

**Job No.**

5352

**File Ref.**

5352/19294v

**Calc. No.**

**Calc. Title**

Analysis of the rate of roll of a truck (3 or 4 wheels) turning on a slope

**Author**

Kin Chan

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**Approved**

A.F. Wylie

**Date**

27 January 2000

**Date**

28/3/2000

## Analysis on the rate of roll of a truck (3 or 4 wheels) turning on a slope

Version : 1.2

### Assumptions:

traction is large enough to stop skidding  
negligible wheel width  
constant slope during the movement of the vehicle  
turning velocity remains constant during rolling  
the plane of roll is perpendicular to the slope at all time  
reaction force at impact of bump stop on ground is negligible for worst case analysis

### Note:

all parameters are in SI units

### List of symbols:

$a$  - horizontal offset of CG from centre line of vehicle (m)  
 $acc$  - average acceleration (radian/sec<sup>2</sup>)  
 $h$  - height of CG from ground (m)  
 $I$  - roll inertia of vehicle about CG  
 $l$  - track width (front and rear) (m)  
 $L$  - perpendicular distance between CG and pivot edge (m)  
 $M$  - mass of vehicle (kg)  
 $r$  - radius of turn of CG before the start of roll (m)  
 $R$  - radius of turn of CG after the start of roll (m)  
 $T$  - turning moment (Nm)  
 $v$  - tangential velocity at midpoint of front track of vehicle (m/sec)  
 $w$  - turning velocity of vehicle (radian/sec)  
 $x$  - moment arm of weight from CG (m)  
 $y$  - moment arm of centripetal force from CG (m)  
 $\alpha$  - angle of roll relative to the starting position (radian),  $0 \leq \alpha \leq \pi/2$   
 $\beta$  - angle of CG from the pivot line relative to the base of vehicle (radian),  $0 \leq \beta \leq \pi/2$   
 $\theta$  - angle of the slope (radian),  $0 \leq \theta \leq \pi/2$   
 $\phi$  - incremental roll angle (radian),  $0 \leq \phi \leq \pi/2$

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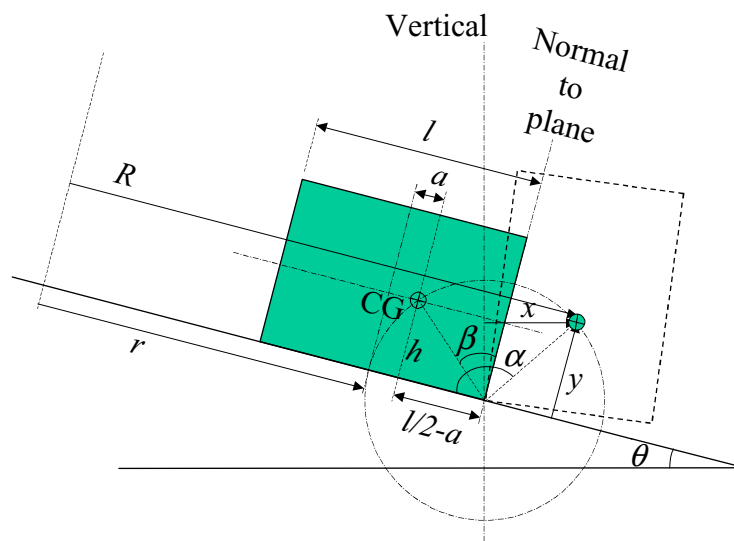
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When front and rear track widths are equal and assume skidding does not occur

$$\tan(\beta) = \frac{\frac{l}{2} - a}{h}$$

Radius of turn of CG

$$R = r + \frac{l}{2} + \sqrt{h^2 + \left(\frac{l}{2} - a\right)^2} \cdot \sin(\alpha - \beta)$$

Moment arm of weight from pivot (i.e. wheel)

$$x = \sqrt{h^2 + \left(\frac{l}{2} - a\right)^2} \cdot \sin[\alpha - (\beta - \theta)]$$

