Evaluating the performance and effectiveness of ladder stability devices. Technical annexe. RSU Ref: 4343/R53.187

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This annexe contains the technical and mathematical background supporting the development of the stability predictor models discussed in the main report. It is provided as a separate annexe since the content is highly specialised and is intended for use by engineering practitioners rather than the general dissemination of the main report. Elements of the main report are duplicated here to enable this annexe to be read as a stand-alone document, although the specific conclusions and recommendations are omitted. This report and the work it describes were funded by the Health and Safety Executive. Its contents, including any opinions and/or conclusions expressed, are those of the author(s) alone and do not reflect HSE policy.

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1.0 TECHNICAL SECTION

This project aims to establish an understanding of leaning ladder stability when used in conjunction with additional devices fitted for the purpose of either extra usability or enhanced user safety, a system referred to as a Device Augmented Ladder (DAL). The process undertaken to achieve this is a natural extension to the author's earlier work reviewing the stability and performance of stepladders. This work is intended to collectively identify and define potential modes of performance failure of three major classes of ladder configuration – together being ladders, DALs, and large Tripods. In doing this it is necessary to consider the mechanical physics involved, and formally model each system as stability prediction algorithms.

These algorithms take a key set of specific system measures from any particular configuration, and produce normalised indices which clearly demonstrate the prevailing stability status. Central to the concept is the employment of a Standard Load Vector (SLV) and an Applied Load Point (ALP). This is a parametric load which, when applied at the magnitudes and locations defined, will duress a ladder at a qualified maximal level, and will push the ladder system similarly towards a stability limit. A large section of the work involves the practical determination of the correct SLV and ALP through extensive user trials involving reasonable but demanding tasks and this is detailed in Section 8 of the main report.

The theoretical models are designed to be capable of correct predictive representations of real world ladder configurations, with high duress users. The models are not prescriptive in that no constraint is placed on any particular constructional parameter, rather they define containment envelopes within which any ladder is qualified as adequate with regard to stability. They will allow designers or testers to conceptually manipulate a system and generate a range of optimisations, and to better understand the relationships practically governing ladder and device performance. Theoretical predictions are directly supported by practical workshop loading regimes, and straightforward pass-fail tests are described which should tally with the underlying model.

The effect of fitting any auxiliary device to an otherwise simple or nominal ladder, will in general shift the baseline stability index values to new levels.

This will arise from either geometrical rearrangement of the active grounding locations, modifications to friction limit indices at the active grounding locations, or facilitated gross positional adjustment of the user causing a shift in the ALP. Such devices could be intended to enhance usability during certain tasks, or to improve safety. The modelling algorithm is unconcerned with the intended purpose of any device, and will simply indicate the level of stability safety impartially. It should be fully realised that an arbitrary device may well improve stability in one mode, whilst also eroding the same in another mode.

At the most basic level it is essential that the act of fitting or implementing some device shall not reduce one or other safety margins below or outside the critical safety envelope. Provided this is assured, then a device should be permissible. Where safety enhancement is claimed by a manufacturer, then the additional requirement of measurable increase in stability index should be demonstrable.

The general objectives are:

- To provide a sound analysis of ladder stability
- To identify and explain the crucial issues
- To generate adequate mathematical tools to predict, measure and optimise the prevailing safety envelopes of the ladder classes DAL and Tripod.

The core of this process is standard mechanical dynamics and the theories of structures, hence the process should be transparent and unequivocal. The subtlety however is in isolating the correct magnitude of the SLV and the associated applied Load Point ALP. These are proposed as universal parameters and have great bearing on the theoretical and practical performance of a ladder. If these are set high then designers are more constrained, or utility is wasted. If set low then clearly there is erosion of real safety levels. Care is therefore exercised to pitch these standard values correctly at statistically valid levels.

The main additional elements can be identified thus:

- The development of a proposed classification system for DAL devices. The discussion of the performance of various arrangements, highlighting the most pertinent kinematic and dynamic functionality.
- The reasoning and utility of the manual footing of leaning ladders are evaluated and the effect of this strategy is extensively measured, including the different footing techniques.

- The effect of ground slope on the performance duress on the leaning ladder is examined and the benefit of devices intended to address this problem is evaluated.

A large database of useful data is generated, generally quantifying dynamic activity on the laboratory reference apparatus. A large pool of information pertaining to high shock loading during the mounting phase is developed but not used in the greater analysis, as this is not a stability issue. This will have significance, however, regarding strength and durability of components, and is therefore available for later contribution to such usage if appropriate.

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2.0 DYNAMOMETER RIG

Figure 1 shows the principle on which the dynamometer rig is based, and Figure 2 the mechanical geometry.

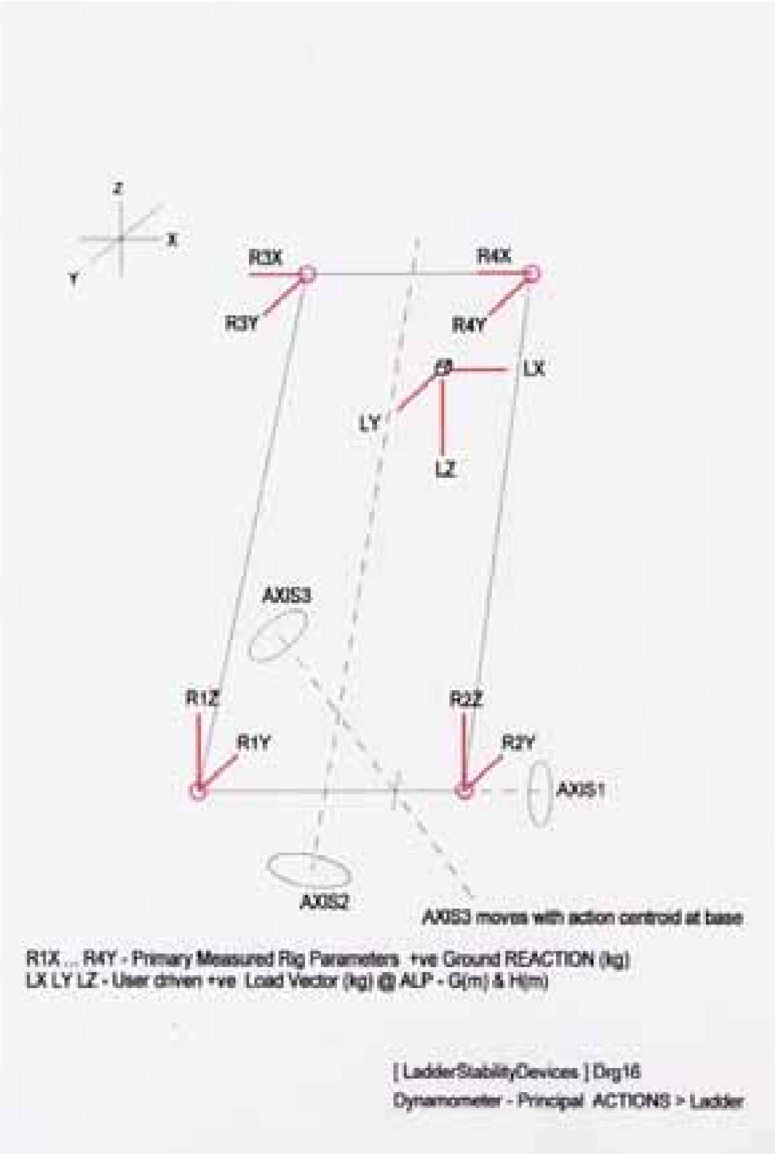

Figure 1

Dynamometer principal actions

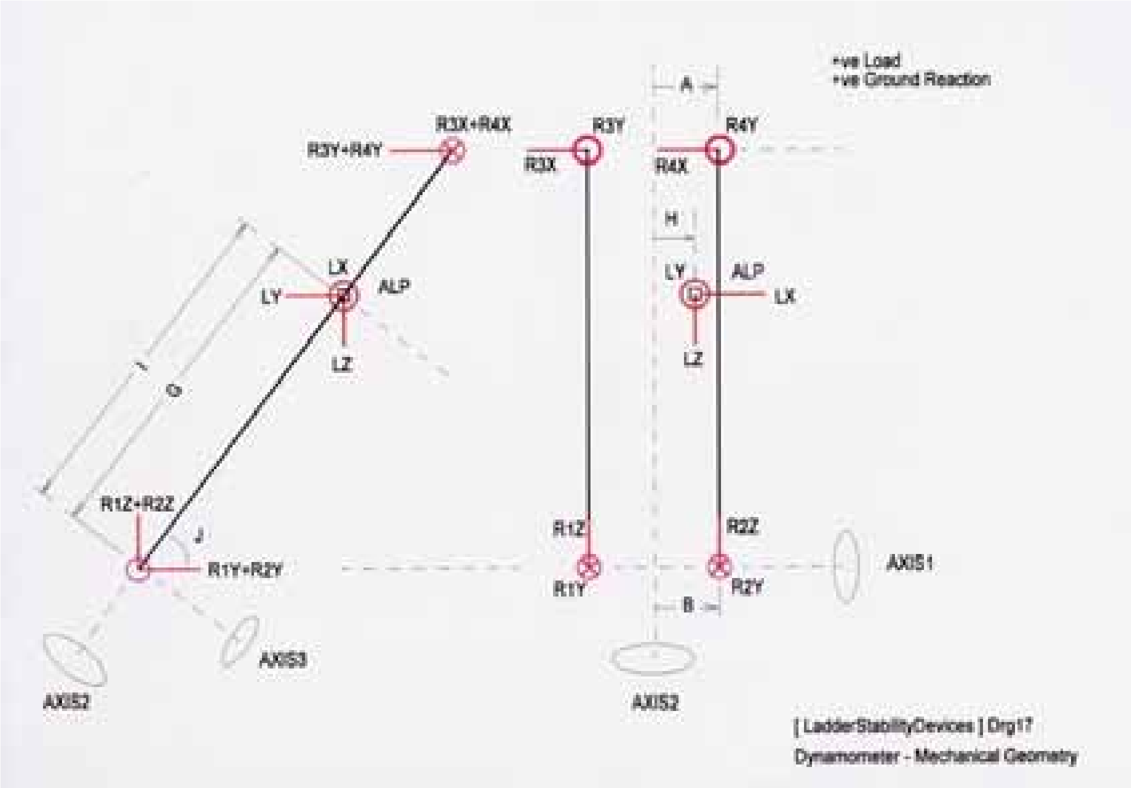

Figure 2

Dynamometer mechanical geometry

An essential component of this research was the collection of accurate data regarding the real world use of ladders. In order to acquire this information it was necessary to run extensive user trials utilising a bespoke data collection tool. This draws on the same technology as the device developed to appraise stepladder safety in the author's previous work, but the configuration is altered to the new task of appraising leaning ladders.

A full-size force dynamometer was developed which fully supported a trial reference ladder. The specification of the reference ladder itself is unimportant, but the model was an extending ladder comprising three sections and was drawn from the range of a well-known manufacturer. The ladder was purchased new. A flat steel plate at each of the four ladder endpoints carried the structure, and supplied ground reaction as necessary. These reaction responses were continuously monitored by dual-axis steel cantilever transducers, producing eight dimensions of action measurement in total. Vertical forces at the ladder top and lateral forces at the base were ignored, since these are generally near to zero at all times. Whilst this is clearly obvious from the most basic mechanical considerations, previous work of this type has also generated these measures, and they have been seen to be miniscule. The array of measured actions is sufficient to fully define the total system dynamic as required in this work.

The electronic signals were recorded at 50 Hz on a PC based data logger as raw trial data files. These were later corrected for zero tare and scaled to engineering units in the appropriate spreadsheet analysers. This conditioned data thereafter represents additional ladder loading entirely due to a user activity, above any system pre-stress or standing load due to ladder weight. Appropriate mechanical modelling generates the final time variant data set of dynamics parameters, pertaining to user driven load magnitude and concentration point of application.

2.1 CONSTRAINT OF LADDERS

The trial regime operated such that extreme usage may well occur, and accordingly it was possible that the ladder may become destabilised on occasion. Whilst there was no interest in observing any actual structural failure in stability, it was essential to ascertain the natural operational limits achieved by users. By constraining the ladder it is therefore possible to allow users to seek and act at whatever limit they reach, irrespective of the fact that stability may from time to time be technically breached. This also affords some real enhancement in actual user safety and comfort in the trials, and ensures that the ladder geometry and rig attitude is held truly constant for the duration, which facilitates continuous high accuracy measurement.

The tethering took the form of nylon ratchet ties at the grounding points. Whilst they prevented catastrophic failure of the ladder system, they were sufficiently loose so as to provide normal feedback to the user of the ladder's condition. In this way it was possible to ensure that the behaviour of the users was not altered through the provision of a rigid or pinned structure.

2.2 PRIMARY RIG SENSORY PARAMETERS

Refer to Figures 1 and 2

The dynamometer generates a set of independent electronic channel signals corresponding to 8 point contact loading vectors, and arising from compound transducers at each of the 4 ground contact points. These signals present as time variant analogue voltage levels which modulate linearly with the particular action magnitude. A tare correction for standing zero is made and engineering scaling adjusts to kg force units. Subsequent recorded data is directly due to user activity therefore, and is not responding to the standing weight of the trial ladder.

The raw and engineering scaling engineering parameters are identified thus:

- **V1Z ... V4X (V)** Instantaneous signal level – Time variant at 20 ms – 8 Channels

- **Zero1Z ... Zero4X (V)** Estimated standing zero level – No load (V)
  Average of initial 1 second of data – prior to user activity

- **K1Z ... K4X (kg/V)** Calibrated scaling factors

The structural dimensional constants are identified thus:

- **I (m)** Ladder length

- **A (m)** Ladder half-width - Top
- **B (m)** Ladder half-width - Base

- **J (deg)** Ladder altitude angle

Sensory parameters are generated and serve as raw data for all subsequent engineering measurements and extended analysis. These are instantaneous values updating at sampling frequency, and evaluated over the active period – specifically from first to last ladder contact by the user. The general analysis following and the parameters obtained are based on standard dynamics principals, assuming static balanced linear forces within the ladder structure, and balanced torsions in axis denoted 1 .. 3.

