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Development of Parametric Equations for MK-Factors for Semi-Elliptic Cracks in T-Butt Welds

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DEVELOPMENT OF PARAMETRIC EQUATIONS FOR MK-FACTORS FOR SEMI-ELLIPTIC CRACKS IN T-BUTT WELDS

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SUMMARY

Parametric equations have been developed to describe the Mk-factor for a semi-elliptic crack located at the toe of a T-butt weld under membrane and bending loading conditions. The equations are based on stress intensity factor data, which has been derived by University College London from the results of a series of finite element analyses using weight function methods. The parametric equations give Mk-factors at both the crack tip and at the surface positions for a wide range of weld attachment lengths, toe radii and fillet angles.

The equations have been fitted to the data using standard non-linear regression analysis methods. A conservative fit to the data has been achieved by using an asymmetric loss function to weight unconservative predictions. Analysis of the residual errors indicates that the values predicted by the parametric equations are unconservative by no more 2% and over-conservative by a maximum of 21% over the region of interest. The mean error between the predicted values and the data is less than 6%.

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1. INTRODUCTION

The new draft British Standard for the assessment of defects in structures, BS 7910: 1997 (1), includes new stress intensity factor solutions for a semi-elliptic crack in a T-butt welded joint. The new solutions provide stress intensity factors at the crack tip and at the surface for welds with different attachment sizes, fillet angles and weld toe radii. The stress intensity factors have been derived from the results of a large number of finite element analyses by University College London using approximate weight functions for a semi-elliptic crack (2). The data has been fitted to a series of parametric equations, which are given in Appendix J of the draft standard.

In principle, these solutions should improve the accuracy of fracture mechanics fatigue crack growth calculations and brittle fracture assessments performed on semi-elliptic cracks in T-butt welded joints. However, the parametric equations given in the draft standard are not only complex and difficult to use, but give an unconservative fit to the underlying data, which can underestimate the stress intensity factor by up to 30%.

This report describes the development of a new set of simplified parametric equations which are intended to provide an improved fit to the data and eliminate unconservative predictions. The new equations are expressed in terms of Mk-factors which represent the ratio of the stress intensity factor for a semi-elliptic crack in a welded joint to that for a similar crack in a flat plate. The stress intensity data on which the equations are based is described in Section 2. The analysis methods used to develop the parametric equations are outlined in Section 3. The resulting parametric equations and the goodness of fit to the data are described in Sections 4 and 5. Finally, Section 6 gives the conclusions of the study.

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2. STRESS INTENSITY DATA

Stress intensity factors have been derived for semi-elliptic cracks, located at the toe of a T-butt weld with a non-load carrying attachment, under both membrane and bending loading (2). Stress intensity factors are available at the deepest point of the crack and at the surface for weld geometries with a range of attachment lengths, toe radii and fillet angles as shown in Figure 1 below:

Weld fillet angle	$\alpha = 30°, 45°, 60°$
Attachment width ratio	$L/T = 0.1577$ to 4.0
Weld toe radius ratio	$\rho/T = 0.01$ to 0.066

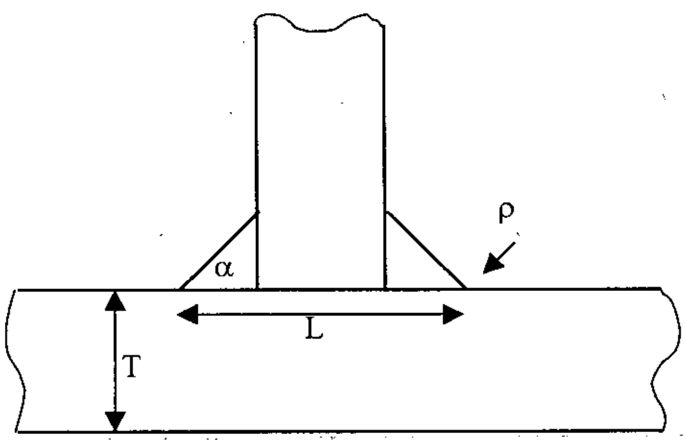

Figure 1
T-butt weld geometry

Data is available for the following crack aspect ratios and depths:

Aspect ratio	$a/c = 0$ to 1
Crack depth ratio	$a/T = 0.01$ to 1.0

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3. DATA ANALYSIS METHOD

The stress intensity data for the T-butt welds has been normalised with respect to the Raju and Newman stress intensity solutions for semi-elliptic cracks given in BS 7910: 1997. This procedure eliminates much of the variation in the data due to differences in crack aspect ratio and differences between membrane and bending loading. The resulting Mk-factors under membrane and bending loading, Mkm and Mkb, are given by expressions of the type:

$$Mk = \frac{Y_w}{Y_p}$$

where

Mk = Mk – factor

Y_w = stress intensity factor for weld

Y_p = stress intensity factor for flat plate

Examination of the stress intensity data indicates that Y_w can be less than Y_p resulting in Mk-factors which are less than unity. Although a degree of 'undershoot' would be expected from the through-thickness stress distribution in the weld, it is possible that the approximate weight function method used to derive the stress intensity factors has exaggerated this tendency. Consequently, it is recommended that Mk-factors, which are less than one, are ignored and a minimum Mk-factor of one is used in these cases.

The stress intensity factor at the crack tip becomes negative for deep cracks in both T-butt welds and flat plates, which are loaded in bending, if the crack aspect ratio is greater than or equal to 0.4. As the stress intensity factor approaches zero, the Mk-factor at the crack tip under bending, Mkb, becomes unstable. In order to resolve this problem, the Mk-factor data has been truncated at a/T~0.45 prior to the onset of instability and has been assumed to remain constant at greater crack depths. This approximation is of little practical significance for real defects because fatigue cracks in welded joints tend to develop aspect ratios close to 0.2 as they increase in depth.