Moment arm of centripetal force from pivot (i.e. wheel)

$$y = \sqrt{h^2 + \left(\frac{l}{2} - a\right)^2} \cdot \cos(\alpha - \beta)$$

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Total turning moment

$$T = M \cdot (w^2 \cdot R \cdot y + g \cdot x)$$

Angular acceleration of rolling

$$\frac{d^2 \alpha}{dt^2} = \frac{T}{I + M \cdot L^2}$$

$$\text{new} \left( \frac{d}{dt} \alpha \right)^2 = \text{last} \left( \frac{d}{dt} \alpha \right)^2 + 2 \cdot \frac{d^2}{dt^2} \alpha \cdot \phi$$

The angle of roll can be found by a series of steps

$$\phi = \frac{d}{dt} \alpha \cdot t + \frac{1}{2} \cdot \frac{d^2}{dt^2} \alpha \cdot t^2$$

$$\frac{1}{2} \cdot \frac{d^2}{dt^2} \alpha \cdot t^2 + \frac{d}{dt} \alpha \cdot t - \phi = 0$$

Or the time required to turn an angle of  $\phi$  is

$$t = \frac{-\left(\frac{d}{dt} \alpha\right) + \sqrt{\left(\frac{d}{dt} \alpha\right)^2 - 4 \cdot \left(\frac{1}{2} \cdot \frac{d^2}{dt^2} \alpha\right) \cdot (-\phi)}}{2 \cdot \left(\frac{1}{2} \cdot \frac{d^2}{dt^2} \alpha\right)}$$

$$t = \frac{-\left(\frac{d}{dt} \alpha\right) + \sqrt{\left(\frac{d}{dt} \alpha\right)^2 + 2 \cdot \left(\frac{d^2}{dt^2} \alpha\right) \cdot (\phi)}}{\frac{d^2}{dt^2} \alpha}$$

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**When the front and rear track widths are different and there are bump stops on both side of the rear track (assume the pivot edge is perpendicular to the slope).**

*c* - distance of CG from the rear (m)

*d* - rear track width (m)

*e* - wheel base (m)

*f* - distance of CG from the front (m)

*h0* - initial height of CG from ground (m)

*h1* - height of CG from ground when the bump stop hit the ground (m)

*j* - width of bump stop (m)

*k* - height of bump stop (m)

*l* - front track width (m)

*L0* - perpendicular distance between CG from the old pivot edge (m)

*L1* - perpendicular distance between CG from the new pivot edge (m)

*n* - length of the side of the vehicle (m)

*p0* - base distance of CG from the old pivot edge (m)

*p1* - base distance of CG from the new pivot edge (m)

*R0* - radius of turn of CG after the start of roll before bump stop hit the ground (m)

*R1* - radius of turn of CG after the bump stop hit the ground (m)

*s* - base distance of CG from the pivot edge (m)

$\beta_0$  - angle of CG from the old pivot edge relative to the ground (radian),  $0 \leq \beta_0 \leq \pi/2$

$\beta_1$  - angle of CG from the new pivot edge relative to the ground (radian),  $0 \leq \beta_1 \leq \pi/2$

$\gamma$  - angle of roll before bump stop hit the ground (radian)

$\delta_0$  - angle of vehicle before bump stop hit the ground (radian),  $0 \leq \delta_0 \leq \pi/2$

$\delta_1$  - angle of vehicle after bump stop hit the ground (radian),  $0 \leq \delta_1 \leq \pi/2$

$\zeta$  - angle between new and old pivot edges (radian)

