The use of Operating Deflection Shapes (ODS) to model the vibration of sanders and polishers

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Science Group: Human Factors

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EXECUTIVE SUMMARY

Operating Deflection Shape (ODS) analysis is a relatively easy method used for visualisation of the vibration pattern of a machine or structure as influenced by its own operating forces.

Objectives

The investigation employs a software package that allows the user to observe, analyse and document the dynamic behaviour of machines and mechanical structures. It works in conjunction with a pre-built frequency analysis project using a real time multi-channel frequency analyser, to enable the user to investigate the operating deflection shapes of the machines under test. By establishing the way in which the machines are vibrating we intend to identify the most appropriate mounting locations for transducers to evaluate the risk of vibration exposure i.e. to avoid nodes of vibration.

A representative sample of the different types of sanders and polishers were investigated using this technique.

Main Findings

Two configurations of accelerometer were used:

1. A tri-axial reference and three roving response transducers
2. A single axis reference and five roving reference transducers

The second configuration created a more defined ODS, because more measurements were taken as a result of the configuration.

Recommendations

For those machines with a front and rear handle the most appropriate location at the operating frequency was found to be on the front handle.

For those machines without handles the most appropriate location at the operating frequency was found to be predominantly on the side of the machine but in some cases on the top of the machine.
1 INTRODUCTION

1.1 BACKGROUND

Operating Deflection Shapes (ODS) are used for visualisation of the vibration pattern of a structure under real life operating conditions. Vibration measurements are performed at different points and directions on the structure known as degrees of freedom (DOFs) and the vibration pattern can be shown in a number of formats including an animated geometry model of the structure. Figure 1 shows an example of a geometry model before animation. Unlike modal analysis techniques which only help visualize the inherent resonant characteristics of a product, ODS is a very powerful tool that can solve problems related to forced vibrations [1].

![An unanimated geometry model of a random orbital sander](image)

Figure 1 – An unanimated geometry model of a random orbital sander

Traditionally, ODS have been defined as the deflection of a structure at a particular frequency. However, ODS can be defined more generally as any forced motion of two or more DOFs on a structure.

All vibration is a combination of both forced and resonant vibration. Forced vibration can be due to:

1. Internally generated forces
2. Unbalances
3. External loads

An operating deflection shape contains the overall vibration for two or more DOFs on a structure. An ODS therefore contains both forced and resonant vibration components whereas a mode shape characterizes only the resonant vibration at two or more DOFs [2].

Real continuous structures have an infinite number of DOFs and an infinite number of modes. From a testing point of view, a real structure can be sampled spatially at as many DOFs as desired but there is a limitation when looking at small structures such as sanders. The more we spatially sample the surface of the structure by taking more measurements within the limitations, the more definition we will give to its ODS as less interpolation will be required.
between measured points. However, the frequency range of interest for hand-arm vibration is only up to 1500Hz and so fewer measurement points will be required for adequate resolution. Figure 2 shows all but one of the measurement locations on a model of a palm sander. The last measurement location is opposite and in the same direction as the DOF marked with a black arrow.

Figure 2 – A geometry model of palm sander showing measurement locations. Each arrow corresponds to a measurement in that particular direction at that particular DOF

It has been discussed that ODS analysis is a method to model the motion of a structure as influenced by its own operating forces and/or those from external sources. They can be viewed for a specific moment in time or a specific frequency. For our investigations the most relevant frequency is the operating frequency under real conditions. Unlike modal testing, measurements are obtained during normal operation.

1.2 SIMPLE EXAMPLE OF A STEEL BAR FIXED AT ONE END

In order to show exactly how the software works and to show that it outputs true ODS, a simple example of a steel bar is used with the following procedure followed:

1. Calculate the expected natural frequencies of the bar via a known equation.

2. Fasten the bar to a shaker and attach 6 transducers. The transducer closest to the pivot will be the reference and so all vibration will be relative to this transducer.

3. Tap the steel bar with a hammer close to the free end and observe the FFT using a real time multi-channel frequency analyser to show that the calculated frequencies are present in the bar.