The Mk-factor data has been fitted to an equation consisting of simple functions of variables defining the geometry of the T-butt weld (ρ/T, α, L/T) and variables defining the crack size and shape (a/T, a/c). For a given weld geometry and crack aspect ratio, the Mk-factor at any crack depth has been described by terms representing the maximum and minimum Mk-factors, Mk_{max} and Mk_{min}, and a further crack depth dependent term which interpolates between these two extreme values as shown below:

$$Mk = \left(Mk_{max} - Mk_{min}\right) \cdot F\left(\frac{a}{T}\right) + Mk_{min}$$

The maximum and minimum Mk-factors have been defined in terms of the weld geometry and the aspect ratio of the crack. Additional functions and cross terms in more than one variable have been added as required to improve the fit of the equations.

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The constant terms in the equation were evaluated using standard non-linear regression analysis techniques. The non-linear regression analysis was carried out using the Gauss Newton method implemented in a commercially available statistics program (3). This method minimises the difference between the observed values and the values predicted by the parametric equation by iteratively calculating the first and second differentials of the loss function until convergence is achieved.

The loss function was constrained to ensure that the parametric equations gave conservative predictions by weighting losses that resulted in predicted values, which were less than the observed values:

$$\boldsymbol{Loss} = (\boldsymbol{Obs} - \boldsymbol{Pred})^2 \qquad \text{if } \boldsymbol{Obs} \le \boldsymbol{Pred}$$

$$\boldsymbol{Loss} = \boldsymbol{W} \cdot (\boldsymbol{Obs} - \boldsymbol{Pred})^2 \qquad \text{if } \boldsymbol{Obs} > \boldsymbol{Pred}$$

where

$\boldsymbol{Obs}$ = observed value of Mk - factor

$\boldsymbol{Pred}$ = predicted value of Mk - factor

$\boldsymbol{W}$ = weighting constant

The exponent of the loss function was progressively increased to minimise the maximum difference between observed and predicted values.

4. RESULTS

4.1 MK-FACTOR AT THE CRACK TIP

Parametric equations for the Mk-factors at the deepest point of the crack front are given below for membrane and bending loading. Parameter values are defined in Table 1.

$$Mk = (F_1 - F_2) \cdot F_4 + F_2 + F_3$$

where

$$F_1 = 1 + P_1 \cdot \left(\frac{\rho}{T}\right)^{\left(P_2 + P_3 \cdot \left(\frac{a}{c}\right)\right)} \cdot \sin(\alpha)^{\left(P_4 + P_5 \cdot \left(\frac{a}{c}\right)\right)} \cdot \left\{1 - \exp\left[\left(P_6 + P_7 \cdot \left(\frac{a}{c}\right)\right) \cdot \left(\frac{L}{T}\right)\right]\right\} \cdot \left(1 + P_8 \cdot \left(\frac{a}{c}\right)^{P_9}\right)$$

$$F_2 = F_1 \cdot \left(1 - P_{10} - P_{11} \cdot \left(\frac{a}{c}\right)\right) - P_{12} - P_{13} \cdot \left(\frac{a}{c}\right)$$

$$F_3 = \left(1 + P_{14} \cdot \left(\frac{a}{c}\right) + P_{15} \cdot \left(\frac{a}{c}\right)^2 + P_{16} \cdot \left(\frac{a}{c}\right)^3 + P_{17} \cdot \left(\frac{a}{c}\right)^4\right) \cdot \left(P_{18} \cdot \left(\frac{a}{T}\right) + P_{19} \cdot \left(\frac{a}{T}\right)^2 + P_{20} \cdot \left(\frac{a}{T}\right)^3 + P_{21} \cdot \left(\frac{a}{T}\right)^4\right)$$

$$F_4 = P_{22} \cdot \exp\left[-P_{23} \cdot \left(\frac{a}{T}\right)^{P_{24}}\right] + P_{25} \cdot \left(\frac{a}{c}\right) \cdot \exp\left[-\left(\frac{a}{T}\right)^{P_{24}} \cdot \left(P_{23} + P_{26} \cdot \left(\frac{a}{c}\right)\right)\right]$$
$$+ P_{27} \cdot \left(\frac{L}{T}\right) \exp\left[-\left(\frac{a}{T}\right)^{P_{24}} \cdot \left(P_{23} + P_{28} \cdot \left(\frac{L}{T}\right)\right)\right] + P_{29} \cdot \left(\frac{T}{\rho}\right) \cdot \exp\left[-\left(\frac{a}{T}\right)^{P_{24}} \cdot \left(P_{23} + P_{30} \cdot \left(\frac{T}{\rho}\right)\right)\right]$$
$$+ P_{31} \cdot \sin(\alpha) \cdot \exp\left[-\left(\frac{a}{T}\right)^{P_{24}} \cdot (P_{23} + P_{32} \cdot \sin(\alpha))\right]$$

$$\textit{If } Mk < 1, \quad Mk = 1$$

The basic form of the parametric equation is similar to that described in the previous section; however, an additional term (F_3) has been included to account for the small increase in Mk-factor at large crack depths.

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The function F_1, which corresponds to the Mk-factor at the smallest crack depth-to-thickness ratio of 0.01, is the product of individual terms for the weld toe radius-to-thickness ratio, the weld fillet angle, the attachment length-to-thickness ratio and the crack aspect ratio. The fillet weld angle is expressed as a sine function since this gives asymptotic values of 0 and 1. Although simple power-law functions are adequate for the weld toe radius and fillet angle terms, a more complex expression incorporating an exponential function is required to give the correct asymptotic behaviour for the attachment length-to-thickness ratio term.

Crack aspect ratio effects have been included through a single multiplicative function and by introducing an aspect ratio term in the exponents of the geometrical variables.

The minimum Mk-factor was found to be proportional to the maximum Mk-factor at a/T=0.01. This dependence has been included in the function corresponding to the minimum Mk-factor (F_2).

The crack depth dependent term, F_4, has been expressed as the sum of separate exponential functions of the crack depth and each of the other geometrical variables. This form was required to model the rapid fall-off in Mk-factor beneath the weld toe.