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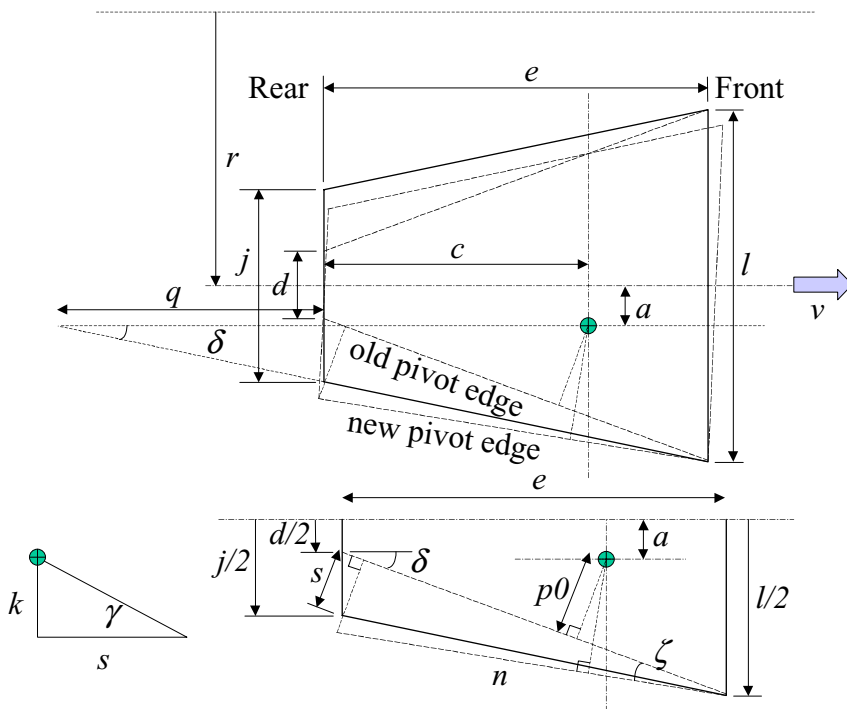
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Before the bump stop hit the ground

$$\tan(\delta) = \frac{l - d}{2 \cdot e}$$

$$s = \left( \frac{j - d}{2} \right) \cdot \cos(\delta)$$

Roll angle before the bump stop hit the ground and the pivot edge shift

$$\tan(\gamma) = \frac{k}{s}$$

If s is zero when j = d, i.e. the bump stop has the same width as the rear track,  $\gamma = \pi/2$

Base (perpendicular) distance between the CG and the pivot edge before the bump stop hit the ground

$$p_0 = c \cdot \sin(\delta) + \left( \frac{d}{2} - a \right) \cdot \cos(\delta)$$

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$$\tan(\beta_0) = \frac{p_0}{h_0}$$

$$L_0 = \sqrt{h_0^2 + p_0^2}$$

Radius of turn of CG

$$R_0 = r + a + p_0 + L_0 \cdot \sin(\alpha - \beta_0)$$

Moment arm of weight from pivot (i.e. wheel)

$$x = L_0 \cdot \sin[\alpha - (\beta_0 - \theta)]$$

Moment arm of centripetal force from pivot (i.e. wheel)

$$y = L_0 \cdot \cos(\alpha - \beta_0)$$

Total turning moment

$$T = M \cdot (w^2 \cdot R_0 \cdot \cos(\delta_0) \cdot y + g \cdot x)$$

Angular acceleration of rolling

$$\text{acc} = \frac{T}{I + M \cdot L_0^2}$$

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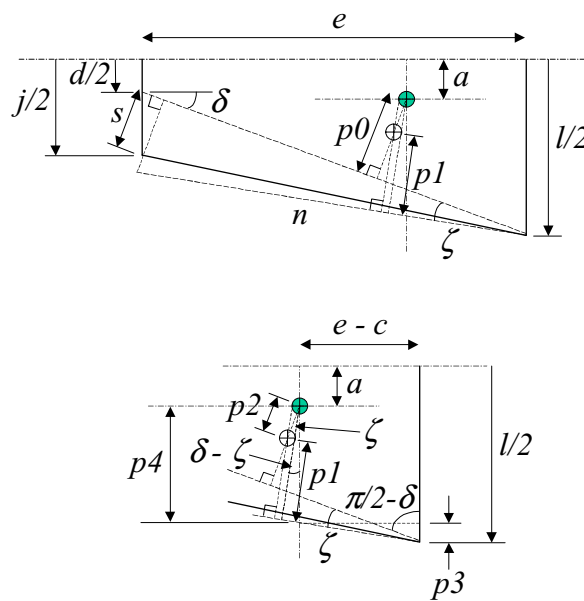
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When the bump stop hit the ground