The ground contact reaction parameters are identified thus:

- **R1Z = K1Z (V1Z – Zero1Z)** Calibrated +ve Ground REACTIONS > Ladder (kg)
- **R1Y = K1Y (V1Y – Zero1Y)**
- **R2Z = K2Z (V2Z – Zero2Z)**
- **R2Y = K2Y (V2Y – Zero2Y)**
- **R3Y = K3Y (V3Y – Zero3Y)**
- **R3X = K3X (V3X – Zero3X)**
- **R4Y = K4Y (V4Y – Zero4Y)**
- **R4X = K4X (V4X – Zero4X)**

The composite ground reactions are identified thus:

- $\mathbf{R_{base}Z = R1Z + R2Z}$ Total Base Z Ground +ve REACTION > Ladder (kg)
- $\mathbf{R_{base}Y = R1Y + R2Y}$ Total Base Y Ground +ve REACTION > Ladder (kg)
- $\mathbf{R_{top}Y = R3Y + R4Y}$ Total Top Y Ground +ve REACTION > Ladder (kg)
- $\mathbf{R_{top}X = R3X + R4X}$ Total Top X Ground +ve REACTION > Ladder (kg)

The frictional demand is identified thus:

- $\mathbf{U_{base} = R_{base}Y / R_{base}Z}$ Base Frictional Demand (#)
- $\mathbf{U_{top} = R_{top}X / R_{top}Y}$ Top Frictional Demand (#)

The user generated equivalent load vector is identified thus:

- $\mathbf{LZ = R_{base}Z}$ **(Note1)** User generated load at ALP - Z (kg)
- $\mathbf{LY = R_{base}Y - R_{top}Y}$ **(Note 2)** User generated load at ALP - Y (kg)
- $\mathbf{LX = R_{top}X \times I/G}$ **(Note 3)** User generated load at ALP - X (kg)

$$L = \sqrt{LZ^2 + LY^2 + LX^2}$$ Total user generated load magnitude (kg)

The user generated ALP co-ordinates are defined thus:

$$G = \frac{R_{top}YISinJ}{LZCosJ - LYSinJ}$$ **(Note4)**

$$H = \frac{(R2Z - R1Z)CosJB + (R4Y - R3Y)SinJA + (R1Y - R2Y)SinJB}{LZCosJ - LYSinJ}$$ **(Note5)**

Note (1) – Vertical static balance – z-axis

Note (2) – Horizontal static balance – y-axis

Note (3) – Axis3 is allowed to float laterally according to the centriod acting between the two ground contact points 1 & 2. This mimics the natural dynamic at the ladder base where imbalance in base reactions is normal and responding to the arbitrary load, and there is in kinematic effect a single pivotal point contact. With the axis so placed, the torsional balance is as given to yield LX.

Note (4) – Torsional balance about Axis1

Note (5) - Torsional balance about Axis2

For the footing trials (Task H only) – The user drive vector is identified thus:

- **FZ = R_{base}Z** +ve User ACTION > Ladder – Base Z – Footing activity (kg)
- **FY = - R_{base}Y** +ve User ACTION > Ladder – Base Y – Footing activity (kg)

2.3 CALIBRATION METHODOLOGY

Four identical transducers, each capable of responding to two action vectors simultaneously, constitute the dynamometer. By design these transducers are reliably linear in response, and naturally free of cross-talk effects.

Point loading of individual transducers was initially undertaken with known loads at the time of their manufacture, and the amplifier channel gains were individually set to achieve a nominal 125 kg/V.

Upon completion of the rig proper, and with the trial ladder registered, a series of reference loads were applied to the ladder at strategic positions. Initial nominally correct computed values produced by the trial mechanical model were fine corrected in the spreadsheet analysers to closely tally with the known references in terms of force or displacement. The corrections required were of the order of low single figure kg and low single figure centimetres, and arose due to positioning tolerances during dyno assembly and ladder mounting, and small settling asymmetries. Results in measurable dynamometer output are produced, accurate generally to about +/- 1 % of scale range.

Exact calibration records exist but are not provided with this report.

3.0 ANALYTIC AND DATA PROCESS METHODS

Raw volt level signals were initially captured at the time of the trial with a PC based data logger, and were stored as bulk data files for post process. The files were named according to user ID number (00...99), Task type (A ... G or H) and repetition number (1 or 2) e.g. 07F2.

A pair of major Microsoft Excel analyser spreadsheets were constructed to process raw instrumented data sets into endpoint parameters as necessary. Spreadsheet ‘Analyser 1’ handled task types A through to G (the general task types), while spreadsheet ‘Analyser 2’ handled task type H (manual footing).

Initially raw time variant volt level data was baseline adjusted to zero then scaled to engineering units through prior set calibration factors, and produced eight elemental ground contact point reaction levels as previously described. These were processed through the particular modelling algorithm to generate time variant user load and action location parameters. Data was then segmented into epochs equivalent to the mount / dismount phase and the actual task phase, allowing better resolution of the various endpoint measures needed. Each raw data block was embedded in turn in the analyser, and the full set of endpoint trial characterisation parameters so generated are each copied and tabulated within a respective archive sheet. This is a transient process whereby the bulk analysis result is discarded prior to accepting the next cycle of data.

Master analysis sheets exist as ‘Collation 1’ for Task type A through to G (general tasks), and ‘Collation 2’ for task type H (manual footing). These contain the entire data sets of all trials, and are the basis of a graded statistical breakdown yielding the various universal standard parameters as defined.

3.1 SINGLE-TRIAL ANALYSIS PARAMETERS

Raw instrument recorded data blocks were successively embedded into the analyser spreadsheet which initially produces usable primary rig parameters – calibrated and scaled to engineering units. These are time variant values updating at a sampling rate interval of 20 ms, and extending over the whole activity period.

The primary rig parameters are identified thus:

- **LZ (kg)** Instantaneous LZ
- **LY (kg)** Instantaneous LY
- **LX (kg)** Instantaneous LX

- **L (kg)** Instantaneous total vector magnitude

- **G (m)** Instantaneous ALP co-ordinate
- **H (m)** Instantaneous ALP co-ordinate

- **$R_{base}Z$ (kg)** Instantaneous Base Reaction Z
- **$R_{base}Y$ (kg)** Instantaneous Base Reaction Y

- **$R_{top}Y$ (kg)** Instantaneous Top Reaction Y
- **$R_{top}X$ (kg)** Instantaneous Top Reaction X

- **U_{base} (#)** Instantaneous Base Frictional Demand
- **U_{top} (#)** Instantaneous Top Frictional Demand

This data is segmented into two major blocks corresponding to the ascent/decent phase and the task phase separately. Thereafter they are managed separately producing appropriate endpoint trial parameters for both phases. These are single value measurement results which collectively characterise and quantify a given trial.

Task parameters fairly numerate the pure activity phase of usage on the ladder, without data skew or dilution through arbitrary ascent delays or styles, and are the basis of the greater determination of a standard load vector and standard applied loading point – SLV and ALP. Ascent/descent parameters characterise the typically high shock or transient loading at the base arising from a user mounting the ladder, more or less harshly. This is predominantly due to hard initial foot impact, and vertical user mass acceleration reactions. While this information is not relevant to stability in the forms expressed, it is highly relevant to structural durability issues.

The choice of measures used are arranged to extract clear boundary limit information derived from typically erratic and volatile source data. The users are highly active and in constant motion, and understandably produce similar patterning in the signal stream. However, the trials were established to determine the extremes of reasonable use, and particularly the fall-off of parameters near critical boundaries. Much use is made of containment percentiles. In this case they effectively reflect progressive limit envelopes. It is this data which is the kernel of much of this work, since standard loading levels and action locations which represent qualified maximal duress are sought, and reliably mimic extreme utility by real users.

Data condition flags CONTACT & TASK section out the relevant phases into Ascent/decent or Task thus:

- **CONTACT Flag** TRUE IF $R_{base}Z$ > Limit1 (Set at 10kg)
- **TASK Flag** TRUE IF $R_{base}Y$ > Limit2 (Set at 0.66 x 90thPercentile all $R_{base}Y$)

Single trial analysis parameters are identified thus:

Qualified by : TASK=TRUE & CONTACT=TRUE

- **LZ_{max} (kg)** Maximum of all LZ
- **LZ_{99} (kg)** 99th Percentile of all LZ
- **LZ_{95} (kg)** 95th Percentile of all LZ
- **LZ_{avg} (kg)** Average of all LZ
- **LZ_5 (kg)** 5th Percentile of all LZ
- **LZ_1(kg)** 1st Percentile of all LZ
- **LZ_{min} (kg)** Minimum of all LZ

- **LY_{max} (kg)** Maximum of all LY
- **LY_{99} (kg)** 99th Percentile of all LY
- **LY_{95} (kg)** 95th Percentile of all LY

- **LX_{max} (kg)** Maximum of all LX
- **LX_{99} (kg)** 99th Percentile of all LX
- **LX_{95} (kg)** 95th Percentile of all LX

- **L_{max} (kg)** Maximum of all L
- **L_{99} (kg)** 99th Percentile of all L
- **L_{95} (kg)** 95th Percentile of all L
- **L_{avg} (kg)** Average of all L
- **L_{5} (kg)** 5th Percentile of all L
- **L_{1} (kg)** 1st Percentile of all L
- **L_{min} (kg)** Minimum of all L

- **G_{max} (m)** Maximum of all G
- **G_{99} (m)** 99th Percentile of all G
- **G_{95} (m)** 95th Percentile of all G
- **G_{med} (m)** Median of all G
- **G_{5} (m)** 5th Percentile of all G
- **G_{1}(m)** 1st Percentile of all G
- **G_{min} (m)** Minimum of all G

- **H_{max} (m)** Maximum of all H
- **H_{99} (m)** 99th Percentile of all H
- **H_{95} (m)** 95th Percentile of all H
- **H_{med} (m)** Median of all H

- **$R_{base}Z_{max}$ (kg)** Maximum of all $R_{base}Z$ (kg)
- **$R_{base}Z_{99}$ (kg)** 99th Percentile of all $R_{base}Z$ (kg)
- **$R_{base}Z_{95}$ (kg)** 95th Percentile of all $R_{base}Z$ (kg)

- **$R_{base}Y_{max}$ (kg)** Maximum of all $R_{base}Y$ (kg)
- **$R_{base}Y_{99}$ (kg)** 99th Percentile of all $R_{base}Y$ (kg)
- **$R_{base}Y_{95}$ (kg)** 95th Percentile of all $R_{base}Z$ (kg)

- **$R_{top}Y_{max}$ (kg)** Maximum of all $R_{top}Y$ (kg)
- **$R_{top}Y_{99}$ (kg)** 99th Percentile of all $R_{top}Y$ (kg)
- **$R_{top}Y_{95}$ (kg)** 95th Percentile of all $R_{top}Y$ (kg)
- **$R_{top}Y_{avg}$ (kg)** Average of all $R_{top}Y$ (kg)
- **$R_{top}Y_{5}$ (kg)** 5th Percentile of all $R_{top}Y$ (kg)
- **$R_{top}Y_{1}$ (kg)** 1st Percentile of all $R_{top}Y$ (kg)
- **$R_{top}Y_{min}$ (kg)** Minimum of all $R_{top}Y$ (kg)

- **$R_{top}X_{max}$ (kg)** Maximum of all $R_{top}X$ (kg)
- **$R_{top}X_{95}$ (kg)** 99th Percentile of all $R_{top}X$ (kg)
- **$R_{top}X_{95}$ (kg)** 95th Percentile of all $R_{top}X$ (kg)

- **$U_{basemax}$ (kg)** Maximum of all U_{base} (#)
- **U_{base99} (kg)** 99th Percentile of all U_{base} (#)
- **U_{base95} (kg)** 95th Percentile of all U_{base} (#)

- **U_{topmax} (#)** Maximum of all U_{top} (#)
- **U_{top99} (#)** 99th Percentile of all U_{top} (#)
- **U_{top95} (#)** 95th Percentile of all U_{top} (#)

Ascent / Descent analysis parameters are identified thus:

Qualified by : TASK=FALSE & CONTACT=TRUE

- **$S_{base}Z_{max}$ (kg)** Maximum of all $R_{base}Z$ (kg)
- **$S_{base}Z_{99}$ (kg)** 99th Percentile of all $R_{base}Z$ (kg)
- **$S_{base}Z_{95}$ (kg)** 95th Percentile of all $R_{base}Z$ (kg)

- **$S_{base}Y_{max}$ (kg)** Maximum of all $R_{base}Y$ (kg)
- **$S_{base}Y_{99}$ (kg)** 99th Percentile of all $R_{base}Y$ (kg)
- **$S_{base}Y_{95}$ (kg)** 95th Percentile of all $R_{base}Y$ (kg)

- **$V_{basemax}$ (#)** Maximum of all Ubase (#)
- **V_{base99} (#)** 99th Percentile of all U_{base} (#)
- **V_{base95} (#)** 99th Percentile of all U_{base} (#

3.2 DATA FILE AND DIRECTORY STRUCTURE

The following information identifies the data file and directory structure pertaining to the trials. Whilst not of immediate value, it will be of use should subsequent need arise to revisit the data collected.

[Docs]	Microsoft Word documents
Technical Report 1	The original technical report
[Xldocs]	Microsoft Excel Spreadsheets
Analyser1	Raw trial data processor – Task A ...G - General Tasks
Analyser2	Raw trial data processor – Task H – Manual Footing
Collation1	Collation & Analysis of all Single -Trial parameters - Task AG
Collation2	Collation & Analysis of all Single -Trial parameters - Task H – Footing
Archive1	Full set Trial Data Parameters – Not data quality verified - Task A..G
Archive	Full set Trial Data Parameters – Not data quality verified - Task H - Footing
MacroSheet	Data management macros
StabilityPredictor1	DAL Stability Modeller – Interactive spreadsheet
StabilityPredictor2	Tripod Stability Modeller – Interactive spreadsheet
Optimiser	SLV & ALP Selector – Master data field from Collation1
SlopeEffect	Ground slope analyser - Interactive spreadsheet

Note : All spreadsheets are maintained in active mode and should be manipulated with care.

[RawDataPLW]	Raw PICO ADC22 files – approximately 760 files
***.PLW**	

[RawDataTXT] ***.TXT**	Raw data files – TXT format – approximately 760 files
[SampleSheets] ***.XLS**	EXCEL Spreadsheets
[CalibrationFiles] **CalA1.TXT** ..…	Raw Data Arrays
[PicoPrograms]	Pico ADC22 Data Logger - Control Programs
MonitorDyno.PSS	Real-time display – Diagnostic - Channels 1 .. 8
RecordDyno.PLS	Record @ 20 ms – Channels 18

3.3 TYPICAL EXAMPLE TRIAL ANALYSIS SPREADSHEETS

A number of the single trial analysis spreadsheets are preserved for full inspection if required. The bulk being generated on a transitory basis only to generate various trial characterisation parameters which are harvested and collated, and processed statistically later. Trial types A through to G are processed within Analyser 1 and represent practical user tasks . Trial type H is manual footing trial results and processed within Analyser 2. They contain the full data extraction algorithm and generate results in numerical and graphical formats, and are available as active spreadsheets. Examination of these files will give deeper insight into the true dynamic and qualitative nature of the mechanics, and the various performance parameters of central interest.