4. In turn drive the bar at the calculated frequencies via the shaker, observe the mode visually by viewing the bar, and then observe the ODS produced by the software and see if they appear the same.
1.2.1 Calculating the natural frequencies

The natural frequency of a beam according to Broch 1980 is given by:

\[ f_n = \frac{A}{2\pi} \sqrt{\frac{EI}{\rho s l^4}} \]

where \( A \) is the constant given in Figure 3 assuming clamped-free, \( E \) is the Young’s modulus (for steel \( \approx 210\text{GPa} \)), \( I \) is the area moment of inertia of the beam cross section (for this beam, \( I = \frac{bh^3}{12} \), where \( b \) is the breadth (40mm) and \( h \) is the height (5mm)), \( \rho \) is the density (for steel \( \approx 7800\text{Kgm}^{-3} \)), \( s \) is the cross sectional area of the beam (40x5mm) and \( l \) is the length of the beam (280mm).

![Figure 3 – A table showing the values of the constant A for the first four modes of different configurations](image)

Using the values of \( A \) in 1), 2) and 3) in Figure 3 gives the frequencies of the first three modes of the bar to be 54Hz, 341Hz and 938Hz respectively. Detailed calculations can be found in Appendix A.

1.2.2 Assigning the transducers and drawing the geometry model

The geometry model is drawn in the software and involves drawing a long, thin box with the correct dimensions and has the right number of points to assign the transducers. Figure 4 shows a real picture of the bar and a geometry model of the bar.

The transducers are assigned to the DOFs (cross over points) as shown in Figure 4.
1.2.3 Acquiring the Fast Fourier Transform (FFT)

Figure 4 – A picture of the steel bar and the geometry model

Figure 5 – The FFT showing the frequencies of the first three modes
The FFT is acquired via the real time multi-channel frequency analyser. The steel bar is tapped repeatedly close to the free end and the FFT from one of the closest transducers is recorded. The resulting FFT is displayed in Figure 5 and shows three dominant peaks corresponding to the first three modes of the bar. The frequencies of the modes are shown to be 52Hz, 355Hz and 916Hz, which are very close to the calculated values. The calculations do not take into account the mass of the six transducers and so this could explain the difference.

### 1.2.4 Obtaining the ODS

The steel bar is driven in turn at each of the three frequencies and a single measurement set is recorded. The data is then exported into the display software where the ODS are viewed. The geometry model is interpolated and viewed at the modal frequency. Interpolation involves the computation of points or values between the ones that have been measured, using the data from the surrounding points or values.

Figure 6 – *From Left to Right, Top to Bottom: The ODS of the first mode of the bar at 52Hz*

Figure 6 shows the ODS of the steel bar at 52Hz. This is the first mode of the bar with the shape being directly compared to picture 1 in Figure 3. It should be noted that the interpolation of the bar is not quite correct at the free end of the bar in Figures 6-8. The extreme edge of the bar is following the adjacent point, as a transducer was not located right at the edge of the bar. No data were collected at this point.

Figure 7 shows the ODS at 355Hz, the second mode of the bar. This should be directly compared with picture 2 in Figure 3.
Figure 7 – *From Left to Right, Top to Bottom: The ODS of the second mode of the bar at 355Hz*

Figure 8 shows the ODS at 916Hz, the third mode of the bar. This should be compared with picture 3 in Figure 3. This is not as clear as the other two modes as more measurement points would be needed in order to gain more definition at this mode. Less interpolation would be needed between measured points if more measurements were taken.

Figure 8 – *From Left to Right, Top to Bottom: The ODS of the third mode of the bar at 916Hz*

It can be seen from Figures 6-8 that the software is outputting true and expected ODS, giving confidence when the software is applied to more complex structures such as sanders.