Length of new pivot edge (the length of the side of the vehicle):

$$n = \sqrt{e^2 + \left(\frac{1-j}{2}\right)^2}$$

Height of the CG from ground when the bump stop hit the ground

$$h_1 = L_0 \cdot \sin\left(\frac{\pi}{2} - \beta_0 + \gamma\right)$$

Displacement of the CG along the slope

$$p_2 = p_0 - L_0 \cdot \cos\left(\frac{\pi}{2} - \beta_0 + \gamma\right)$$

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Base (perpendicular) distance between the CG and the pivot edge after the bump stop hit the ground

$$\sin(\zeta) = \frac{\sqrt{k^2 + \left(\frac{j-d}{2}\right)^2}}{n}$$

$$p3 = \frac{e - c}{\tan\left(\frac{\pi}{2} + \zeta - \delta\right)}$$

$$p4 = \frac{1}{2} - p3 - a$$

$$p1 = p4 \cdot \cos(\delta - \zeta) - p2 \cdot \cos(\zeta)$$

$$\tan(\beta1) = \frac{p1}{h1}$$

$$L1 = \sqrt{h1^2 + p1^2}$$

**After the bump stop hit the ground**

Radius of turn of CG

$$R1 = R0 + L1 \cdot \sin(\alpha - \beta1)$$

Moment arm of weight from pivot (i.e. wheel)

$$x = L1 \cdot \sin(\alpha - \gamma - \beta1 - \theta)$$

Moment arm of centripetal force from pivot (i.e. wheel)

$$y = L1 \cdot \cos(\alpha - \gamma - \beta1)$$

Total turning moment

$$T = M \cdot (w^2 \cdot R1 \cdot \cos(\delta1) \cdot y + g \cdot x)$$

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Angular acceleration of rolling

$$\text{acc} = \frac{T}{I + M \cdot L_1^2}$$

Just after the bump stop hit the ground, the plane of roll changes and the average roll acceleration is the average over the new plane.

$$\text{acc} = \frac{\text{acc}0 \cdot \cos(\zeta) + \text{acc}1}{2}$$

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**Work example**

$$\begin{array}{llll}
 M := 2500 & a := 0 & \theta := 0 & c := 1 \\
 I := 225 & h_0 := 0.5 & w := 1 & j := 0.9 \\
 g := 9.81 & l := 0.9 & r := 10 & k := 0.05 \\
 \alpha := 0 & d := 0 & e := 1.5 & 
 \end{array}$$

$$j := \begin{cases} j & \text{if } d < j \\ d & \text{otherwise} \end{cases}$$

$$j = 0.9$$

**Before the bump stop hit the ground**

$$\delta_0 := \operatorname{atan}\left(\frac{1-d}{2 \cdot e}\right)$$

$$\delta_0 = 0.291$$

$$s := \left(\frac{j-d}{2}\right) \cdot \cos(\delta_0)$$

$$s = 0.431$$

Roll angle before the bump stop hit the ground and the pivot edge shift

$$\gamma := \begin{cases} \frac{\pi}{2} & \text{if } s \leq 0 \\ \operatorname{atan}\left(\frac{k}{|s| + 0.00001}\right) & \text{otherwise} \end{cases}$$

$$\gamma = 0.115$$

Base (perpendicular) distance between the CG and the pivot edge before the bump stop hit the ground

$$p_0 := c \cdot \sin(\delta_0) + \left(\frac{d}{2} - a\right) \cdot \cos(\delta_0)$$