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4.0 THEORETICAL STABILITY MODEL – DAL

Figures 3 to 7 are provided to illustrate the theoretical stability model used in this evaluation.

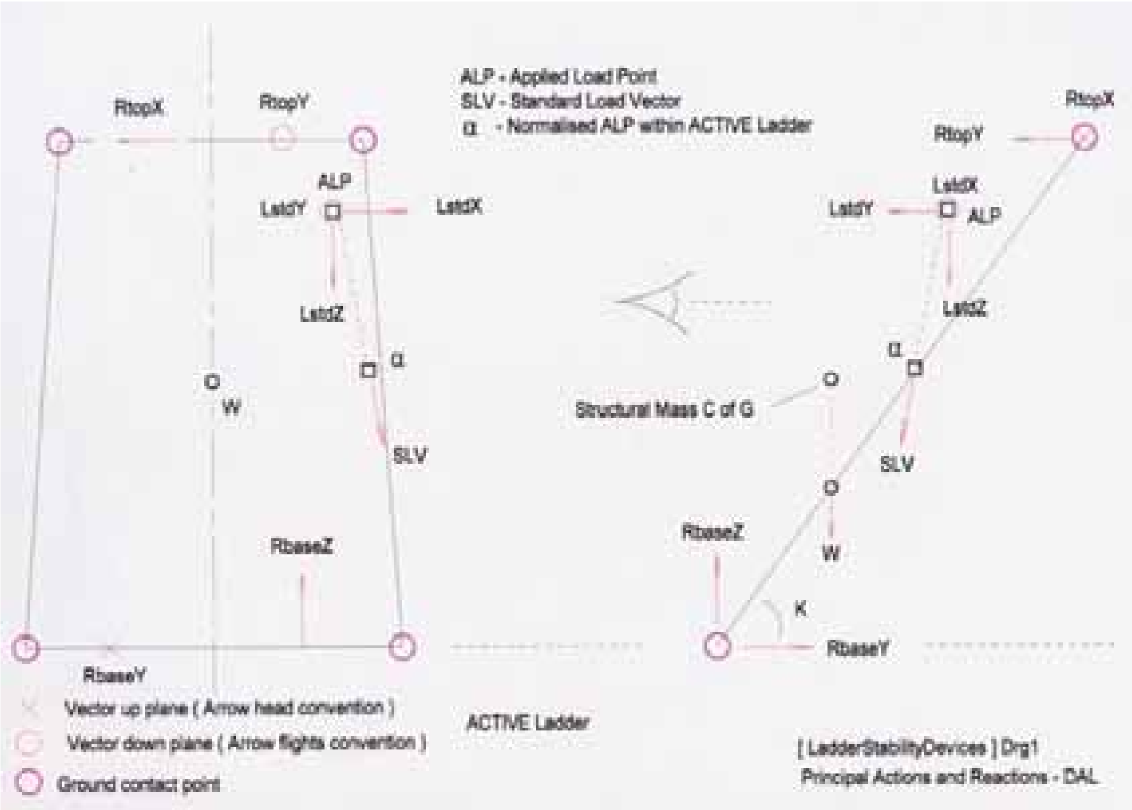

Figure 3

Principal actions and reactions

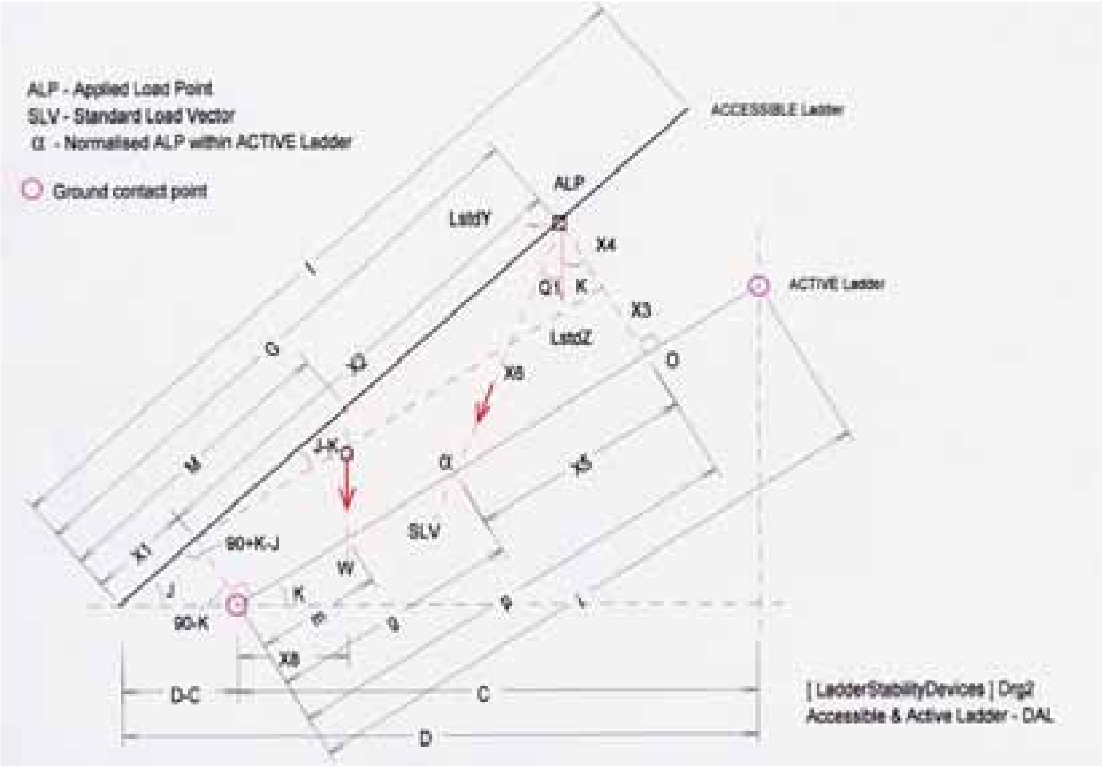

Figure 4

Accessible and active ladders

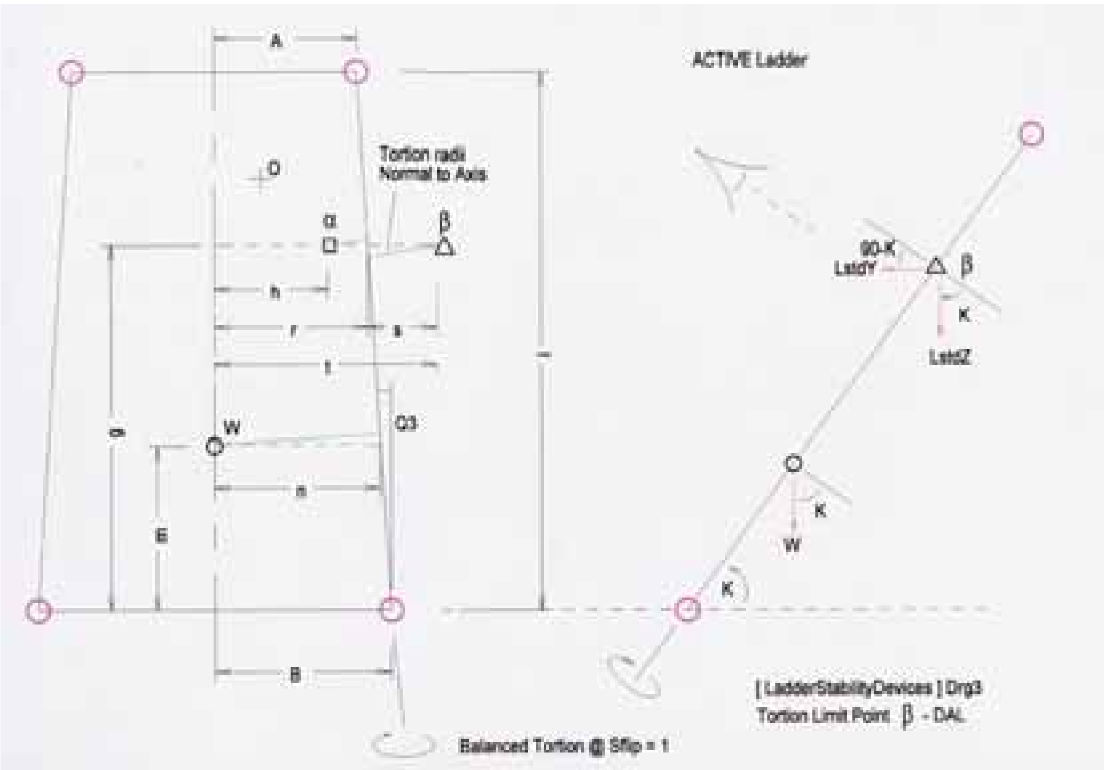

Figure 5

Tortion limit points

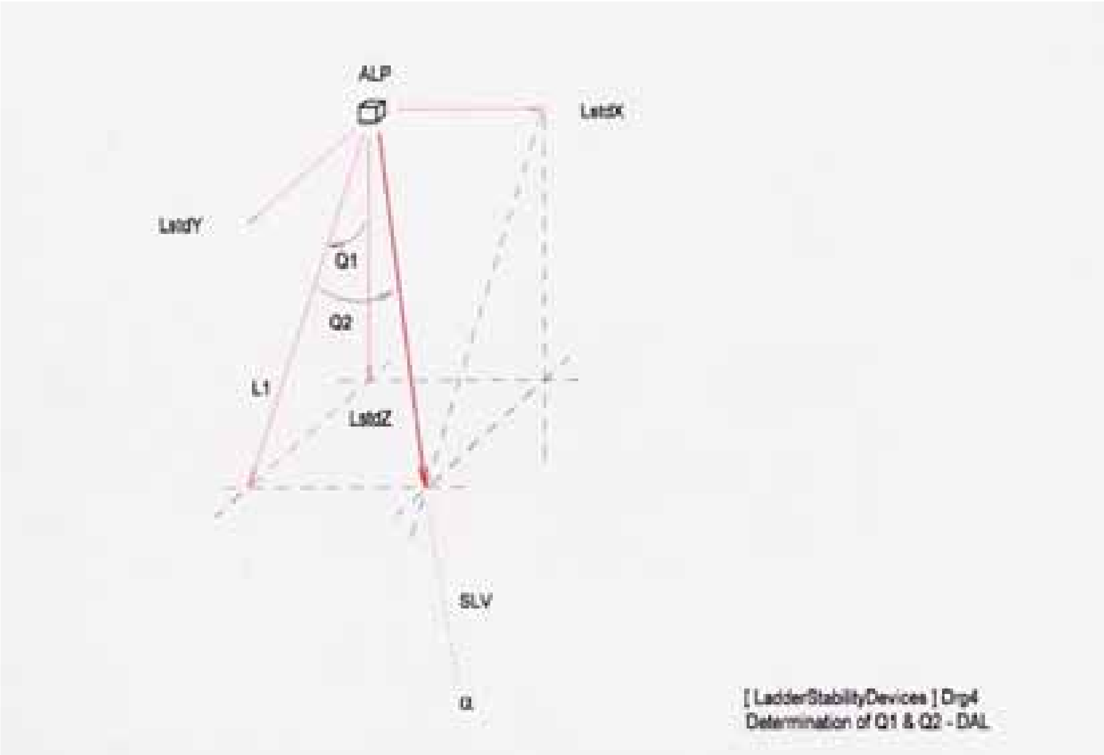

Figure 6

Determination of Q1 and Q2

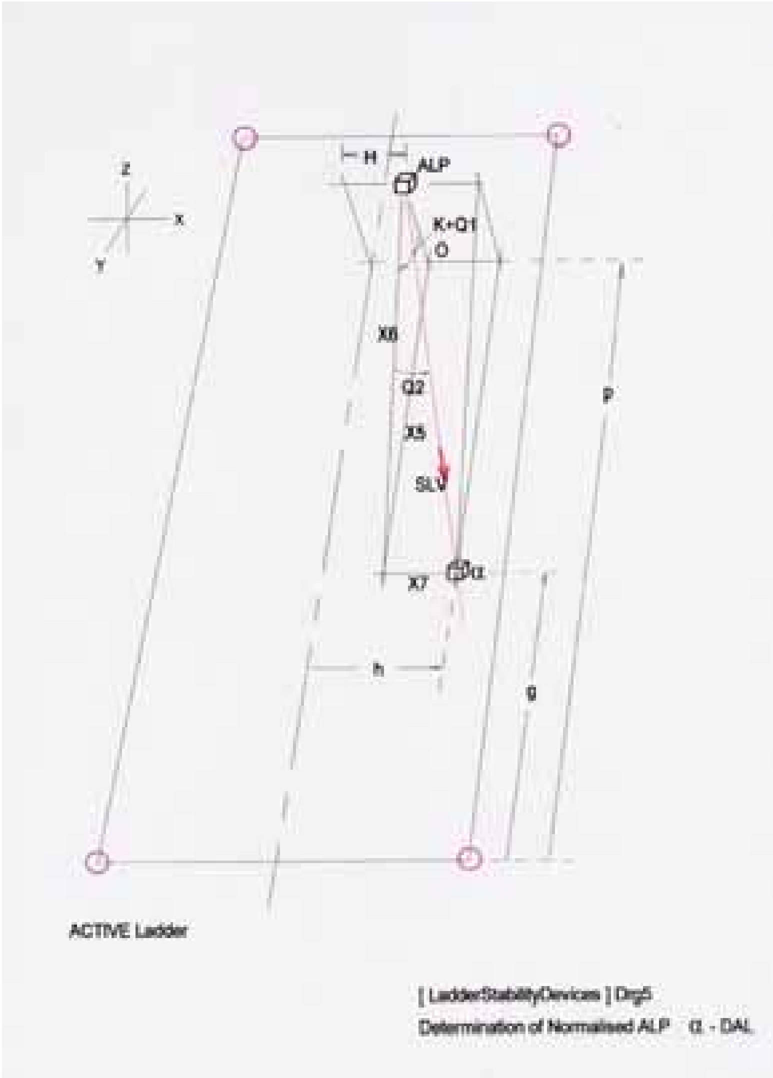

Figure 7

Determination of normalised ALP

Part of the analysis required the development of a numerical modeller of a generalised DAL structure. The technical model assumes a pair of potential point contacts in the horizontal ground plane, and a similar pair in the vertical plane. Using key measurable structural dimensions, information defining structural mass Centre of Gravity and defined loading vectors, the dynamic status of the total frame can be determined, and ultimately expressed as normalised stability indices.