4.2 MK-FACTOR AT THE SURFACE

The parametric equation for the Mk-factor at the surface under membrane and bending is given by a similar expression to that for the Mk-factor at the crack tip. However, it was found necessary to use a different function to describe the change in Mk-factor with crack depth. The decease in the Mk-factor at the surface with increasing crack depth is described by a single exponential function of all the variables as shown below:

$$Mk = (F_1 - F_2)\cdot F_4 + F_2 + F_3$$

where

F_1, F_2, F_3 are defined in Section 4.1 above

$$F_4 = \exp\left\{-\left(\frac{a}{T}\right)^{P_{22}} \cdot \left[\begin{array}{l} P_{23} + P_{24}\cdot\left(\frac{a}{c}\right)^{P_{25}} + \left(P_{26} + P_{27}\cdot\left(\frac{a}{c}\right)^{P_{28}}\right)\cdot\left(\frac{L}{T}\right) + \left(P_{29} + P_{30}\cdot\left(\frac{a}{c}\right)^{P_{31}}\right)\cdot\left(\frac{T}{\rho}\right) \\ + \left(P_{32} + P_{33}\cdot\left(\frac{a}{c}\right)^{P_{34}}\right)\cdot\sin(\alpha) \end{array}\right]\right\}$$

$$+\left(P_{35} + P_{36}\cdot\left(\frac{a}{c}\right)^{P_{37}}\right)\cdot\left(\frac{a}{T}\right)^{P38}$$

The parametric equation given above for the Mk-factor at the surface is only valid for crack aspect ratios greater than or equal to 0.1.

Table 1
Parameters for Mk-factor equations

Function	Parameter	Crack tip		Surface	
		Mkm	Mkb	Mkm	Mkb
F1	P_1	1.04424	1.19137	0.47722	0.52011
	P_2	-0.09627	-0.14198	-0.46228	-0.36027
	P_3	0.03790	0.038086	0.19046	0.12547
	P_4	0.54616	0.86676	0.39777	0.54940
	P_5	-0.12508	-0.24951	0.22176	-0.098759
	P_6	-2.43313	-2.03967	-3.25447	-2.80066
	P_7	-0.07251	0.20231	-0.63489	-0.71090
	P_8	0.18353	0.40094	2.85835	1.50561
	P_9	0.87051	0.94855	1.95878	1.86540
F2	P_{10}	0.99924	1.00095	0.40489	0.96775
	P_{11}	0.04125	0.10217	0.34526	-0.21496
	P_{12}	-0.75765	-0.95780	-0.29917	-0.82377
	P_{13}	-0.000426	-0.075004	-0.77810	-0.25998
F3	P_{14}	-0.05692	-0.68779	41.72046	-8.77203
	P_{15}	1.19362	-8.67636	-78.8175	+24.27778
	P_{16}	-1.43325	16.16166	34.10390	-28.1240
	P_{17}	0.61335	-8.14948	2.73640	11.4415
	P_{18}	1.05721	-0.152293	0.030034	2.64087
	P_{19}	-2.4052	-0.148843	-0.13126	-10.4940
	P_{20}	2.61759	1.77150	0.11538	12.8098
	P_{21}	-0.98207	-1.27776	0.040551	-5.98773
F4	P_{22}	1.06748	1.78291	0.53107	0.78365
	P_{23}	7.74090	8.37239	0.26223	-0.24718
	P_{24}	0.47714	0.41021	-0.24730	1.55530
	P_{25}	-0.21542	-0.95097	12.2781	0.049054
	P_{26}	-1.08081	1.64652	-0.059328	0.040332
	P_{27}	-0.002871	3.52508	-0.002740	-0.000146
	P_{28}	0.89122	31.9326	1.04175	-2.41618
	P_{29}	0.008454	0.000011	0.050788	0.002455
	P_{30}	0.14155	0.010084	-0.039354	0.013053
	P_{31}	0.48533	0.93093	1.39315	0.57026
	P_{32}	-2.12357	-2.52809	-10.8442	0.40172
	P_{33}	-	-	16.6945	0.35095
	P_{34}	-	-	0.12542	0.55589
	P_{35}	-	-	-1.39604	0.047656
	P_{36}	-	-	1.21456	0.042067
	P_{37}	-	-	0.69694	1.07535
	P_{38}	-	-	0.42960	-0.48462

5. DISCUSSION

The extent to which the parametric equations fit the data is dependent on the number of cross terms involving more than one variable in the equation, as well as the form of the functions themselves. However, a compromise must be reached between improving the fit of the equations by the addition of further terms and increasing the complexity of the equations. The parametric equations for the Mk-factor at the crack tip and at the surface described in Section 4 include cross terms involving two and three variables respectively. The addition of further cross terms did not result in a significantly better fit to the data and, in some cases, caused premature convergence. Similarly, simplifying the equations by the removal of cross terms was found to adversely effect the fit to the data.

The fit of the parametric equations to the data has been examined by plotting the Mk-factors predicted by the parametric equations against the observed values. Figures 2 to 5 show that the predicted Mk-factors lie close to and slightly above the observed values indicating that the parametric equations give a reasonable fit to the data and are conservative in most cases. Figures 6 to 9 show the residual error between the predicted and observed values, which is defined as:

$$\textbf{\textit{Residual Error}}\ (\%) = \frac{(\textbf{\textit{Obs}} - \textbf{\textit{Pred}})\cdot 100}{\textbf{\textit{Obs}}}$$

Figure 6 shows that the parametric equation for the Mk-factor at the crack tip under membrane loading gives an excellent fit to the data with the residual error lying between +1% and –10% of the observed values. The fit of the parametric equation for the Mk-factor at the crack tip under bending, which is shown in Figure 7, is not quite so good due to the divergence of the Mk-factor values as the stress intensity factors approach zero. However, the residual error in the region of interest (i.e. Mkb>1) is still within +1% and –20% of the observed values. Figures 8 and 9 show that the residual errors for the Mk-factors at the surface under both membrane and bending loading conditions are within +2% and –21% of the observed values.