$$p_0 = 0.287$$

$$\beta_0 := \operatorname{atan}\left(\frac{p_0}{h_0}\right)$$

$$\beta_0 = 0.522$$

$$L_0 := \sqrt{h_0^2 + p_0^2}$$

$$L_0 = 0.577$$

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Initial radius of turn of CG

$$R0 := r + a + p0 + L0 \cdot \sin(\alpha - \beta0)$$

$$R0 = 10$$

Moment arm of weight from pivot (i.e. wheel)

$$x := L0 \cdot \sin[\alpha - (\beta0 - \theta)]$$

$$x = -0.287$$

Moment arm of centripetal force from pivot (i.e. wheel)

$$y := L0 \cdot \cos(\alpha - \beta0)$$

$$y = 0.5$$

Total turning moment

$$T := M \cdot (w^2 \cdot R0 \cdot \cos(\delta0) \cdot y + g \cdot x)$$

$$T = 4.926 \times 10^3$$

Angular acceleration of rolling

$$\text{acc1} := \frac{T}{I + M \cdot L0^2}$$

$$\text{acc1} = 4.663$$

After turning an angle of  $\alpha$ 

$$\alpha := \frac{\pi}{100}$$

$$\alpha = 0.031$$

Radius of turn of CG

$$R0 := r + (a + p0 + L0 \cdot \sin(\alpha - \beta0))$$

$$R0 = 10.016$$

Moment arm of weight from pivot (i.e. wheel)

$$x := L0 \cdot \sin[\alpha - (\beta0 - \theta)]$$

$$x = -0.272$$

Moment arm of centripetal force from pivot (i.e. wheel)

$$y := L0 \cdot \cos(\alpha - \beta0)$$

$$y = 0.509$$

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Total turning moment

$$T := M \cdot (w^2 \cdot R0 \cdot \cos(\delta0) \cdot y + g \cdot x)$$

$$T = 5.544 \times 10^3$$

Angular acceleration of rolling

$$\text{acc2} := \frac{T}{I + M \cdot L0^2}$$

$$\text{acc2} = 5.248$$

Average roll acceleration

$$\text{acc} := \frac{(\text{acc1} + \text{acc2})}{2}$$

$$\text{acc} = 4.955$$

$$\phi := \frac{\pi}{100}$$

$$\phi = 0.031$$

$$u := 0$$

$$v := \sqrt{u^2 + 2 \cdot \text{acc} \cdot \phi}$$

$$v = 0.558$$

The angle of roll can be found by a series of steps

Average roll velocity

$$v := \frac{v}{2}$$

$$v = 0.279$$

$$z(t) := \frac{1}{2} \cdot \text{acc} \cdot t^2 + u \cdot t - \phi$$

$$\text{root}(z(t), t, 0, 5) = 0.113$$

**After the bump stop hit the ground**

$$\alpha := \gamma$$

$$\alpha = 0.115$$

$$R0 := r + a + p0 + L0 \cdot \sin(\alpha - \beta0)$$

$$R0 = 10.06$$

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Length of new pivot edge (the length of the side of the vehicle):

$$n := \sqrt{e^2 + \left(\frac{1-j}{2}\right)^2}$$

$$n = 1.5$$

Height of the CG from ground when the bump stop hit the ground

$$h1 := L0 \cdot \sin\left(\frac{\pi}{2} - \beta0 + \alpha\right)$$

$$h1 = 0.53$$

Displacement of the CG along the slope

$$p2 := p0 - L0 \cdot \cos\left(\frac{\pi}{2} - \beta0 + \alpha\right)$$

$$p2 = 0.06$$

Base (perpendicular) distance between the CG and the pivot edge after the bump stop hit the ground

$$\zeta := \begin{cases} \operatorname{asin}\left[\frac{\sqrt{k^2 + \left(\frac{j-d}{2}\right)^2}}{n}\right] & \text{if } d < j \\ 0 & \text{otherwise} \end{cases}$$