The user's activity was modelled as a point action vector, designated the standard load vector, or SLV. This is considered to act at a known position in space, designated the applied load point, or ALP.

In reality the user is placing the ladder structure under duress in distributed fashion, they are applying forces at numerous locations and at variable magnitude. This drive arises due to the following variables:

- Their weight and gross attitude
- The additional weight of tools or other objects
- Contact reactions to ground – being solid surfaces about the ladder
- Inertial actions due to user motion or attitude change

Although the forces acting on the ladder are distributed as described, they can be resolved into a single directed vector at a single point. This vector and point of application is, in effect, an equivalent or parametric drive which places the ladder under duress, from a rigid systems point of view, without distinction. The extensive user trials provide the practical values of SLV and ALP for both modelling and workshop stability verification tests. Standard dynamics and rigid structure mechanics are used to develop the formal model.

A virtual construct termed the active ladder is extensively used. This is a simple flat ladder equivalent, which is, by definition, co-planar with the true DAL ground contact points and is weightless. This serves as a normalised action frame against which various action drives are applied. These drives arise from the SLV itself, the structural mass Centre of Gravity, and the responding ground reactions, and are technically mapped onto the active ladder. It is generally the case that both the location of the ALP and the mass Centre of Gravity will be displaced from the active ladder by a measurable degree. A standardised algorithm can thereafter fully determine the system dynamics referenced to the active ladder, and will produce appropriate stability status indices.

The SLV and ALP are referenced to the accessible ladder at all times, and defined by the parameters $L_{std}Z$, $L_{std}Y$, $L_{std}X$(kg), and G_{set}(m) and H_{set}(m). This action vector is mapped into the active ladder at a position denoted α, hence is a referred or normalised ALP. A limit position is calculated and denoted β, which indicates an extreme point where the referenced ALP must reach to achieve critical torsional balance, or instability. This permits a scaling calculation for flip mode failure.

Four concurrent modalities are identified by which the DAL structure can destabilise or fail. Base and top slip modes arise as friction limit failures, where demanded drive exceeds the inherent capability of the particular materials at the contact interface.

Flip and top contact failure are rotations, and occur when there is a critical loss of ground contact points, below the mandatory three. This occurs when a magnitude of destabilising torsion meets a stabilising torsion about a crucial axis, that is they are in critical balance.

Friction is considered as both a demand level and as a reliable limit failure. We define frictional demand in terms of the ratio between planar and normal load, in the conventional manner. Physics gives us that this parameter is insensitive to total area of contact, but is sensitive to the materials in contact, and the state of surface finish. Experimentally, upper frictional limits can be found with simple test rigs, progressively loaded until cohesion failure is observed. These values are typically erratic, however very reliable lower limits can be found which translate into reliable working limits in the model – these parameters being $U_{baselim}$(#) and U_{toplim}(#). These must be supplied to the model as input values, and are taken as ratiometrics when calculating S_{int}Base(#) and S_{int}Top(#).

Top contact failure is defined as a reduction of the top normal reaction – R_{top}Y(kg) – below a critical value defined by $R_{contact}$(kg). The value here is debatable, but could reasonably be set at about 3 kg based on previous measurements and allowing for transient loading. Hence the parameter S_{int}Contact(#) will descend to value 1 when R_{top}Y reduces to 3 kg, in this case.

The model determines the mechanical duress which arises in the arbitrary DAL structure, and measures the drive towards instability expressed as proximity to prevailing critical limits. The stability indices themselves are fully normalised and dimensionless parameters, and indicate stability integrity as a number which is value 1.0 (unity) precisely at a maximal limit or a critical balance. Collectively the four parameters S_{int}Base(#), S_{int}Top(#), S_{int}Flip(#) and S_{int}Contact(#), demark an envelope of qualified safety. Provided none are less than value 1, then the structure is compliant to the minimal requirement for stability, under the specified standard loading condition. In practice the standard load is adjusted to maximally duress the DAL for each failure mode in turn. There are strong natural interlinks between drives to stability failure as a function of applied load magnitude and varying geometry, and some can increase while others decrease. There are therefore optimums to be found in terms of DAL design.

4.1 ACCESSIBLE AND ACTIVE LADDER – MODELLING DISTINCTION - DAL

The accessible ladder is the ordinary or nominal ladder as made available to the user. This ladder is therefore the physical structure upon which a user will work and apply load to the DAL as a whole. The SLV is reckoned to be applied on, and registered at, the ALP with respect to the accessible ladder.

Generally the basic ladder will be displaced in space through intervening devices, and will finally contact the ground through some arbitrary construction or arrangement, with modified ground contact geometry or modified frictional capability, or both. This final configuration of ground contacting is germane to the stability status of the total structure.

A notional or virtual ladder is created which underlies the real structure, and is termed the active ladder. This is a simple plane which contains the four potential grounding contact points. This construction behaves as a normalised action frame, functioning as a dynamic equivalent to an arbitrarily complex real system, and allows a tractable method of analysis based on a standardised dynamics sub-system. The process is a type of mapping, and is allowed according to the principals of dynamics and rigid structures.

From the viewpoint of the active ladder, the ALP lies at some displacement in space which is fully determined by the system geometry. Also the mass Centre of Gravity is likewise removed by some amount. By figuring the drives on the active ladder arising from the standard load, the effect of weight Centre of Gravity, and the true grounding geometry of the structure, the propensity to instability can be directly determined.

4.2 ANALYTIC MODEL PARAMETERS - DEFINITIONS - DAL

Refer to Figures 3 to 7

In this section there are listed the formal parameters utilised in the stability modelling algorithm. They consist of various classes of parameter type as described, and produce a normalised set of stability indices via the defined algorithm.

Measured Structural Parameters

The measured structural parameters can be identified thus:

- **I (m)** Total length of Accessible Ladder

- **A (m)** Active Ladder – Upper Semi-width
- **B (m)** Active Ladder – Lower Semi-width
- **C (m)** Ground contact planar displacement of Active Ladder
- **D (m)** Ground planar displacement of Accessible Ladder

- **F (m)** Access Limit dimension at G

- **W (kg)** Total Weight – combined Ladder + Devices

- **M (m)** Weight position of C of G referenced within Accessible Ladder

- **J (deg)** Base Elevation Angle – Accessible Ladder
- **K (deg)** Base Elevation Angle – Active Ladder

User Specified Parameters :

The user specified parameters can be identified thus:

- **$U_{baselim}$ (#)** Maximum reliable frictional limit - Base
- **U_{toplim} (#)** Maximum reliable frictional limit - Top

Prescribed Standard Parameters :

The prescribed standard parameters can be identified thus

- **$L_{std}X$ (kg)** Standard applied load vector (SLV) - X axis
- **$L_{std}Y$ (kg)** Standard applied load vector (SLV) - Y axis
- **$L_{std}Z$ (kg)** Standard applied load vector (SLV) - Z axis

- **G_{set} (m)** Standard Offset dimension to determine ALP parameter G (m)
- **H_{set} (m)** Standard Offset dimension to determine ALP parameter H (m)

- **$R_{contact}$ (kg)** Minimum permissible $R_{top}Y$(kg)

Modelled Performance Parameters :

The modelled performance parameters can be identified thus:

- **G (m)** Applied Load Point (ALP) – co-ordinate
- **H (m)** Applied Load Point (ALP) – co-ordinate

- **S_{int}Base (#)** Normalised Intrinsic Stability Index – Base slip mode
- **S_{int}Top (#)** Normalised Intrinsic Stability Index – Top slip mode
- **S_{int}Flip (#)** Normalised Intrinsic Stability Index – Flip mode
- **S_{int}Contact (#)** Normalised Intrinsic Stability Index – Top contact mode

- **U_{base} (#)** Friction Demand - Base
- **U_{top} (#)** Friction Demand – Top

- **R_{base}Y (kg)** Total Reaction – Base - Y axis
- **R_{base}Z (kg)** Total Reaction – Base - Z axis

- **R_{top}X (kg)** Total Reaction – Top - X axis
- **R_{top}Y (kg)** Total Reaction – Top - Y axis

Intermediate Modelling Parameters – Transient usage only :

The intermediate modelling parameters can be identified thus:

- **I, p, g, h, m, n, r ,s, t (m)** Virtual dimensions defining Active Ladder

- **X1 .. X8(m)** Temporary construction dimensions

- **Q1 .. Q3(deg)** Temporary construction angles

4.3 ANALYTIC MODEL PARAMETERS – FORMAL DERIVATIONS - DAL

In this section the formal derivations of the analytical model are presented. They are submitted in conventional algebraic terms utilising the expressions defined in the previous sections.

Geometric identities used in the algebraic process

The geometric identities used in the algebraic process necessary to devise the model are given as:

1. Sin (90 +/- Q) = Cos Q
2. Cos (-Q) = Cos Q
3. General Sin Rule

$$\frac{Sin(90-K)}{X1}=\frac{Sin(90+K-J)}{D-C}$$

$$X1=\frac{(D-C)CosK}{Cos(J-K)} \qquad \textbf{(1)}$$

$$X2=G-X1 \qquad \textbf{(2)}$$

$$\frac{SinJ}{X3}=\frac{Sin(90+K-J)}{D-C}$$

$$X3=\frac{(D-C)SinJ}{Cos(J-K)} \qquad \textbf{(3)}$$

$$Sin(J-K)=\frac{X4}{X2}$$

$$X4=X2Sin(J-K) \qquad \textbf{(4)}$$

$$Tan(K+Q1))=\frac{X5}{X3+x4}$$

$$X5=(X3+X4)Tan(K+Q1) \qquad \textbf{(5)}$$

$$Sin(K+Q1)=\frac{X5}{X6}$$

$$X6=\frac{X5}{Sin(K+Q1)} \qquad \textbf{(6)}$$

$$TanQ2 = \frac{X7}{X6}$$

$$X7 = X6TanQ2 \quad \textbf{(7)}$$

$$CosJ = \frac{(D-C)+X8}{M}$$

$$X8 = MCosJ - (D-C) \quad \textbf{(8)}$$

$$TanQ1 = \frac{L_{std}Y}{L_{std}Z}$$

$$Q1 = ArcTan\frac{L_{std}Y}{L_{std}Z} \quad \textbf{(9)}$$

$$TanQ2 = \frac{L_{std}X}{\sqrt{(L_{std}Y^2 + L_{std}Z^2)}}$$

$$Q2 = ArcTan\frac{L_{std}X}{\sqrt{(L_{std}Y^2 + L_{std}Z^2)}} \quad \textbf{(10)}$$

$$TanQ3 = \frac{B-A}{i}$$

$$Q3 = ArcTan\frac{B-A}{i} \quad \textbf{(11)}$$

$$CosK = \frac{C}{i}$$

$$i = \frac{C}{CosK} \qquad \textbf{(12)}$$

$$Cos(J-K) = \frac{P}{X2}$$

$$P = X2Cos(J-K) \qquad \textbf{(13)}$$

$$CosK = \frac{X8}{m}$$

$$m = \frac{X8}{CosK} \qquad \textbf{(14)}$$

$$TanQ3 = \frac{B-n}{m}$$

$$n = B - mTanQ3 \qquad \textbf{(15)}$$

$$g = p - X5 \qquad \textbf{(16)}$$

$$h = H + X7 \qquad \textbf{(17)}$$

$$r = B - gTanQ3 \qquad \textbf{(18)}$$

Torsion balance condition – Location of limit point β :

The calculation of the torsion balance condition and the location of the limit point β can be identified thus:

$$sCosQ3L_{std}ZCosK = sCosQ3L_{std}YSinK + WnCosQ3CosK$$

$$sL_{std}ZCosK = sL_{std}YSinK + WnCosK$$

$$s\left(L_{std}ZCosK - L_{std}YSinK\right) = WnCosK$$

$$s = \frac{WnCosK}{L_{std}ZCosK - L_{std}YSinK} \qquad \textbf{(19)}$$

$$t = r + s \qquad \textbf{(20)}$$

Ground reactions – Resolving Horizontal – Balance condition :

The horizontal ground reaction balance condition can be identified thus:

$$R_{base}Y = L_{std}Y + R_{top}Y \qquad \textbf{(21)}$$

Ground reactions – Resolving Vertical - Balance condition:

The vertical ground reaction balance condition can be identified thus:

$$R_{base}Z = W + L_{std}Z \qquad \textbf{(22)}$$

Moments about Base - Balance condition :

The balance condition of the moments around the base can be identified thus:

$$gL_{std}ZCosK + mWCosK = gL_{std}YSinK + iR_{top}YSinK$$

$$R_{top}Y = \frac{gL_{std}ZCosK - gL_{std}YSinK + mWCosK}{iSinK} \qquad \textbf{(23)}$$

$$gL_{std}X = iR_{top}X$$

$$R_{top}X = \frac{gL_{std}X}{i} \qquad \textbf{(24)}$$

Frictional Demand Parameters :

The frictional demand parameters can be identified thus:

$$U_{base} = \frac{R_{base}Y}{R_{base}Z} \qquad \textbf{(25)}$$

$$U_{top} = \frac{R_{top}X}{R_{top}Y} \qquad \textbf{(26)}$$

Normalised Intrinsic Stability Indices :

The normalised intrinsic stability indices can be identified thus:

Stable if S > 1.0

$$S_{\text{int}}Base = \frac{\mu_{base\,\text{lim}}}{\mu_{base}} \qquad \textbf{(27)}$$

$$S_{\text{int}}Top = \frac{\mu_{top\,\text{lim}}}{\mu_{top}} \qquad \textbf{(28)}$$

$$S_{\text{int}}Flip = \frac{t}{h} \qquad \textbf{(29)}$$

$$S_{\text{int}}Contact = \frac{R_{top}Y}{R_{contact}} \qquad \textbf{(30)}$$

Applied Load Point - ALP :

The applied load point can be identified thus:

$$G = I - G_{set} \qquad \textbf{(31)}$$

$$H = F + H_{set} \qquad \textbf{(32)}$$

4.4 STANDARD LOAD VECTOR SLV AND APPLIED LOAD POINT ALP - DAL

These parameters are obtained from key results within the 'Optimisation' spreadsheet and are set at qualified high levels as defined :

- $\mathbf{L_{std}X}$ - 95th Percentile of all LX95 = 22.6 kg (23 kg)

- $\mathbf{L_{std}Y}$ - 95th Percentile of all LY95 = 12.3 kg (13 kg)

This parameter set at qualified low level as defined :

- $\mathbf{L_{std}Z}$ – 5th Percentile of all LZ5 = 60.0 kg (60 kg)

This parameter set at qualified high level as defined :

- $\mathbf{L_{std}Z}$ – 95th Percentile of all LZ95 = 128.1 kg (128 kg)

These parameters set at qualified high levels as defined :

- $\mathbf{G_{set}}$ = I(1) - 95th Percentile of all G95
  = 4.25 – 3.86
  = 0.39 m (390 mm)

- $\mathbf{H_{set}}$ = 95th Percentile of all H95 – F(1)
  = 0.28 – 0.145
  = 0.135 m (135 mm)

Note – I(1) is measured total ladder length of trial ladder = 4.25 m
Note – F(1) is measured Access Limit Dimension for trial ladder at G_{set} =0.39 m

Each of the four defined stability failure modes is determined by the particular simultaneous combination of standard load vector components, since this is three-dimensional. Table 1 illustrates the maximal duress in each failure mode in terms of the relative SLV vector magnitudes.