A test was also carried out with the reference at the free end of the bar as opposed to the fixed end. The result of the test was the same indicating that all measurements are compared relatively to the value at the reference. If the reference is located at the point of lowest vibration all other points are looked at in a relatively positive sense. If the reference is located at the point
of highest vibration all other points are looked at in a relatively negative sense and the absolute value is taken, with the final outcome always being the same.
2 MEASUREMENTS

The method of investigation employs a software package that allows the user to observe, analyse and document the dynamic behaviour of machines and mechanical structures. The software displays spatially acquired vibration on a 3D model of the test structure. Measurements on the structure are carried out using a real time multi-channel frequency analyser and exported into the display software where the ODS are viewed. All of the geometrical drawing and the measurement process are carried out in a pre-built real time multi-channel frequency analysis project.

The software enables the user to investigate the operating deflection shapes of the machine under test. By establishing the way in which the machines are vibrating we will be able to identify the most appropriate mounting locations for transducers to evaluate the greatest risk of vibration exposure. Locating the transducers on a node where there is little or no vibration will result in an underestimate of the vibration magnitude associated with the use of the tool.

A representative sample of the different types of sanders and polishers was investigated using this technique. Each axis of the machine will in turn be investigated, to build up a picture of how the machine vibrates. The machine was operated and held lightly in a static position, flat on the surface of a sanding block for testing to BS EN ISO 8662-8:1997 [4].

2.1 EQUIPMENT

The instrumentation used to make the measurements was as follows:

1. Brüel & Kjær (B&K) 3560C Pulse acquisition and frequency analysis system (Max. six channels)
2. Six B&K 4393 piezoelectric accelerometers
3. Six B&K 2635 charge amplifiers
4. Laptop PC to use in conjunction with 1.

More details on the instrumentation are given in Appendix B.

2.2 MACHINES TESTED

The sample of the different types of sanders and polishers used are described in Table 1 and shown in Figure 9.

<table>
<thead>
<tr>
<th>TOOL</th>
<th>SHAPE/TYPgE/SIZE</th>
<th>POWER</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>5 inch circular, random orbital palm sander</td>
<td>Pneumatic</td>
</tr>
<tr>
<td>D</td>
<td>Orbital, rectangular sander</td>
<td>Pneumatic</td>
</tr>
<tr>
<td>G</td>
<td>5 inch circular, random orbital angle sander</td>
<td>Electric</td>
</tr>
<tr>
<td>I</td>
<td>6 inch circular, random orbital palm sander</td>
<td>Pneumatic</td>
</tr>
<tr>
<td>K</td>
<td>Rotary angle polisher</td>
<td>Electric</td>
</tr>
</tbody>
</table>
Differences in the ODS might be expected between orbital and random orbital, 5 inch and 6 inch and circular and rectangular machines due to the way they operate.

![Figure 9](image)

**Figure 9** – Top Left: Tool C, Top Right: Tool D, Middle Left: Tool G, Middle Right: Tool I and Bottom: Tool K

### 2.3 ODS TESTS

For each machine, a geometrical model was constructed in the frequency analysis project. The models were approximately to scale, an example of which is shown in Figure 1. The reference and response transducers were then assigned to the appropriate locations.

In the case of Tools C and D a tri-axial reference was located on the top of the machine (fixed) leaving three response transducers to rove on the top, side and front of the machine creating three measurement sets.

For Tools G, I and K the reference was single axis leaving five response transducers to rove therefore creating a more defined ODS. Tool I also had three measurement sets; top, side and front.

Tools G and K are different in the fact that they involve a two handed operation. They have a side and a rear handle. The response transducers are located on the front and top of each handle, creating four measurement sets and making the measurement process a little longer.

An example of the two different reference transducer configurations is shown in Figure 10.
For each measurement set, the data needs to be verified. This is done using the coherence function in the analysis project. For the data to be valid, the coherence at the peak in the phase assigned spectrum corresponding to the operating frequency of the machine, needs to be very close or equal to 1. Figure 11 shows such a spectrum for one measurement set on Tool I. The dominant peak in the spectrum corresponds to the operating frequency of the machine at 155Hz. At this frequency the coherence is very close to 1 and so the data is valid for this particular measurement set.

Having validated all of the measurements, ratio based phase assigned spectra are used to give the average ODS in cases with slightly varying operating frequencies, when using roving transducers. The data is then exported into the animation software where the ODSs can be viewed.

