability distribution of a crest height, $H_c$, for a given $H_s$ sea state….

$$P(H_c \mid H_s)_{\text{RAYLEIGH}} = \left(16 \left(\frac{H_c}{H_s}\right)\right) \exp \left(-8 \left(\frac{H_c}{H_s}\right)^2\right)$$  \[A2\]

10.2 FORRISTALL 2D / 3D CREST DISTRIBUTIONS

Probability of a crest height, $H_c$, exceeding a threshold, $c$, for a given $H_s$ sea state….

$$P(H_c > c \mid H_s)_{\text{FORRISTALL}} = \exp \left(-\left(\frac{H_c}{\alpha H_s}\right)^\beta\right)$$  \[A3\]

This is a form of equation \[A1\] where $\alpha$ and $\beta$ are now functions of the wave steepness and the Ursell number (both measures of the non-linearity of the waves).

Forristall (2000) defines $\alpha$ and $\beta$ thus for his 2D and 3D formulations:

$$\begin{align*}
\alpha_2 &= 0.3536 + (0.2892 \times S_1) + (0.1060 \times Ur) \\
\beta_2 &= 2 - (2.1597 \times S_1) + (0.0968 \times Ur^2)
\end{align*}$$

$$\begin{align*}
\alpha_3 &= 0.3536 + (0.2568 \times S_1) + (0.0800 \times Ur) \\
\beta_3 &= 2 - (1.7912 \times S_1) - (0.5302 \times Ur) + (0.2840 \times Ur^2)
\end{align*}$$

where

$$S_1 = \frac{2 \pi H_s}{g T_1^2} \quad \text{and} \quad Ur = \frac{H_s}{k_1^2 d^3} \quad k_1 = \text{the wave number,} \quad \frac{1}{T_1}$$

As depth $\to \infty$, $\alpha = \frac{1}{\sqrt{8}} = 0.3536$ and $\beta = 2$ and equation \[A3\] reverts to \[A1\]

Probability distribution of a crest height, $H_c$, for a given $H_s$ sea state….

$$P(H_c \mid H_s)_{\text{FORRISTALL}} = \frac{\beta}{\alpha} \left(\frac{H_c}{\alpha H_s}\right)^{\beta-1} \exp \left(-\left(\frac{H_c}{\alpha H_s}\right)^\beta\right)$$  \[A4\]
If for simplicity we let $\psi$ represent the Jahns and Wheeler non-linear transformation term

$$
\psi = \left[1 - \left(4.37 \left(\frac{H_c}{d}\right) \left(0.57 - \left(\frac{H_c}{d}\right)\right)\right)\right]
$$

where

$d = \text{the water depth}$

and the coefficients 4.37 and 0.57 are from Haring and Heideman (1978)

then….

Probability of a crest height, $H_c$, exceeding a threshold, $c$, for a given $H_s$ sea state….

$$
P(H_c > c \mid H_s)_{J \& W} = \exp \left(-8 \left(\frac{H_c}{H_s}\right)^2 \psi \right)
$$

Probability distribution of a crest height, $H_c$, for a given $H_s$ sea state….

$$
P(H_c \mid H_s)_{J \& W} = \left(16 \psi \left(\frac{H_c}{H_s}\right)\right) \exp \left(-8 \psi \left(\frac{H_c}{H_s}\right)^2 \right)
$$
11. APPENDIX B: WEIBULL PLOTS USED IN ISO ANALYSES

EVA Plot
Extreme Value Analysis Plot

TVM Storm Picking

Primary Data Table
Data Source: WUC18I1_RHE_BHE Raw Data/Processed Data/Processed Data/Processed Data
Lastest Processed: Subsets 19 vs. 2000
WUC18I1_RHE_BHE_Processed Data.csv

Time Series Table Converter
Raw Count: 17722
Frequency Distribution:
Source Raw Count: 17722
Class Set: Ns