Table 1

Loading to determine stability failure in different modes (values in parenthesis)

Failure mode test	$L_{std}Z$	$L_{std}Y$	$L_{std}X$
Test1 – S_{int}Base (#) & S_{int}Contact (#)	LOW (60 kg)	HIGH (13 kg)	ZERO (0 kg)
Test2 – S_{int}Top (#)	LOW (60 kg)	ZERO (0 kg)	HIGH (23 kg)
Test3 – S_{int}Flip (#)	HIGH (128 kg)	ZERO (0 kg)	HIGH (23 kg)

Evidently, to test all failure contingencies, two levels of $L_{std}Z$ are required, with single high strength vectors in X and Y sufficient.

The correct set of SLV component values should be entered into the predictive model, and the relevant intrinsic stability parameter determined at this point. This will represent the worst case, producing in effect the lowest reliable stability index pertaining to that failure mode. The operation is three stage therefore, to deduce the full set of four stability indices.

In a similar fashion the workshop stability proving tests call for a specified set of load vectors per each of the three tests 1 to 3.

It is important to make it clear that the SLV value for both modelling purposes and workshop stability proving tests are identical.

4.5 INTRINSIC STABILITY INDICES – SINTBASE, SINTTOP, SINTFLIP & SINTCONTACT – DAL

The primary function of the stability modeller is to predict the global safety assurance of any given DAL by numerated values indicating proximity to stability failure. The stability indices are dimensionless quantities normalised to value 1.0 at criticality, and can be universally compared across arbitrary configurations. Stability pertaining to a given failure mode is assured if the relevant parameter attains a value equal to or greater than 1. Total system stability integrity occurs if all indices meet the criteria.

The stability modeller requires categories of information in the following classes :

1. Structural geometric – Directly measurable key dimensions
2. Weight – W(kg)
3. Prescribed standard parameters – SLV , G_{set}(m) and H_{set}(m) and $R_{contact}$(kg)
4. User supplied parameters – $U_{baselim}$(#) and U_{toplim}(#)

This generates output parameters :

1. Intrinsic stability indices - S_{int}Base(#),S_{int}Top(#),S_{int}Flip(#) and S_{int}Contact(#)
2. Computed ALP – G(m) and H(m)
3. Contact reactions – R_{base}Z(kg), R_{base}Y(kg), R_{top}Y(kg) and R_{top}X(kg),
4. Contact frictional demand – U_{base}(#) and U_{top}(#)

There are three verification tests each requiring a specific set of SLV component magnitudes, which sequentially stress the system maximally. When employing the modelling algorithm, each test is done by assigning the prescribed values of SLV components given as elements in L_{std}X, L_{std}Y and L_{std}Z(kg) , and specified in Table 1, and the particular valid stability indices noted.

Test 1 - Maximal duress to base slip and top contact failure – S_{int}Base(#) & S_{int}Contact(#) valid

Test 2 - Maximal duress to top slip failure – S_{int}Top(#) valid

Test 3 - Maximal duress to Flip failure – S_{int}Flip(#) valid

These criteria are automatically and empirically tested during practical workshop performance checks, this also being a three stage process.

4.6 PRACTICAL WORKSHOP STABILITY VERIFICATION TESTS – DAL

Figure 8 illustrates the workshop test for DAL systems.

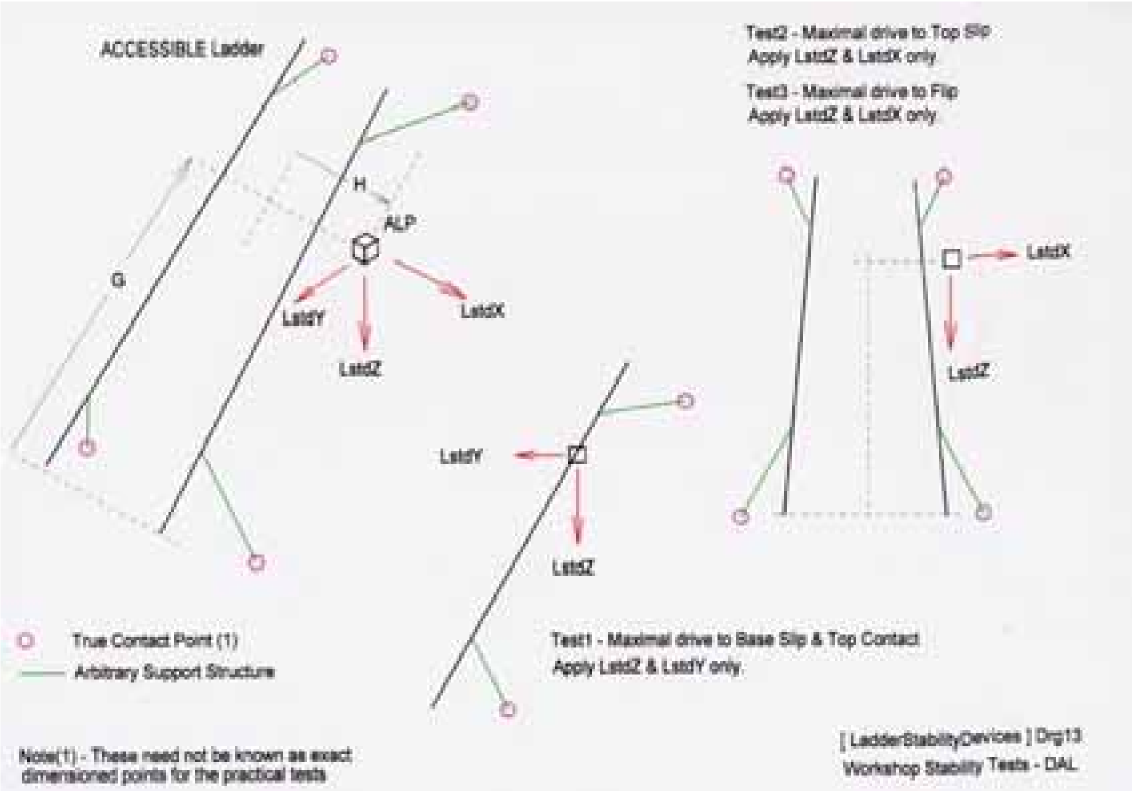

Figure 8

Workshop stability tests

The parameters G_{set}(m) and H_{set}(m) define the ALP for the DAL. These are referenced to the accessible ladder. Note that the access limit dimension F(m) is found directly at G(m), and H(m) is found by $H = F + H_{set}$.

The three tests (1 to 3) should be performed in sequence with the prescribed values of SLV. This varies for each test and specific sets of values of $L_{std}Z$(kg), $L_{std}Y$(kg) and $L_{std}X$(kg) are required and given in Table 1.

Provided the ladder remains upright for all three conditions, the ladder is qualified compliant.

The SLV is applied at the ALP, but the ALP lies outside the physical ladder. Any simple but strong temporary projecting structure will be required to perform this operation.

The DAL is defined and tested in a fully configured form, hence employs a given ladder with a given additional retrofit support system. It follows that a DAL strictly requires a specific ladder as part of the stability qualification regime. It will be generally the case however that DALs are created ad hoc, with arbitrary nominal ladders. It is a fact that the final stability status of any given configuration is sensitive to the nominal ladder in terms of geometry and mass distribution. This is recognised as beyond the control of a DAL stability device manufacturer. For this reason a designer should test with differing types of ladder covering available geometries and weight, and perhaps qualify on this basis. For instance the maximum relevant dimensions for which they have tested which maintain a cross modality index of 1.0 or greater.

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5.0 THEORETICAL STABILITY MODEL – TRIPOD

Figures 9 to 12 are provided in support of the stability model for tripod devices.