Histograms of the residual errors, which are shown in Figures 10 to 13, indicate that the asymmetric loss function has resulted in the distribution of the residual errors being skewed away from the mean value of zero. The average residual error between the observed and predicted values varies between –2.2% to –5.7%.

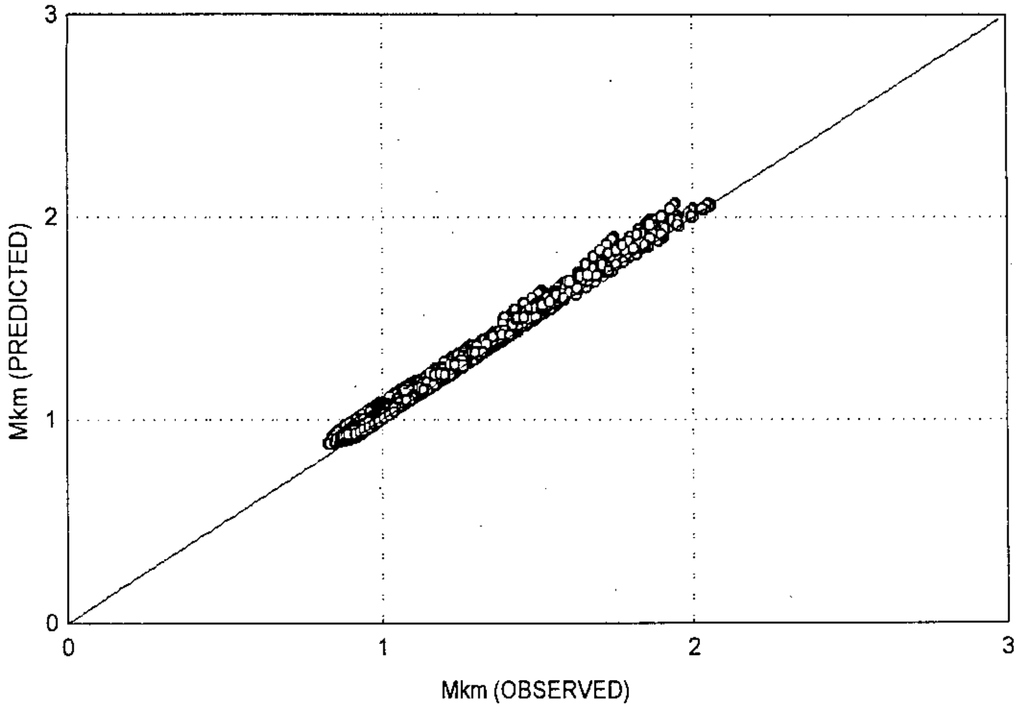

Figure 2
Comparison of predicted and observed values for Mkm at the crack tip

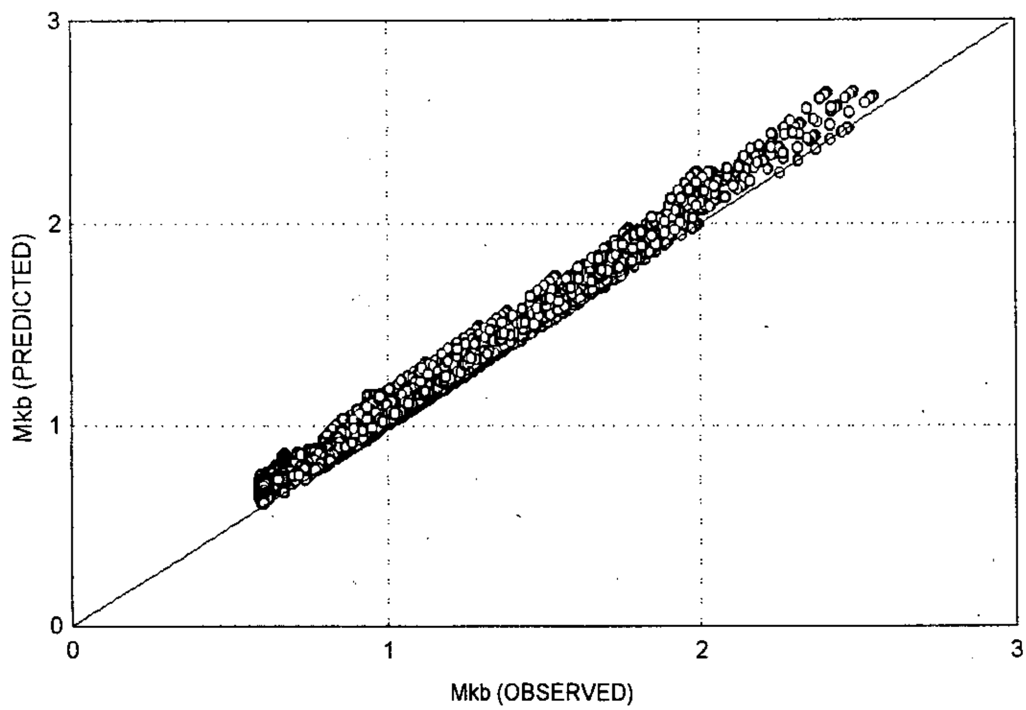

Figure 3
Comparison of predicted and observed values for Mkb at the crack tip

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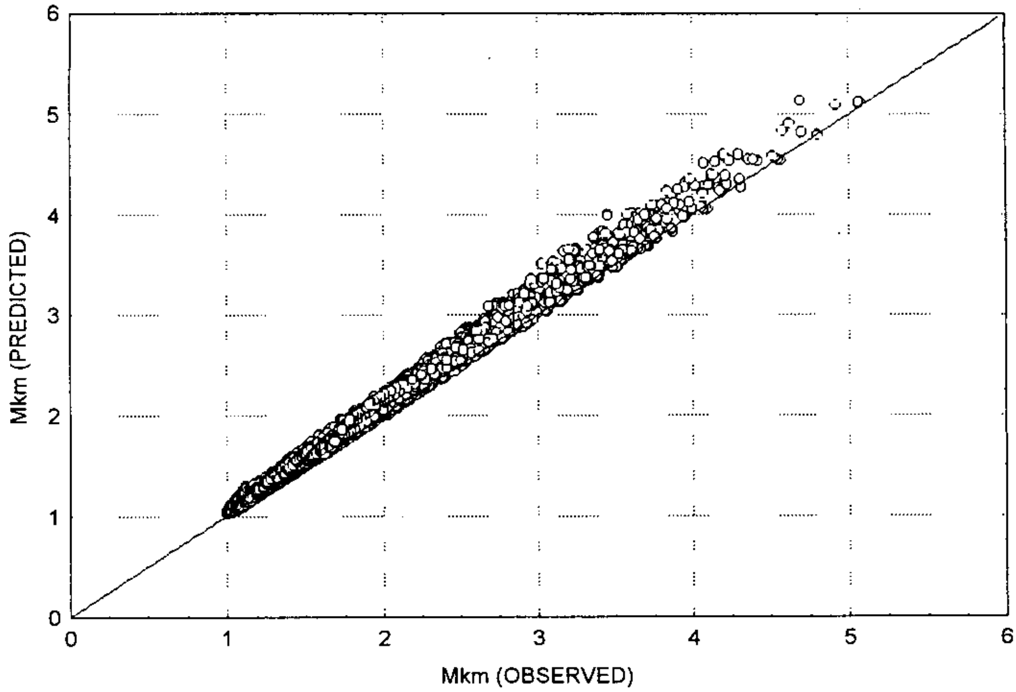