Weibull Analysis
Marginal Probability: Manual
Marginal Probability Mode: Stationary Interval: 1.0000
Minimum Probability: 0.0000
Probability Fitting Type: Linear Least Squares
R-Squared: 0.9990
Fit Location: True
Location Parameter: 0.0000
Shape Parameter: 1.4325
Scale Parameter: 2.3030

<table>
<thead>
<tr>
<th>Return Period</th>
<th>Probability of Non-Exceedance</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.996666666223</td>
<td>11.54</td>
</tr>
<tr>
<td>2</td>
<td>0.99999992614</td>
<td>12.11</td>
</tr>
<tr>
<td>2.6</td>
<td>0.99999961242</td>
<td>12.92</td>
</tr>
<tr>
<td>100</td>
<td>0.9999999992</td>
<td>15.14</td>
</tr>
<tr>
<td>10000</td>
<td>0.99999999999</td>
<td>18.28</td>
</tr>
</tbody>
</table>

EVA Plot
Extreme Value Analysis Plot

TVM Storm Picking

Primary Data Table
Data Source: WUC18I1_RHE_BHE Raw Data/Processed Data/Processed Data/Processed Data
Lastest Processed: Subsets 19 vs. 2000
WUC18I1_RHE_BHE_Processed Data.csv

Time Series Table Converter
Raw Count: 17722
Frequency Distribution:
Source Raw Count: 17722
Class Set: Ns

Weibull Analysis
Marginal Probability: Manual
Marginal Probability Mode: Stationary Interval: 1.0000
Minimum Probability: 0.0000
Probability Fitting Type: Linear Least Squares
R-Squared: 0.9990
Fit Location: True
Location Parameter: 0.0000
Shape Parameter: 1.5666
Scale Parameter: 2.7019

<table>
<thead>
<tr>
<th>Return Period</th>
<th>Probability of Non-Exceedance</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.996666666223</td>
<td>11.46</td>
</tr>
<tr>
<td>2</td>
<td>0.99999992614</td>
<td>12.00</td>
</tr>
<tr>
<td>2.6</td>
<td>0.99999961242</td>
<td>12.21</td>
</tr>
<tr>
<td>100</td>
<td>0.9999999992</td>
<td>14.85</td>
</tr>
<tr>
<td>10000</td>
<td>0.99999999999</td>
<td>17.84</td>
</tr>
</tbody>
</table>
### EVA Plot

**Extreme Value Analysis Plot**

**TVM Storm Picking**

**Primary Data Table**

Data Source: WA0180_99.9% Storm Data

- **Frequency Distribution:** Gumbel
- **Return Period:** 75 yr
- **Location Parameter:** 0.9500
- **Shape Parameter:** 1.5715
- **Scale Parameter:** 2.5712

<table>
<thead>
<tr>
<th>Return Period</th>
<th>Probability of Non-Exceedance</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0000000020</td>
<td>11.75</td>
</tr>
<tr>
<td>2</td>
<td>0.0000003914</td>
<td>11.99</td>
</tr>
<tr>
<td>2.6</td>
<td>0.0000009242</td>
<td>12.06</td>
</tr>
<tr>
<td>100</td>
<td>0.0000009902</td>
<td>14.70</td>
</tr>
<tr>
<td>10000</td>
<td>0.0000009998</td>
<td>17.68</td>
</tr>
</tbody>
</table>

---

### EVA Plot

**Extreme Value Analysis Plot**

**TVM Storm Picking**

**Primary Data Table**

Data Source: WA0180_99.9% Storm Data

- **Frequency Distribution:** Gumbel
- **Return Period:** 75 yr
- **Location Parameter:** 0.9500
- **Shape Parameter:** 1.5715
- **Scale Parameter:** 2.5712