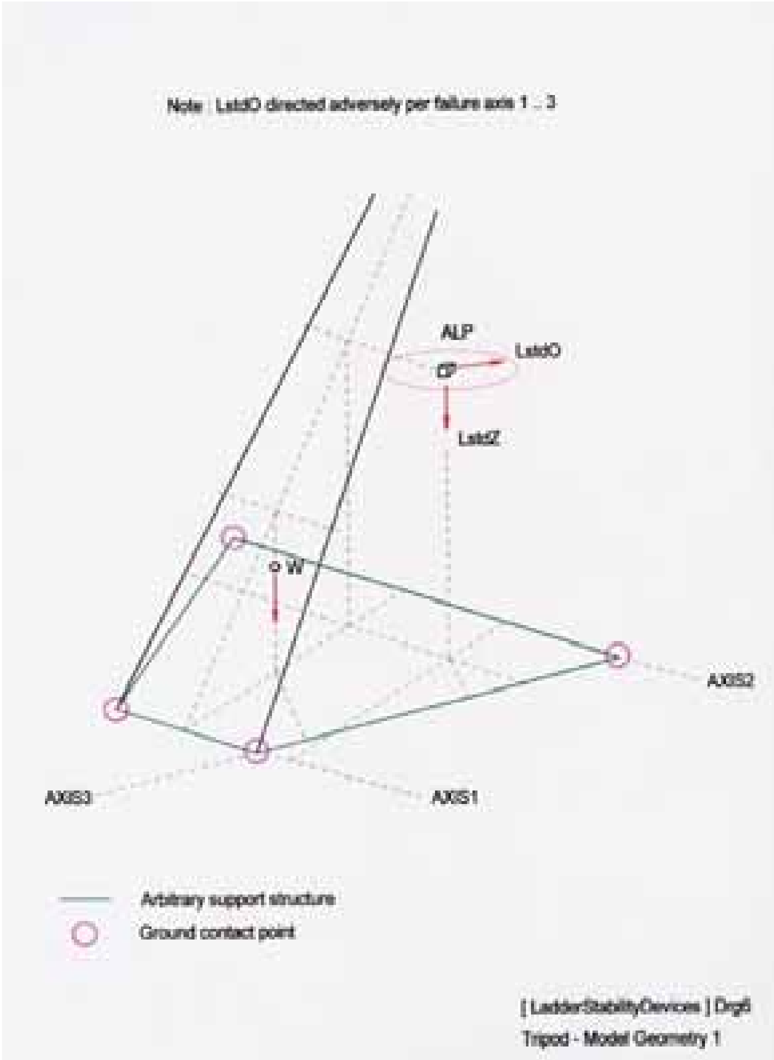

Figure 9

Tripod model geometry 1

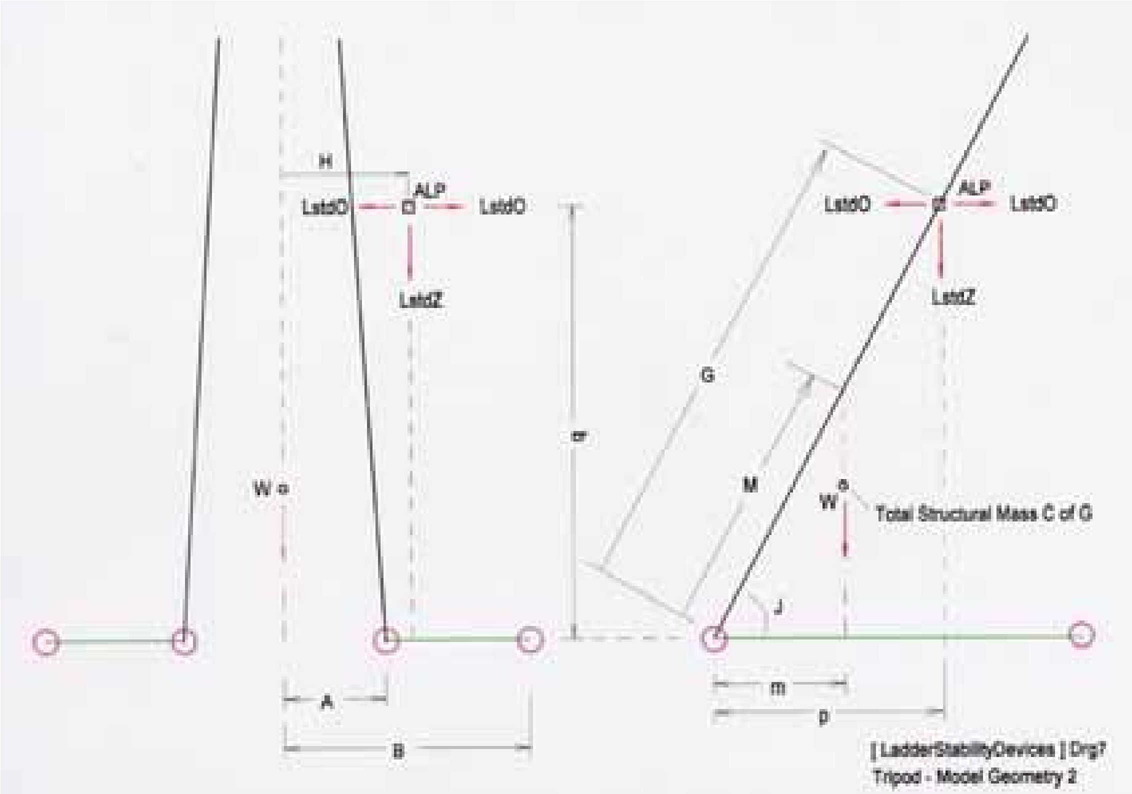

Figure 10

Tripod model geometry - 2

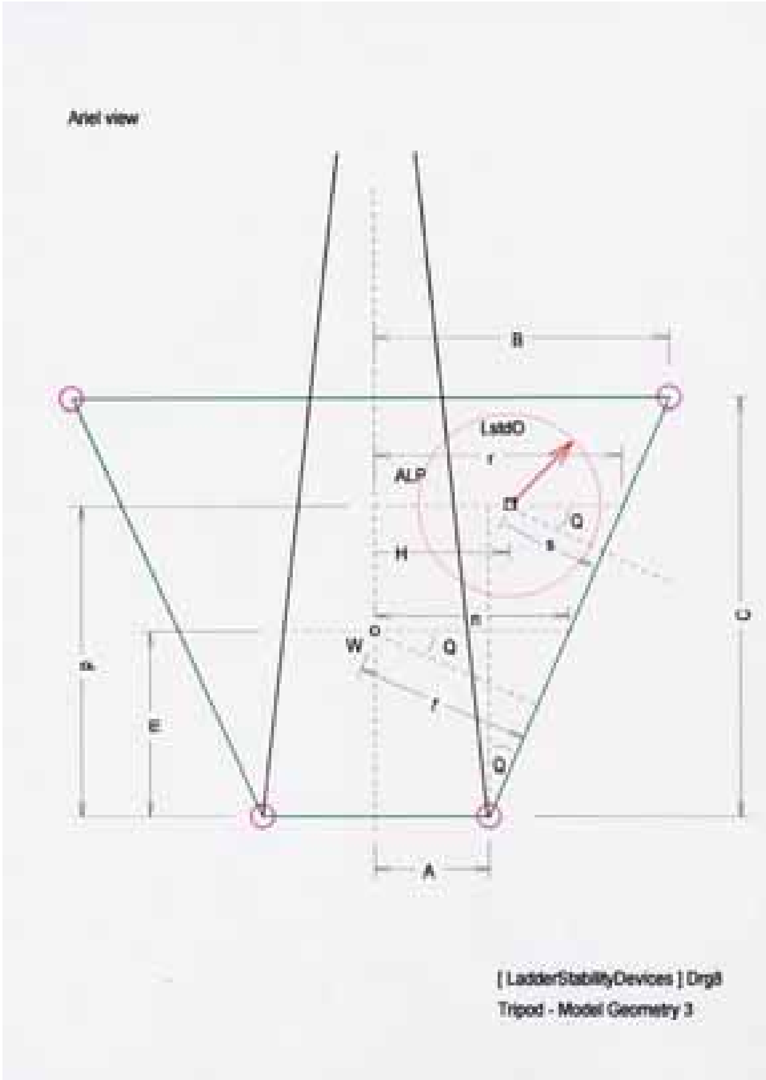

Figure 11

Tripod model geometry - 3

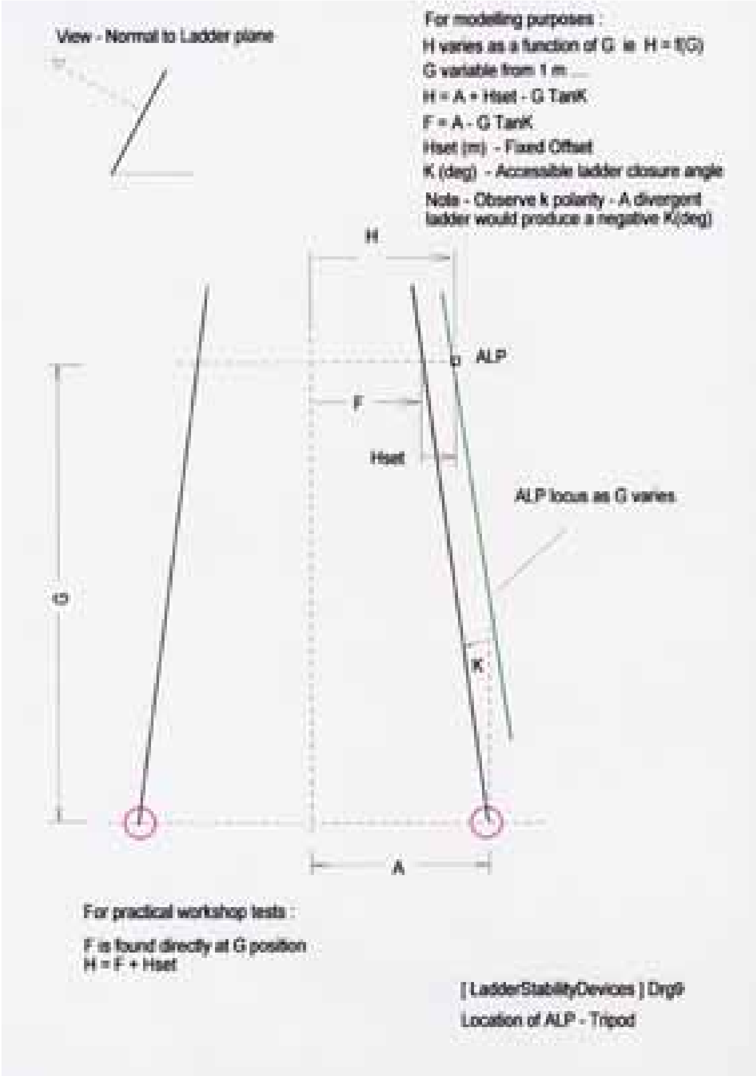

Figure 12

Location of ALP - tripod

The tripod is capable of flip failure about any one of three axis at the base – denoted AXIS 1 to AXIS 3. The mechanical condition where instability occurs is at a point of torsional balance. The combined action drive of the user expressed as a SLV acting at the ALP, the structural weight Centre of Gravity, and the base geometry fully determine the critical points of balance.

The predictive model expresses stability as three indices $S_{int}1(\#)...S_{int}3(\#)$, which are calculations based upon the relative surfeit of corrective torsional drive acting against a destabilising drive. A resultant index value of 1.0 indicates that the highest user duress is just able to reach the natural limit envelope of the system, hence is in a qualified safe status.

The model is crucially dependent upon the SLV magnitude, and this must be determined correctly. There is a strong geometric argument with this ladder configuration, in favour of a polar type of standard action vector, matching a more circular than rectangular symmetry. The authors have not directly measured tripod activity, but can reasonably infer from DAL information, the level of drives in operation.

From a functional point of view the tripod is equivalent to a regular leaning ladder, albeit that the support system is entirely different. However where the leaning ladder tends to offer restricted forward access, typically because of a large planar obstruction (typically a wall), the tripod is somewhat more accessible in this direction. In addition, the natural portability of the tripod configuration readily allows a user to place the structure in any attitude to the work intended, but particularly such that they can work in reverse stance, i.e. work towards the rear of the structure. The leaning ladder DAL data is strongly indicative that lateral (x-axis) user generated drive is much stronger potentially than is forwards (y-axis) drive. If it is considered that this is reasonably an ergonomics issue, in terms of ease of access or user comfort, then a pragmatic argument is to consider that a user on a tripod can, and will, operate at all horizontal planar angles non-preferentially. The use of polarised or axis specific forces for the tripod should therefore be rejected, and a single omni-directional force serving as single maximal SLV drive should be chosen. Hence the use of a standard load vector $L_{std}O$(kg), which is a fixed value horizontal planar drive allowed to operate at any bearing.

The required high duress standards for the Tripod are taken directly from the DAL parameters :

- The maximal value $L_{std}X$ is taken from the DAL parameters and assigned to $L_{std}O$ also.
- The minimal value $L_{std}Z$ is taken from the DAL parameters and assigned to $L_{std}Z$ also.

The action placement of the user, represented by the ALP location, is equivalent to the DAL definition, and is partially specified through the H_{set}(m) parameter.

The ALP height parameter G(m) is allowed to be variable in the formal technical model, with stability indices $S_{int}1$(#)...$S_{int}3$(#) responding accordingly. For a given structural format, the model will allow a maximal value of G(m) to be found, occurring at the first failure of any one of the three axes. This essentially defines a maximal allowable operating height for the user, on any given fixed tripod structure. It should be realised that this limit height is not related directly to the total accessible ladder length, but is simply measured from the ground. In this manner it can be seen to be relational to the stability device and its precise manner of attachment to the ladder.

The spatial location of the ALP is practically close to the user's mass Centre of Gravity, and when determining the operational height limits by defining an equivalent footing height limit, the intervening distance must be allowed for.

5.1 ANALYTIC MODEL PARAMETERS – DEFINITIONS - TRIPOD

Refer to Figures 9 to 12

Measured Structural Parameters :

The measured structural parameters are identified thus:

- **A (m)** Structural dimension
- **B (m)** Structural dimension
- **C (m)** Structural dimension

- **J (deg)** Elevation angle – Accessible ladder
- **K (deg)** Riser Closure angle - Accessible ladder

- **W (kg)** Total Weight – combined structure

- **M (m)** Weight C of G of combined structure within Accessible Ladder

Design Variables :

The design variables are identified thus:

- **G (m)** Applied Load Point (ALP) – Model variable determines H (m) (Set by designer)

Prescribed Standard Parameters :

The prescribed standard parameters are identified thus:

- **$L_{std}O$ (kg)** Standard applied load vector (SLV) – Planar horizontal & Omni-directional

- **$L_{std}Z$ (kg)** Standard applied load vector (SLV) – Z axis

- **H_{set} (m)** Standard set distance offset from Access Limit Dimension F(m) edge limit

Modelled Performance Parameters :

The modelled performance parameters are identified thus:

- **F (m)** Access Limit Dimension at G(m)

- **H (m)** Applied Load Point (ALP) as f(G)

- **$S_{int}1$ (#)** Normalised Intrinsic Stability – Failure pivot axis 1
- **$S_{int}2$ (#)** Normalised Intrinsic Stability – Failure pivot axis 2
- **$S_{int}3$ (#)** Normalised Intrinsic Stability – Failure pivot axis 3

Intermediate Modelling Parameters – Transient usage only

The intermediate modelling parameters are identified thus:

- **p, q, m, n, r ,s, t (m)** Virtual dimensions

- **Q (deg)** Construction angle

5.2 ANALYTIC MODEL PARAMETERS – FORMAL DERIVATIONS - TRIPOD

This section contains the formal derivations of the analytical model parameters for the tripod structures.

$$TanQ = \frac{B - A}{C}$$

$$Q = ArcTan\frac{B - A}{C} \qquad \textbf{(1)}$$

$$SinJ = \frac{q}{G}$$

$$q = GSinJ \qquad \textbf{(2)}$$

$$CosJ = \frac{m}{M}$$

$$m = MCosJ \qquad \textbf{(3)}$$

$$CosJ = \frac{p}{G}$$

$$p = GCosJ \qquad \textbf{(4)}$$

$$TanQ = \frac{n - A}{m}$$

$$n = A + mTanQ \qquad \textbf{(5)}$$

$$TanQ = \frac{r - A}{p}$$

$$r = A + pTanQ \qquad \textbf{(6)}$$

$$CosQ = \frac{s}{r - H}$$

$$s = (r - H)CosQ \qquad \textbf{(7)}$$

$$CosQ = \frac{t}{n}$$

$$t = nCosQ \qquad \textbf{(8)}$$

$$F = A - GTanK \qquad \textbf{(9)}$$

$$H = F + H_{set} \qquad \textbf{(10)}$$

Balancing Torsions

The balancing torsions are described by the following expressions.

$$S_{\text{int}}1 = \frac{Wm + L_{std}Zp}{L_{std}Oq} \qquad \textbf{(11)}$$

$$S_{\text{int}}2 = \frac{W(c - m) + L_{std}Z(c - p)}{L_{std}Oq} \qquad \textbf{(12)}$$

$$S_{\text{int}}3 = \frac{Wt + L_{std}Zs}{L_{std}Oq} \qquad \textbf{(13)}$$

5.3 STANDARD LOAD VECTOR SLV AND APPLIED LOAD POINT ALP - TRIPOD

These parameters are set at qualified high duress levels as defined in Table 2:

Table 2

Standard Load Vectors and Applied Load Point value for tripods

Parameter	Value
$L_{std}Z$ (kg)	60 kg
$L_{std}O$ (kg)	23 kg
H_{set} (m)	0.135 m

G(m) is left as a design variable and is undefined within the provided stability performance model. However G(m) achieves a natural upper limiting value by restraint in values of $S_{int}1(\#)$ to $S_{int}3(\#)$, each required to be equal to or greater than 1.0 for assured stability in the three tip failure modes.

The stability model as presented will identify for a designer the maximal conditionally safe ALP altitude at G(m), for a given ladder in this configuration class. The corresponding distance H(m) is then directly computed from appropriate measured or prescribed parameters and hence fully defines the ALP.

For reference :

- F = A - G TanK Note – F(m) is the access limit dimension at G(m).
- $H = F + H_{set}$

The ALP will generally lie outside the ladder structure within a line locus, parallel with the side riser and in the plane of the ladder. So with G(m) allowed to be variable, H(m) is forced a distance H_{set}(m) beyond F(m), the access limit dimension.

For practical workshop stability verification purposes F(m) is best found by direct reference at the inner face of the side riser, existing at any given testing altitude G(m). The ALP is then easily located a further distance H_{set}(m) beyond F(m).

The SLV is applied at the ALP as prescribed by both the model definition and workshop procedures, these being directly equivalent. The tripod requires a single set of SLV component magnitude values for all conditions, but the direction of $L_{std}O$(kg) varies with the tripod geometry.

5.4 INTRINSIC STABILITY INDICES – SINT1, SINT2, SINT3 – TRIPOD

The primary function of the stability modeller is to predict the global safety assurance of any given Tripod through numerated quantities indicating proximity to stability failure. The stability indices are normalised to value 1.0 and can be universally compared across arbitrary configurations. Stability pertaining to a given failure mode is assured if the relevant parameter attains a value equal to or greater than 1. Total system stability integrity occurs if all indices meet the criteria.