Figure 4
Comparison of predicted and observed values for Mkm at the surface

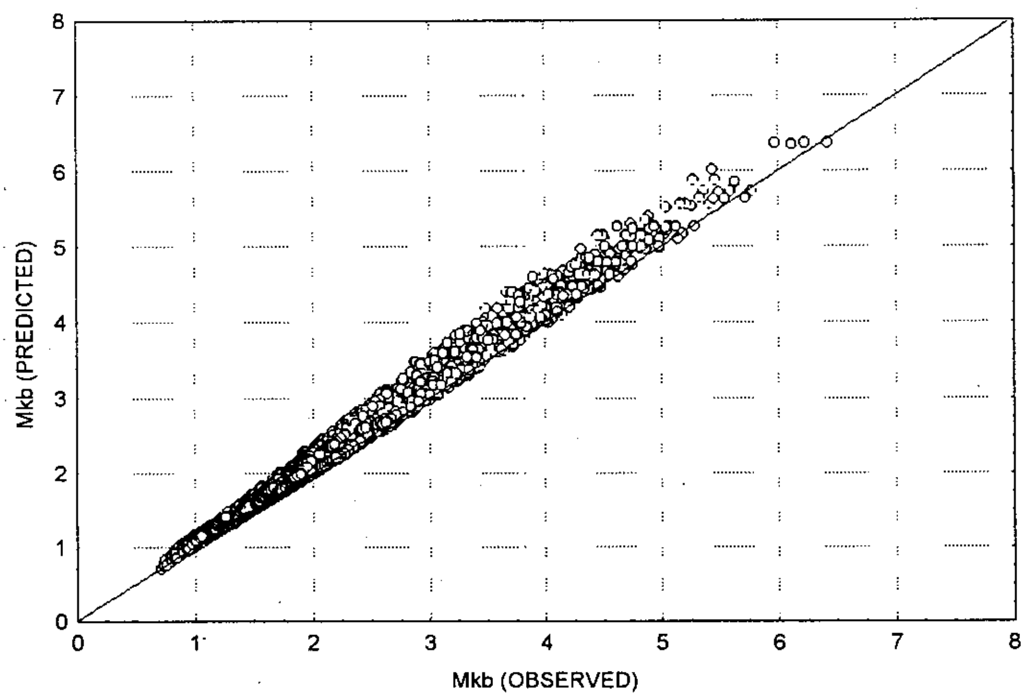

Figure 5
Comparison of predicted and observed values for Mkb at the surface

Contents

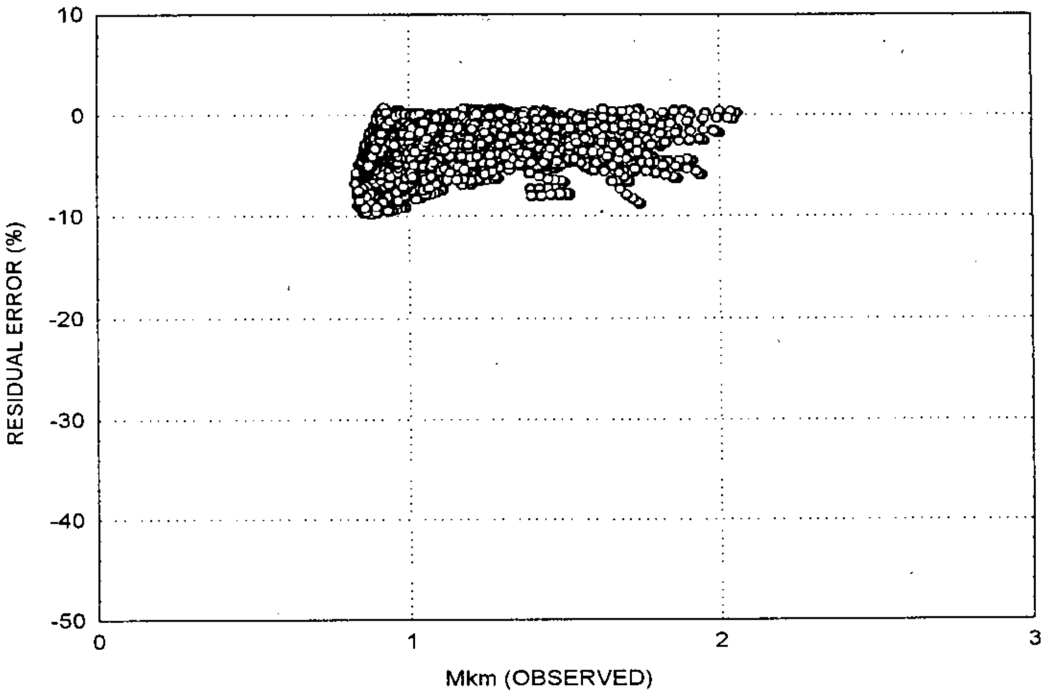