<table>
<thead>
<tr>
<th>Return Period</th>
<th>Probability of Non-Exceedance</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0000000020</td>
<td>11.75</td>
</tr>
<tr>
<td>2</td>
<td>0.0000003914</td>
<td>11.99</td>
</tr>
<tr>
<td>2.6</td>
<td>0.0000009242</td>
<td>12.06</td>
</tr>
<tr>
<td>100</td>
<td>0.0000009902</td>
<td>14.70</td>
</tr>
<tr>
<td>10000</td>
<td>0.0000009998</td>
<td>17.68</td>
</tr>
</tbody>
</table>
12. APPENDIX C: TVM PLOTS

Cormorant A (1-hr TVM Analysis)
### North Cormorant (1-hr TVM Analysis)

<table>
<thead>
<tr>
<th>Return Period</th>
<th>Probability of Non-Exceedance</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.99960000000</td>
<td>12.64</td>
</tr>
<tr>
<td>100</td>
<td>0.99966000000</td>
<td>14.28</td>
</tr>
<tr>
<td>1000</td>
<td>0.99996000000</td>
<td>16.71</td>
</tr>
<tr>
<td>10000</td>
<td>0.99999600000</td>
<td>17.02</td>
</tr>
</tbody>
</table>

### Data Table

**Data Source:** WA:WCB, WA:WSS, WA:WSE - Raw Data/Statistical Results NA:Rigup Linh

<table>
<thead>
<tr>
<th>Return Period</th>
<th>Probability of Non-Exceedance</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.986999004</td>
<td>12.01</td>
</tr>
<tr>
<td>2.6</td>
<td>0.9868109672</td>
<td>12.27</td>
</tr>
<tr>
<td>10</td>
<td>0.9998572369</td>
<td>15.60</td>
</tr>
<tr>
<td>1000</td>
<td>0.9999977769</td>
<td>19.50</td>
</tr>
</tbody>
</table>
Cormorant A (3-hr TVM Analysis)
North Cormorant (3-hr TVM Analysis)
13. APPENDIX D: TOP CREST EVENTS (CORMORANT A)

Cormorant A : Cmax = 12.90 m, 1 hr sea state ending 24/12/2002 12:00 hrs (inverted)

Cormorant A : Cmax = 12.33 m, 1 hr sea state ending 25/01/2008 10:00 hrs (inverted)
Cormorant A: Cmax = 11.03 m, 1 hr sea state ending 24/12/2002 16:00 hrs (inverted)

Cormorant A: Cmax = 11.00 m, 1 hr sea state ending 24/12/2002 20:00 hrs (inverted)
Cormorant A: $C_{\text{max}} = 10.85$ m, 1 hr sea state ending 24/12/2002 10:00 hrs (inverted)

Cormorant A: $C_{\text{max}} = 10.83$ m, 1 hr sea state ending 25/01/2008 09:00 hrs (inverted)
14. APPENDIX E: TOP CREST EVENTS (NTH. CORMORANT)

North Cormorant: $C_{\text{max}} = 13.15$ m, 1 hr sea state ending 01/02/2008 08:00 hrs (inverted)

North Cormorant: $C_{\text{max}} = 12.02$ m, 1 hr sea state ending 01/01/2004 13:00 hrs (inverted)
North Cormorant: Cmax = 11.83 m, 1 hr sea state ending 01/01/2004 06:00 hrs (inverted)

North Cormorant: Cmax = 11.80 m, 1 hr sea state ending 01/01/2004 08:00 hrs (inverted)
North Cormorant: $C_{\text{max}} = 11.47$ m, 1 hr sea state ending 04/12/2001 05:00 hrs (inverted)

North Cormorant: $C_{\text{max}} = 11.23$ m, 1 hr sea state ending 01/01/2004 04:00 hrs (inverted)
Testing of ISO-compliant extreme water level calculations using raw wave data

This report describes a study of raw (2 Hz) wave data with relevance to ISO-compliant derivations of extreme crest elevation and extreme water level.

This report and the work it describes were funded by the Health and Safety Executive (HSE). Its contents, including any opinions and/or conclusions expressed, are those of the author alone and do not necessarily reflect HSE policy.