The stability modeller requires categories of information in the following classes :

1. Structural geometric – Directly measurable key dimensions
2. Weight – W(kg)
3. Prescribed standard parameters – SLV , H_{set}(m)
4. ALP altitude parameter G(m) – Design variable

This generates output parameters :

1. Intrinsic stability indices $S_{int}1$(#), $S_{int}2$(#) and $S_{int}3$(#) as function of G(m)
2. Computed ALP as function of G(m)

There are three verification tests requiring a single universal SLV, which individually stress the system maximally towards the respective failure modes. When employing the modelling algorithm, the test is done once by assigning the prescribed values of SLV given as elements in $L_{std}O$(kg) and $L_{std}Z$(kg), and defined in Table 2, and the prevailing stability indices $S_{int}1$, $S_{int}2$ and $S_{int}3$(#) are simultaneously figured as a function of G(m). These indices are therefore qualified at a defined ALP altitude.

The model ignores results for G(m) < 1m, and stability indices should not be calculated or expected in this altitude range. There is no assigned upper limit to which G(m) can take as a numerical system variable, but the stability criteria itself dictates an upper working limit.

- **Test 1** - Maximal duress to axis 1 failure – $S_{int}1$(#) valid
- **Test 2** - Maximal duress to axis 2 failure – $S_{int}2$(#) valid
- **Test 3** - Maximal duress to axis 3 failure – $S_{int}3$(#) valid

These criteria are sequentially and empirically tested during practical workshop stability performance tests.

5.5 PRACTICAL WORKSHOP STABILITY VERIFICATION TESTS – TRIPOD

Figure 13 illustrates the workshop tests for tripod devices

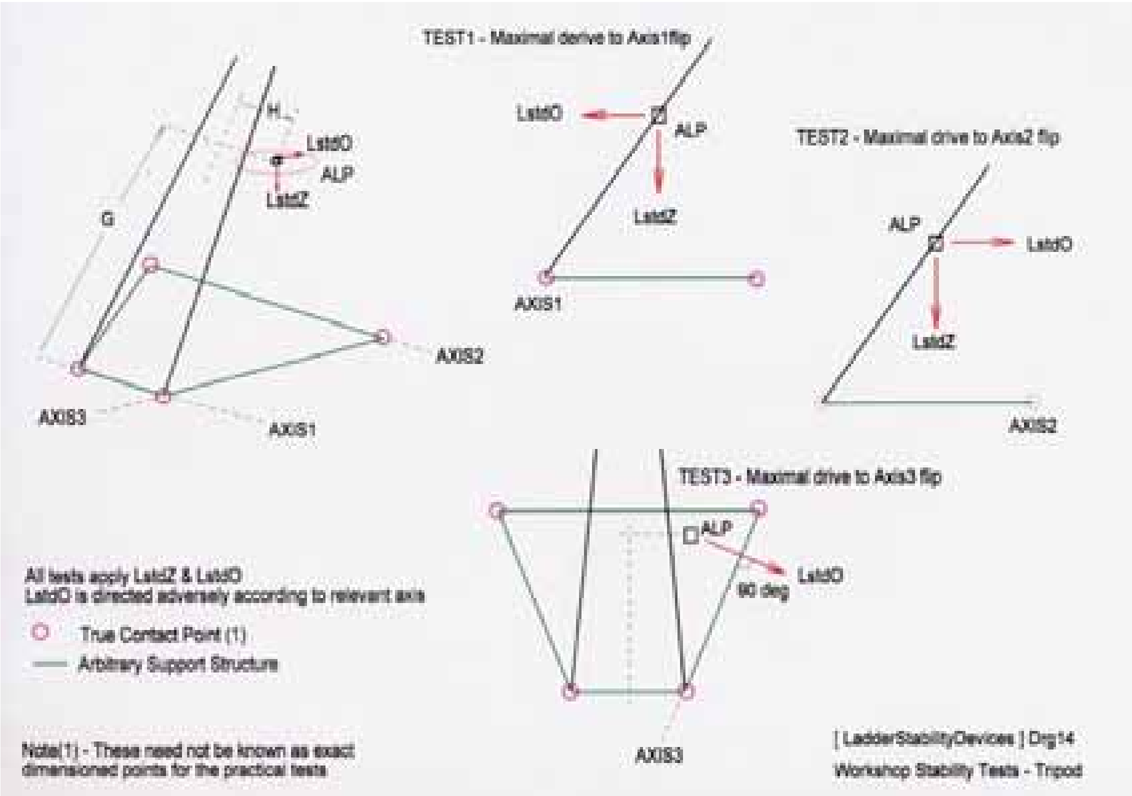

Figure 13

Workshop stability tests - tripod

There is no prescribed height for the ALP, which is equivalent to the maximal working altitude of the user. The parameter G(m) can be chosen at any value and the workshop test performed. However G(m) determines the position where the access limit dimension F(m) should be determined empirically. The variable H_{set}(m) with F(m) now fixes H(m), and hence the ALP is determined.

The three tests 1 to 3 should be performed in sequence with the prescribed values of L_{std}Z(kg) and L_{std}O(kg). Note that L_{std}O(kg) is of fixed magnitude but variable direction. This is always applied in the horizontal plane containing the ALP, but directed adversely to each failure axis in the base. The implication is that some maximal G(m) will be evident for any given structure, hence an arbitrary Tripod configuration will have associated a qualified working height.

Provided the ladder remains upright for all three conditions, the ladder is qualified compliant.

The SLV is applied at the ALP, but the ALP lies outside the physical ladder. Any simple but strong temporary projecting structure will be required to perform this operation.

The Tripod is defined and tested in a fully configured form, hence employs a given ladder with a given stability device system. It follows that a Tripod strictly requires a specific ladder as part of the stability qualification regime. It will be generally the case however that Tripods are created ad hoc, with arbitrary nominal ladders. It is a fact that the final stability status of any given configuration is sensitive to the nominal ladder in terms of geometry and weight Centre of Gravity, which is itself a function of the extended length of the ladder. This is recognised as beyond the control of a tripod stability device manufacturer. For this reason a designer should test with differing classes of ladder covering available geometries and weight, and perhaps qualify on this basis.

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6.0 MODELLING SPREADSHEETS – PREDICTIVE MODEL IMPLEMENTATIONS

The modelling algorithms for DAL and Tripod configurations are fully detailed in Section 4 and Section 5 and can be implemented via any convenient method. The authors utilise two major active spreadsheets which fully implement the specified algorithms, and can be used to explore the stability and general performance of arbitrary ladder configurations in the relevant class.

- **Stability Predictor 1 -** DAL Stability Modeller – Interactive spreadsheet

This software is illustrated in Figure 14.

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DAL Stability Performance Predictor

File [LadderStabilityDevices] StabilityPredictor1

Measured Structural Parameters

I (m)	7.44
A(m)	0.20
B (m)	0.25
C (m)	1.80
D (m)	1.80
F (m)	0.18
M (m)	2.98
J (deg)	75
K (deg)	75
W (kg)	25

Prescribed Standard Parameters

LstdX (kg)	20	35
LstdY(kg)	18	22
LstdZ (kg)	72	72

Gset (m)	0.35
Hset(m)	0.155

Rcontact (kg)	3

User specified parameters

Ubaselim (#)	0.6
Utoplim (#)	0.6

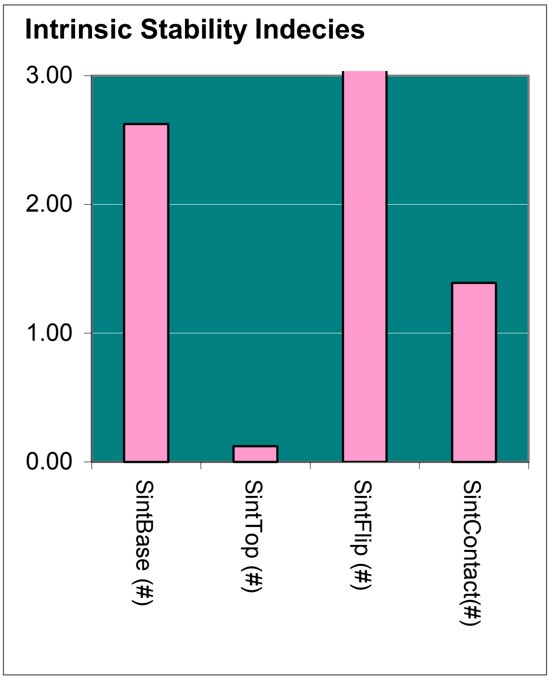

Figure 14 – Example of the Predictor 1 software output

Users should enter parameters in the data fields denoted :

- Measured Structural Parameters
- Prescribed Standard Parameters
- User Specified Parameters

Note that three defined sets of SLV parameters $L_{std}Z$ $L_{std}Y$ & $L_{std}X$(kg) are required to sequentially check for stability compliance in four failure modes.

The sheet will process this data and generate all output parameters in accordance with the specification. This data is presented both numerically and graphically.

- **Stability Predictor 2** - Tripod Stability Modeller – Interactive spreadsheet

This software is illustrated in Figure 15.

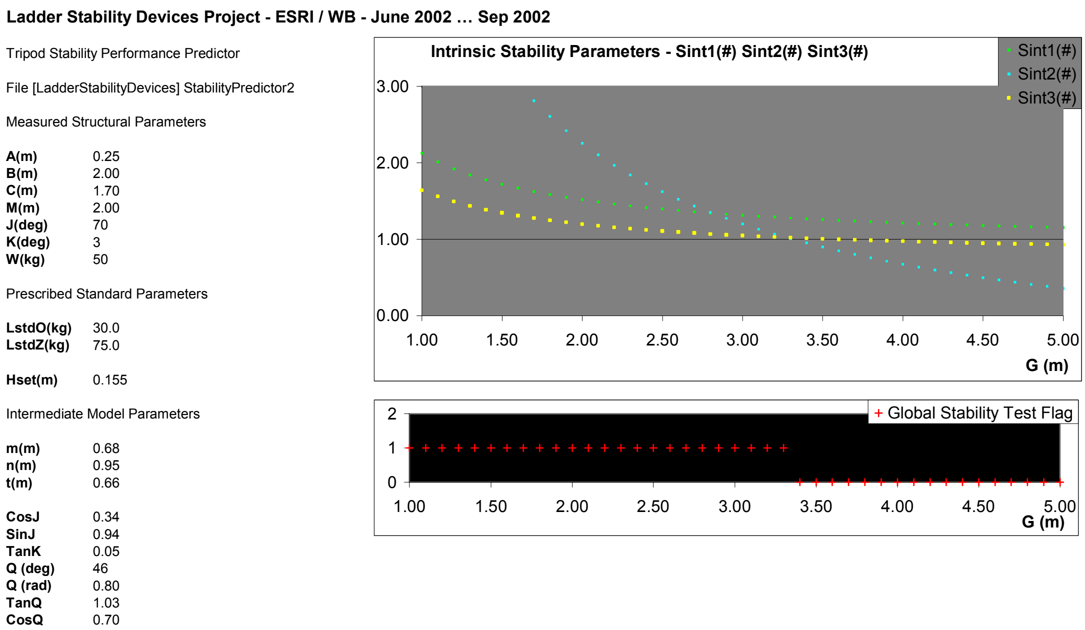

Figure 15 – Example of the Predictor 2 software output

Users should enter parameters in the data fields denoted :

- Measured Structural Parameters
- Prescribed Standard Parameters

The sheet will process this data and generate all output parameters in accordance with the specification. The height parameter G(m) is incrementally considered over the range 1 to 5 m, with stability estimates made over the whole of this range. The parameters $S_{int}1$ to $S_{int}3$ are tested for compliance to be greater than 1.0.

Provided all are compliant then the global stability flag will indicate this. An upper limit of G(m) for the given structure will be determined. This data is presented numerically and graphically.

7.0 MANUAL FOOTING – THEORETICAL CONSIDERATIONS

For manual footing to be of benefit, then an improvement must be afforded in one or more stability indices. Furthermore, this improvement should be worthwhile. Lastly, it should be reliably maintained over the entire period of user activity. It is important to note that improvements in safety envelope margins may be redundant if the user/ladder system is already comfortably placed within the calculated safe zone, expressed numerically as S_{int}Top(#), S_{int}Base(#) S_{int}Flip(#) and S_{int}Contact(#). Additionally, if 'footer' performance is variable or erratic during the event, then the actual additional safety utility afforded will vary from time to time, meaning that it could reduce to zero or even reverse, generally unpredictably. This is at best useless and worse dangerous. A small number of trial participants definitely exhibited such counterproductive actions upon the ladder, meaning that the issue of footing seems more profound than previously thought. An example is shown in Figure 16

Figure 16

Highly asymmetric footing technique

The 'footer' is generally imparting a directed action vector which offers zero torsional action benefiting top-slip failure or top-contact failure. Furthermore the leverage is adverse and imparts very little modified contact action at the top, hence has negligible effect upon U_{top}(#) or R_{top}Y(kg), and does not improve S_{int}Top(#) or S_{int}Contact(#). It can therefore be seen that even with good technique, footing will do nothing to improve ladder stability in:

- Top slip failure mode.
- Top contact failure mode.

The benefits in the remaining failure modes are discussed in sections 7.1 and 7.2 below.

7.1 FLIP MODE

When considering failure in flip mode, it is evident that any nominal footing action could aid this situation through counteractive torsions about the principal ladder axis. **However for this to be of benefit, the 'footer' is required to impart a particularly symmetric force, acting centrally and squarely within the ladder and balanced left to right, amounting to a structured and disciplined footing methodology**. For illustration, even a high force level directed and acting through a ladder side riser will have virtually zero effect on the torsional tension within the ladder, and hence similarly zero effect on S_{int}Flip(#). Due to observable variation in style and stance preference, the drive to the ladder is typically erratic in this particular respect of symmetry, but crucially the action Centre of Gravity is volatile and moving unpredictably between the ladder base contact points. The added torsional utility, acting to enhance flip safety, is instant to instant varying from some potentially useful upper value down to near zero, and could conceivably be negative in effect if the footing drive becomes overly asymmetric. In summary, flip stability can be enhanced in theory by prescribing a specified and disciplined footing technique, but is prone to be unreliable and unpredictable given an undefined or freestyle footing technique, and should currently be considered as non-existent.

No relative flip enhancement utility measure is attempted in this survey for these reasons. However, cursory examination of the two riser base forces and the degree of balance in particular, demonstrates the volatile nature of flip mode protection. A high efficiency manually delivered footing technique could nevertheless, realistically double the effective value of S_{int}Flip(#) at normal operational levels.

7.2 BASE SLIP MODE

Enhancement in base-slip safety margins is markedly less sensitive to footing technique asymmetries, and by contrast are more reliable and predictable. This failure mode is highly sensitive to footing actions, and can reasonably be measured. The mechanism is theoretically considered and numerically analysed, and is expressed as fractional changes in the frictional demand parameters.