Figure 6
Residual errors for Mkm at the crack tip

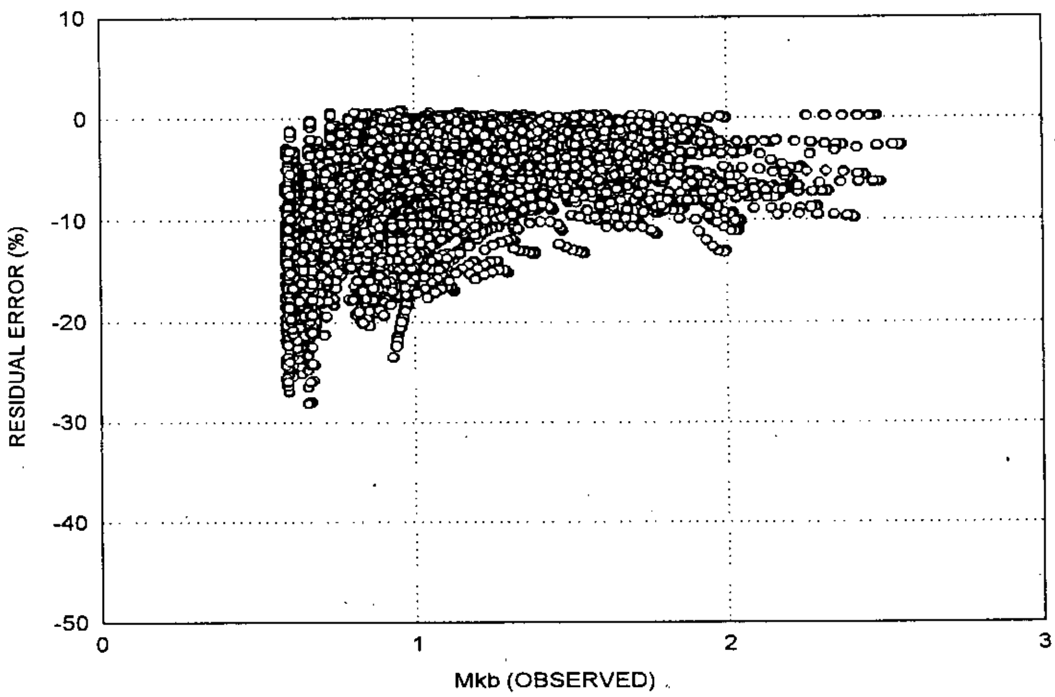

Figure 7
Residual errors for Mkb at the crack tip

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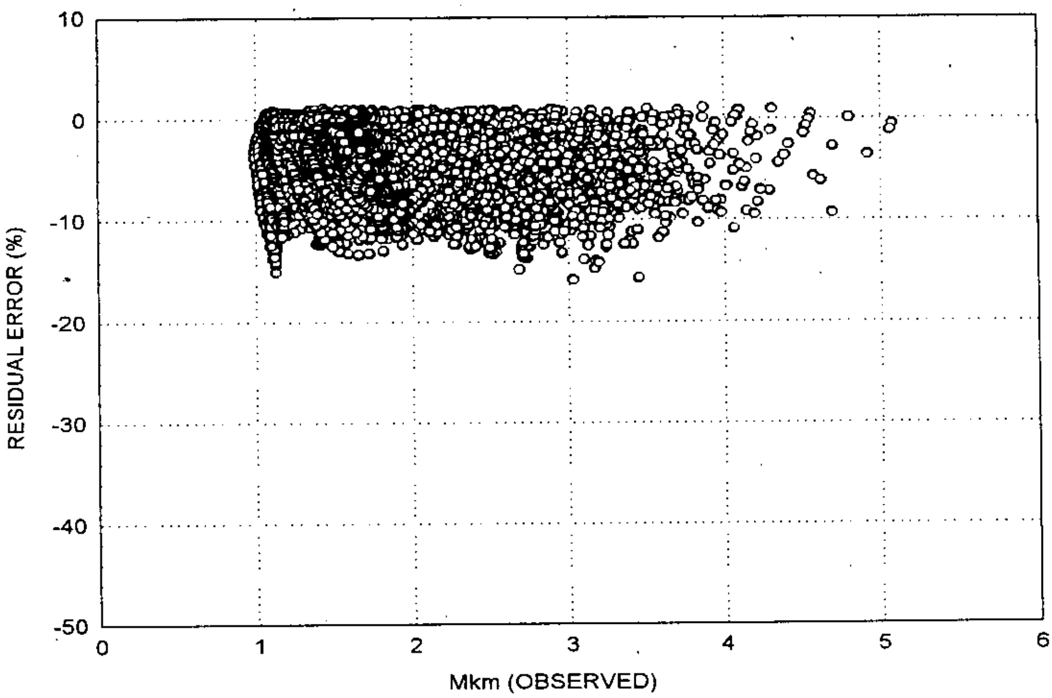