In the animation software the geometry models are interpolated and viewed at the operating frequency. The animated geometries are saved as AVI files. They were then decomposed into still images for displaying in this report.
Figure 11 – The top graph is the phase assigned spectrum for one measurement set showing a dominant peak at 155Hz. The bottom graph is the coherence and shows a value very close to 1 at 155Hz. The measurement is therefore valid at 155Hz.
3 RESULTS AND DISCUSSION

Having verified that the software works correctly on a very simple level with the steel bar example, it was decided to look at a structure a little closer to that of a body of a sander. This was required to try to understand the software on a more complex level to verify whether the ODS of the tested machines were true. The structure under test was the side handle of Tool G. A similar aspect of the steel bar procedure was followed which involved tapping the end of the handle and observing the FFT. The side handle acted like a cantilever and this was observed with the software. However, this was still along way from verifying the motion of the body of the machine.

The ODS show general areas of different magnitudes of vibration. They do not give any other information such as nodal points within areas of similar vibration. This additional information can be obtained but not through the ODS software. The phase and magnitude at each measurement location can be viewed by pulling apart the frequency analysis project. Figures 12-16 give an idea of the general area to locate the transducers but not in any detail in order to avoid the nodal points. Overall motion of the machine or part of the machine such as a handle can be visualised.

In the sequences of images shown in Figures 12-16 the colour sequence dark green, blue, yellow through to red corresponds to the magnitude of displacement from small through to large where dark green is small and red is large. The colour is emphasising where the areas of high displacement are located. The scale is the same in all of the Figures. At a given frequency the vibration is related to the displacement by a constant and so the ODS can be thought of as displaying the vibration magnitudes. A proof of this is shown in Appendix C.
The ODS for Tool C at an operating frequency of 187Hz is shown in Figure 12. Tool C used a tri-axial reference and three measurement sets.

Figure 12 – From Left to Right, Top to Bottom: A 3D view of the ODS of Tool C at 187Hz. The time increment between each image is 0.44ms

The ODS shows that at the operating frequency, a large amount of the vibration emission is located on the side and front of the machine, as these are the locations that turn red during the sine dwell. There is emission on the top but it is lower than the emission on the side, so the best location is on the side. A rocking type motion can be visualised by looking at the skirt of the sander. The pivot is likely to be the compressed air inlet. This could suggest a reason for the variation in vibration magnitude measured across the top of the machine.
Tool D is a rectangular sander with an operating frequency of 120Hz. A tri-axial reference was used. The ODS is shown in Figure 13 at 120Hz from a rear view rather than a 3D view.

The ODS shows that the highest vibration emission is located on the side and top of the machine as these are the only locations that turn red. The front is also red but not quite to the same extent. The side and top are therefore the best locations for measuring the vibration emission. A rocking motion pivoted about the air inlet can again be seen by looking at the bottom of the sanding plate.

Figure 13 – From Left to Right, Top to Bottom: A front view of the ODS of Tool D at 120Hz. The time increment between each image is 0.69ms
A 3D view of the ODS of Tool G at 229Hz is shown in Figure 14. Tool G used a single axis reference and five roving response transducers.

The area of interest in Figure 14 is the side handle. No measurements were taken on the main body of the sander. The handle gets redder the further away from the body. This is expected as the handle is like the simple steel bar example in section 1.2, although the handle is not being driven at a specific mode. The fact that the handle bends in the middle suggests that the points either side are moving in anti-phase at the operating frequency and that there is a node in the middle.

It is best to locate the transducers on this handle, but not in the middle as the vibration exposure will be underestimated due to the node.
The ODS in Figure 15 is a 3D view of Tool I at a frequency of 155Hz. This ODS also used a single axis reference transducer. Tool I is like Tool C but 6 inch rather than 5 inch.

The ODS indicates that the majority of the highest vibration emission is located on the side and the top of Tool I. The vibration emission on the side is not as high as Tool C as the side never becomes red as in Figure 12. The best location is therefore on the side as it gets the closest to red. Again a rocking motion can be visualised by looking at the bottom of the sanding disc.
Tool K is the rotary polisher and the ODS uses a single axis reference. Figure 16 is a 3D view of the ODS at 29Hz.