The Frictional Demand for a non-footed ladder is U_{base}(#) for any given R_{base}Y(kg) and R_{base}Z(kg), and is generally given by :

$$U_{base} = \frac{R_{base}Y}{R_{base}Z}$$

The Frictional Demand for a footed ladder is U_{foot}(#) for any given R_{base}Y(kg), R_{base}Z(kg) & FY(kg) & FZ(kg), and is generally given by :

$$U_{foot} = \frac{R_{base}Y - FY}{R_{base}Z + FZ}$$

Ordinarily $U_{foot} < U_{base}$ and is consistent with an increase in S_{int}Base(#), but it is dependent upon the footing technique. In some trial case instances this was not true, with U_{foot}(#) exceeding U_{base}(#), and effectively reducing the safety status in base-slip mode failure.

7.3 MANUAL FOOTING – MEASURED UTILITY

Figure 17 provides background to the manual footing calculations

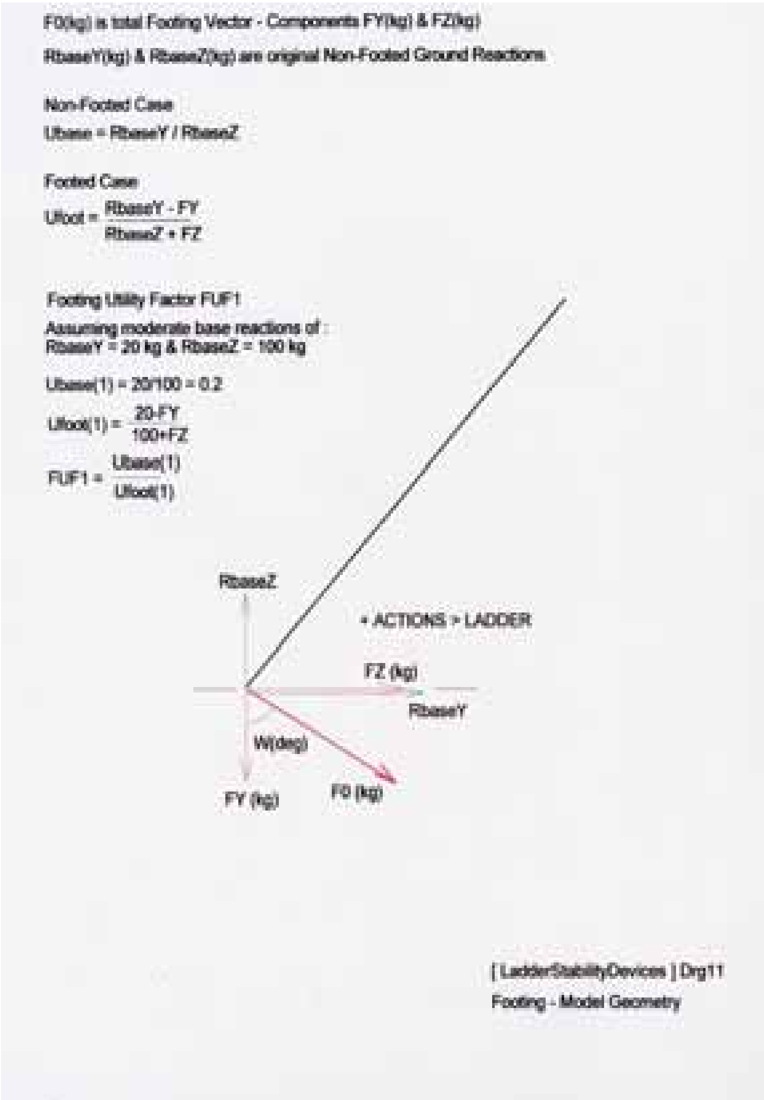

Figure 17

Footing model geometry

Footing trials produce time variant data sets measuring the mechanical drive developed at the ladder base, due to the action of the trial subject. This occurred with a zero loaded ladder, and are therefore direct measurements of the imparted drive only.

- $FZ = R_{base}Z$ +ve Footing ACTION > Ladder (kg)

- $FY = R_{base}Y$ +ve Footing ACTION > Ladder (kg)

- $FO = \sqrt{FY^2 + FZ^2}$ Total Footing Vector – Magnitude (kg)

- $W = ArcTan\left(\frac{FY}{FZ}\right)$ Footing vector argument (deg)

The held force levels are analysed into a single representative action vector in terms of magnitude and direction. These parameters are calculated from samples validated by conditional tests. This yields reliable footing strength measures, independent of user arbitrary settling and release activities.

Data acceptance test - FZ > 0.5 x 90th Percentile of all FZ

- FZ_{med} (kg) Median of all valid FZ
- FY_{med} (kg) Median of all valid FY
- FO_{med} (kg) Median of all valid FO
- W_{med} (deg) Median of all valid W_{med}

The actual modification in U_{base}, and hence S_{int}Base, is a function both of the particular base load imparted by the 'user' on the ladder, and the force actions imparted by the 'footer'. Both are therefore variables in any real situation.

Accordingly, a measure of footing utility has been developed.

Assuming a nominal standing base load of $R_{base}Y$ = 20kg & $R_{base}Z$ = 100kg :

$$U_{base(1)} = \frac{20}{100} = 0.2$$

Revised base demand due to footing is calculated :

$$U_{foot(1)} = \frac{20 - FY_{med}}{100 + FZ_{med}}$$

We produce a normalised Footing Utility Factor FUF1 :

$$FUF1 = \frac{U_{base}(1)}{U_{foot}(1)}$$

FUF1 > 1.0	Greater utility – Reduction in frictional demand – Increase in S_{int}Base(#)
FUF1 = 1.0	No change
FUF1 < 1.0	Reduced utility – Increase in frictional demand– Reduction in S_{int}Base(#)

FUF1 is acceptable as a pragmatic measurement and would be broadly meaningful in all practical and real cases. It correctly ranks the added value of the footing activity with respect to base slip stability.

Arbitrarily high FUF1 values where observed in the trial analysis, with factors easily reaching 5. This corresponds to the frictional demand at the ladder base reducing to 20% of the nominal standing value without footing. A number of subjects yielded negative values for FUF1 indicating overcompensation and beyond, with full reversal of reaction vectors. The utility obtained from footing is evidently a highly erratic parameter within the conditions of this trial, and particularly with the freedoms allowed in style. A good number of trial subjects yielded FUF1 levels close to zero, indicating no measurable effect.

Definition of Footing Activity types :

- **Activity 1** 2 Feet + Arms
- **Activity 2** 1 Foot + Arms
- **Activity 3** Arms only
- **Activity 4** Other

A ladder will normally operate with a base frictional demand at a conservatively high value of 0.25, corresponding to high duress and the qualification tests in Section 8 of the main report.

Also this parameter is not susceptible to high deviation in the form of transients. From previous work measuring friction limit performance levels of nominally loaded ladders on cement surfaces, the U_{base}lim(#) parameter was very reliably observed at 0.5. With less certainty but high probability, values of up to 0.7 prevailed. If this is so, then the clear implication is that while ladder footing can easily improve S_{int}Base(#), it is arguably not required.

A number of trial results indicated very adverse footing actions. Some of these were completely mis-directed actions effectively reducing the prevailing safety envelope. Others pushed so hard that they overcompensated, through the zero condition, and actually reversed the normal ground reaction direction.

7.4 MANUAL FOOTING – OPTIMUM METHODS

Generally, an optimum footing action will deliver a single vector at the ladder, acting square-on and central, and at any convenient but low height. The applied action can be any combination of horizontal force towards the ladder, or down through the ladder. Directed forces outside of this quadrant are actively counterproductive. There is a marked disparity in direction sensitivity in relation to utility. As a rule of thumb, 1 kg horizontal in the y-axis is equivalent to 5 kg vertically down in the z-axis. A 'footer' can easily use their weight to generate z-action, however y-action will ordinarily require constant muscular activity.

A simple method, therefore, of achieving efficient and consistent footing is to stand centrally on the lowest rung with both feet, or via some simple seat, to sit at, or near, the ladder base. Both these approaches utilise only z-action arising through weight, and are therefore continuously sustainable by the person. More importantly, they tend to ensure the required action direction symmetry as previously explained.

It can be shown, therefore, that correctly applied footing can very usefully improve the flip mode stability of a ladder where users will frequently operate in close proximity to the safety envelope, but requires consistent and symmetric action delivery. Base slip safety margin can be technically improved, but is evidently not the most pressing requirement. Top-slip or top-contact mode is not easily modified due to the adverse leverage argument, and is practically independent of footing activity.

8.0 GROUND SLOPE – EFFECT ON BASE FRICTIONAL DEMAND

Figure 18 illustrates the frictional demand mechanics

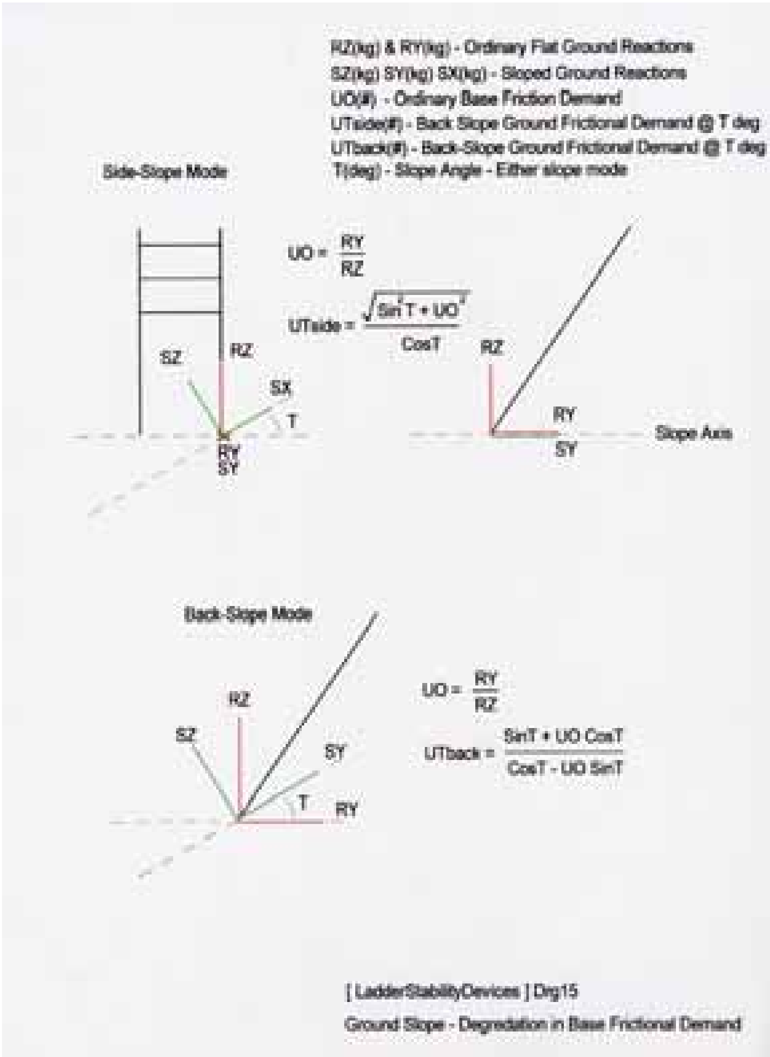

Figure 18

Degradation of base frictional demand from ground slope

Any ground inclination to the side (side slope mode) or the rear (back slope mode) will reduce slip stability margins. The slope alters the base reaction vectors adversely from a friction demand perspective. The additional slip potential in side-slope and back-slope mode was investigated from a theoretical approach, and useful measures which best quantify the effect are identified.

Developed parameters :

- **RY(kg)** Base horizontal reaction – Flat ground equivalent
- **RZ(kg)** Base vertical reaction – Flat ground equivalent

- **U0(#)** Ordinary Frictional Demand - Flat ground equivalent

- **UT_{side}(#)** Elevated Friction Demand – Side-slope mode
- **UT_{back}(#)** Elevated Friction Demand – Back-slope mode

- **T(deg)** Slope Angle – either mode

8.1 SIDE-SLOPE MODE

A side directed ground slope is problematic from a gross stability point of view because the ladder will rotate about a near vertical ladder axis between two distinct stable stances. This swing can be large for even small side slope gradients, and must be checked. A packing piece, or purpose designed extender device, can be arranged at the foot to take up the nominal slack. This will serve to eliminate the gross motion of the ladder, which will then perform equivalent to a ladder on flat ground. Note that it is not required, or possible, to wedge the shortfall since the ladder base will behave as a normal ladder, with load freely transferring between the feet, and hence likely to reach zero load on the packing device during real activity. However any interposed device or material must have reliable limit friction capability, at least equal to the designed foot.

The ordinary flat ground base frictional demand is defined as U0(#) where:

$$U0 = \frac{RY}{RZ}$$

For a given ground incline of T(deg) the base friction demand rises to a new value UT_{side}(#).

It can be shown that :

$$UT_{side} = \frac{\sqrt{(Sin^2T + U0^2}}{CosT}$$

Side-mode frictional demand rises relatively gently with increasing T(deg) initially, but more rapidly increases as T(deg) advances. For a given qualified maximal U0(#) = 0.25 as would reasonably prevail on flat ground, the formula gives us that UT_{side}(#) will reach value 0.5 at about 22 deg – approaching a slip failure condition. However UT_{side}(#) is below 0.4 up to about 16 deg, and which leaves a reasonable operational safety margin.

8.2 BACK-SLOPE MODE

The ordinary flat ground base frictional demand is defined as U0(#) where:

$$U0 = \frac{RY}{RZ}$$

For a given ground incline of T(deg) the base friction demand rises to a new value UT_{back}(#)

It can be shown that :

$$UT_{back} = \frac{SinT + U0CosT}{CosT - U0SinT}$$

Back-mode frictional demand rises aggressively fast with increasing T(deg) initially at approximately 0.02 per deg, and more rapidly still as T(deg) advances. For a given qualified maximal U0(#) = 0.25 as would reasonably prevail on flat ground, the formula gives us that UT_{back}(#) will reach value 0.5 at about 12 deg – approaching a slip failure condition. However UT_{back}(#) is below 0.4 up to about 6 deg, and which leaves a reasonable operational safety margin.

Clearly a rearward incline is the more critical condition with minimal tolerance to angle magnitude. As a rule of thumb for ground inclines, stability degradation in back-mode is broadly three times more sensitive to angle than is side-mode. Within the suggested angular limits given above, the ladders are nevertheless naturally liable to remain stable without additional devices or manual footing.