Figure 8
Residual error for Mkm at the surface

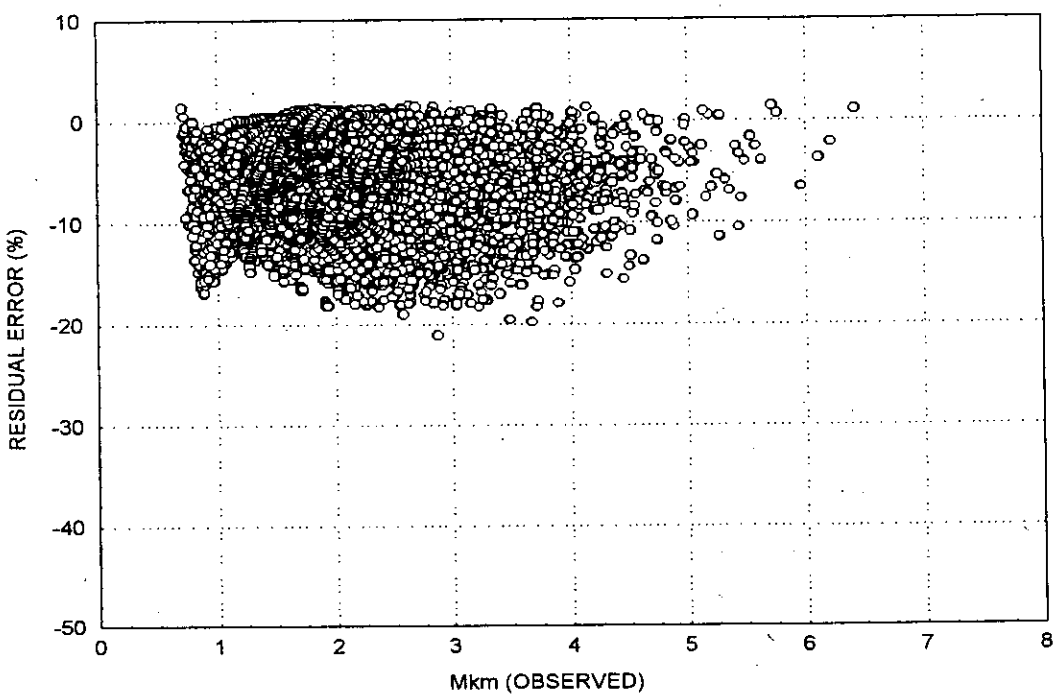

Figure 9
Residual error for Mkb at the surface

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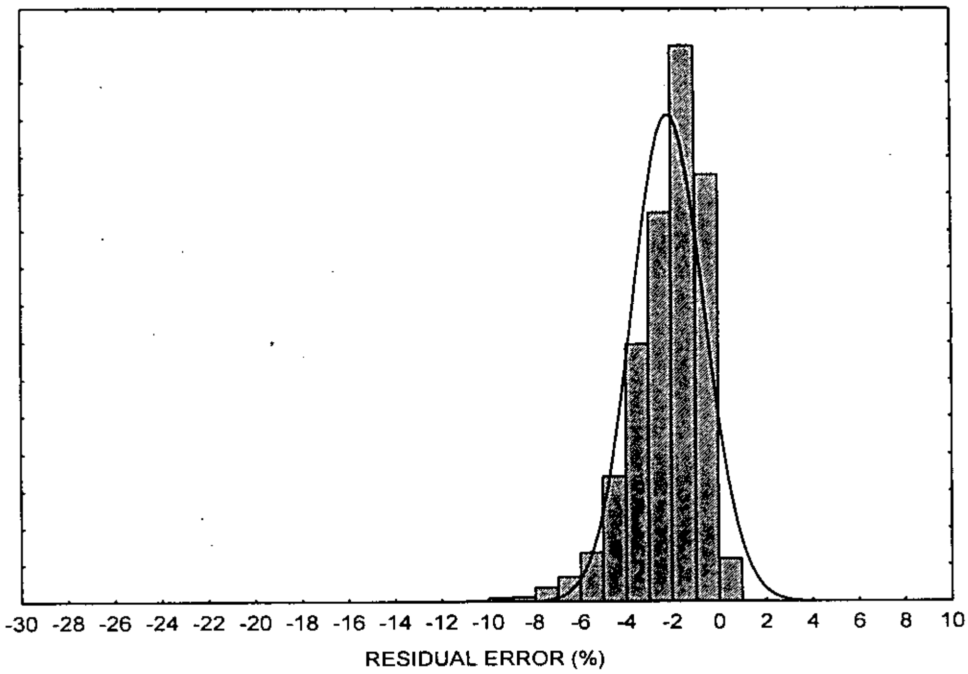

Figure 10
Distribution of residual errors for Mkm at the crack tip
(mean error = -2.2% standard deviation = 1.6%)

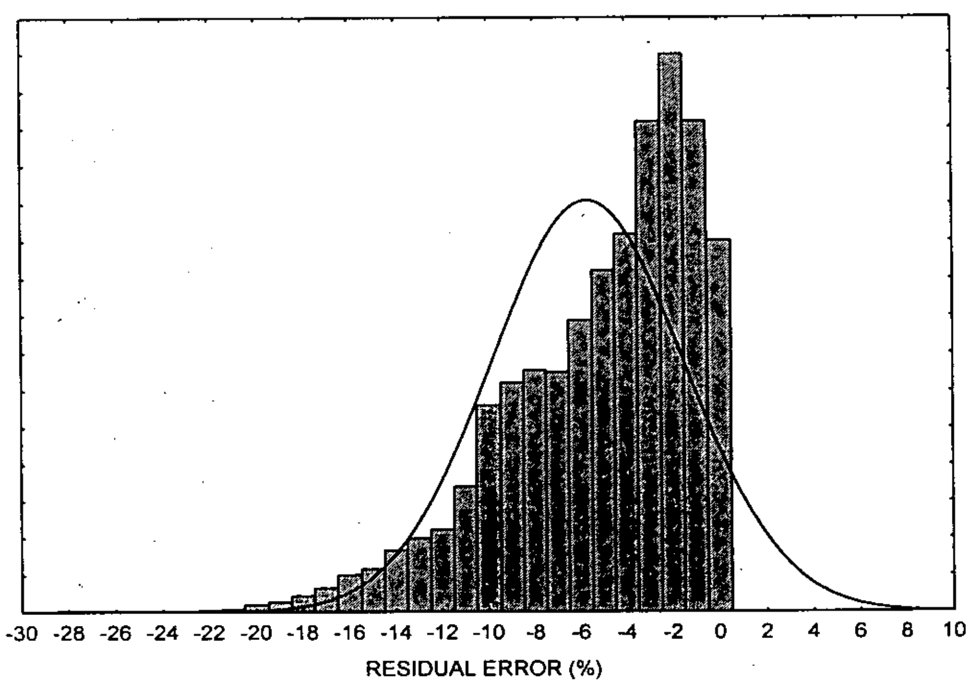

Figure 11
Distribution od residual errors for Mkb at the crack tip
(mean error = -5.7% standard deviation = 4.2%)