As with Tool G, the area of interest is the side handle, where a large proportion of the highest vibration emission is located at the operating frequency. No measurements were made on the main body of the machine. The vibration increases the further away from the body of the machine, as it is again acting like a cantilever. The motion of the handle is similar to a 1st order mode, which contains no nodes. Unlike Tool G, the transducers can therefore be located at the middle of the handle.
4 CONCLUSIONS

With the objective of the investigation being to establish the most appropriate mounting location for the transducers it is concluded that: -

Tool C  Most appropriate location is on the side of the machine although it could also be located on the front.

Tool D  Most appropriate location is also on the side of the machine although it could also be located on the top.

Tool G  Most appropriate location is on the front handle, but not in the middle.

Tool I  Most appropriate location is on the side of the machine although it could also be located on the top.

Tool K  Most appropriate location is at the middle of the front handle.

In summary for the machines tested in the investigation, transducers should be located on front handles for those machines with front and rear handles and on the side of the machine for those machines without.

In the cases of Tool C, Tool D and Tool I, the palm Sanders, a rocking motion can be visualised suggesting a possible reason for the variation of vibration emission measured across the top of the machines. The pivot is likely to be the air inlet.
5 APPENDICES

5.1 APPENDIX A - DETAILED CALCULATIONS OF NATURAL FREQUENCIES

The natural frequency of a beam according to Broch 1980 is given by:

\[ f_n = \frac{A}{2\pi} \sqrt{\frac{EI}{\rho l^4}} \]

where \( A \) is the constant given in Figure 3 assuming clamped-free, \( E \) is the Young’s modulus (for steel \( \approx 210\text{GPa} \)), \( I \) is the area moment of inertia of the beam cross section (for this beam, \( I = \frac{bh^3}{12} \)), where \( b \) is the breadth (40mm) and \( h \) is the height (5mm), \( \rho \) is the density (for steel \( \approx 7800\text{Kgm}^{-3} \)), \( s \) is the cross sectional area of the beam (40x5mm) and \( l \) is the length of the beam (280mm).

Putting in the equation for \( I \) and \( s = bh \)

\[ \Rightarrow f_n = \frac{A}{2\pi} \sqrt{\frac{Eh^2}{12 \rho l^4}} \]

\[ \therefore f_1 = \frac{3.52}{2\pi} \sqrt{\frac{210 \times 10^9 \cdot (5 \times 10^{-3})^2}{12 \cdot 7800 \cdot (280 \times 10^{-3})^4}} = 54\text{Hz} \]

\[ f_2 = \frac{22.4}{2\pi} \sqrt{\frac{210 \times 10^9 \cdot (5 \times 10^{-3})^2}{12 \cdot 7800 \cdot (280 \times 10^{-3})^4}} = 341\text{Hz} \]

\[ f_3 = \frac{61.7}{2\pi} \sqrt{\frac{210 \times 10^9 \cdot (5 \times 10^{-3})^2}{12 \cdot 7800 \cdot (280 \times 10^{-3})^4}} = 938\text{Hz} \]
5.2 APPENDIX B - DETAILS OF EQUIPMENT

### Transducers

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B&K Pulse 3560C Serial # 2423351
B&K Pulse LabShop software v10.1
Vibrant Technology, Inc. ME’scope VES software v4.0
Calibrator B&K 4294 2361765

Calibrator B&K 4294 2361765
September 2005
5.3 APPENDIX C - THE RELATIONSHIP BETWEEN ACCELERATION AND DISPLACEMENT

\[ \text{displacement} = x = x_0 \sin(\omega t) \]
\[ \text{velocity} = v = \frac{dx}{dt} = x_0 \omega \cos(\omega t) \]
\[ \text{acceleration} = a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -x_0 \omega^2 \sin(\omega t) \]

\[ \therefore a = -\omega^2 x \]

At a specific frequency the acceleration is just the displacement multiplied by a constant.
6 REFERENCES