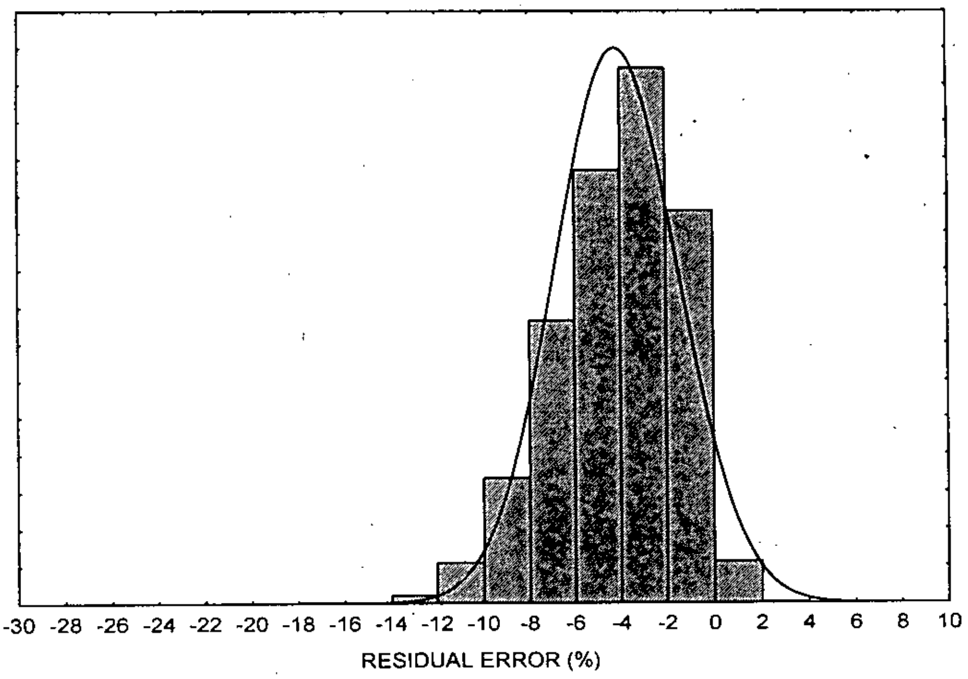

Figure 12
Distribution of residual errors for Mkm at the surface
(mean error = -4.2% standard deviation = 2.7%)

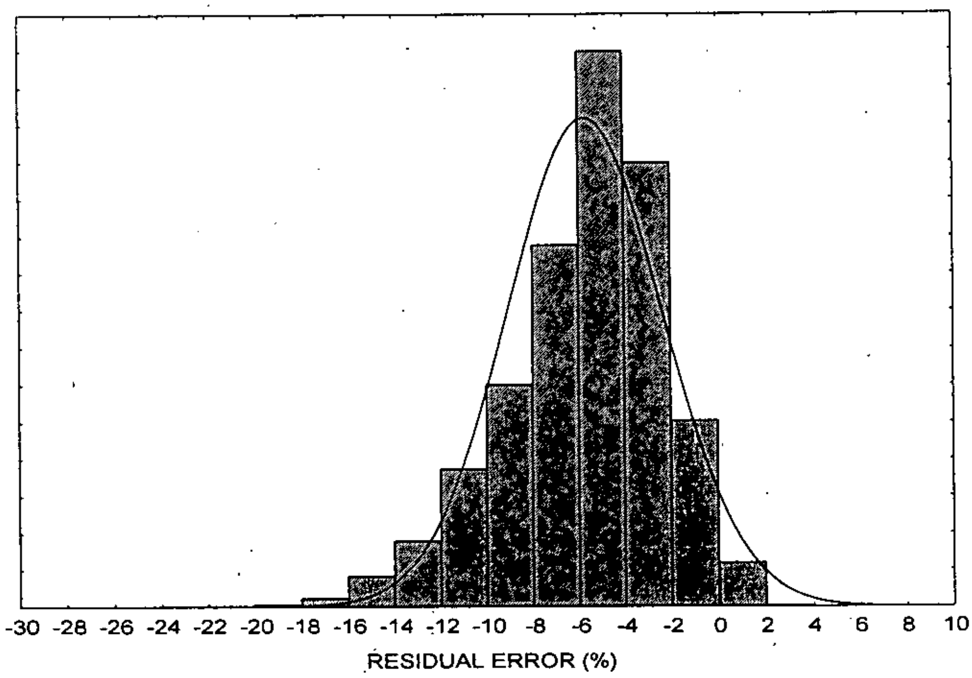

Figure 13
Distribution of residual errors for Mkb at the surface
(mean error = -5.7% standard deviation = 3.4%)

6. CONCLUSION

The parametric equations described in this report provide a relatively simple mechanism for calculating Mk-factors for semi-elliptic cracks in T-butt welds and cover a wide range of crack dimensions and weld geometries. The equations give a good fit to the data and are conservative in most cases. In the small number of instances where the equations underpredict the data, the lack of conservatism is less than 2%. The values predicted by the parametric equations are less than 6% over-conservative on average and the maximum over-conservatism is limited to 21%.

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