$$\zeta = 0.307$$

$$\delta1 := \delta0 - \zeta$$

$$\delta1 = -0.015$$

$$p3 := \frac{e - c}{\tan\left(\frac{\pi}{2} + \zeta - \delta0\right)}$$

$$p3 = -7.586 \times 10^{-3}$$

$$p4 := \left(\frac{1}{2} - p3 - a\right)$$

$$p4 = 0.458$$

$$p1 := p4 \cdot \cos(\delta0 - \zeta) - p2 \cdot \cos(\zeta)$$

$$p1 = 0.401$$

$$\beta1 := \operatorname{atan}\left(\frac{p1}{h1}\right)$$

$$\beta1 = 0.648$$

$$L1 := \sqrt{h1^2 + p1^2}$$

$$L1 = 0.664$$

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After one interval

$$\alpha := \alpha + \frac{\pi}{100}$$

$$\alpha = 0.147$$

Radius of turn of CG

$$R1 := R0 + L1 \cdot (\sin(|\beta1|) - \sin(|\beta1| - \alpha + \gamma))$$

$$R1 = 10.076$$

Moment arm of weight from pivot (i.e. wheel)

$$x := L1 \cdot \sin(\alpha - \gamma - \theta - \beta1)$$

$$x = -0.384$$

Moment arm of centripetal force from pivot (i.e. wheel)

$$y := L1 \cdot \cos(\alpha - \gamma - \beta1)$$

$$y = 0.542$$

Total turning moment

$$T := M \cdot (w^2 \cdot R1 \cdot \cos(\delta1) \cdot y + g \cdot x)$$

$$T = 4.238 \times 10^3$$

Angular acceleration of rolling

$$\text{acc1} := \frac{T}{I + M \cdot L1^2}$$

$$\text{acc1} = 3.191$$

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## **Analysis on the rate of roll of a truck (3 or 4 wheels with high back axle hinge) turning on a slope**

Version : 1.3

### **Assumptions:**

traction is large enough to stop skidding  
negligible wheel width  
constant slope during the movement of the vehicle  
turning velocity of vehicle remains constant during rolling  
conservation of angular momentum  
reaction force at impact of bump stop on ground is negligible for worst case analysis

### **Note:**

all parameters are in SI units  
analysis conducted using vector manipulation

### **List of symbols:**

$a$  - lateral offset of CG from centre line of vehicle (m)  
 $acc$  - average acceleration (radian/sec<sup>2</sup>)  
CG - centre of gravity  
 $d$  - rear track width (m)  
 $e$  - wheel base (m)  
 $f$  - distance of CG from the front (m)  
 $g$  - gravitational acceleration (m<sup>2</sup>/s)  
 $h$  - height of CG from ground before the start of roll (m)  
 $H$  - height of back hinge from ground before the start of roll (m)  
 $I$  - roll inertia of vehicle about CG  
 $l$  - front track width (m)  
 $M$  - mass of vehicle (kg)  
PV - vector of the pivot line  
 $r$  - radius of turn of CG before the start of roll (m)  
 $T$  - turning (roll) moment (Nm)  
 $v$  - tangential velocity at midpoint of front track of vehicle (m/sec)  
 $w$  - turning velocity of vehicle (radian/sec)  
 $\alpha$  - angle of roll relative to the starting position (radian),  $0 \leq \alpha \leq \pi/2$   
 $\gamma$  - angle of the second pivot line on the y-z plane (radian),  $0 \leq \gamma \leq \pi/2$   
 $\delta$  - angle of the second pivot line on the x-y plane (radian),  $0 \leq \delta \leq \pi/2$   
 $\zeta$  - angle of the first pivot line on the x-y plane (radian),  $0 \leq \zeta \leq \pi/2$   
 $\theta$  - angle of the slope (radian),  $0 \leq \theta \leq \pi/2$

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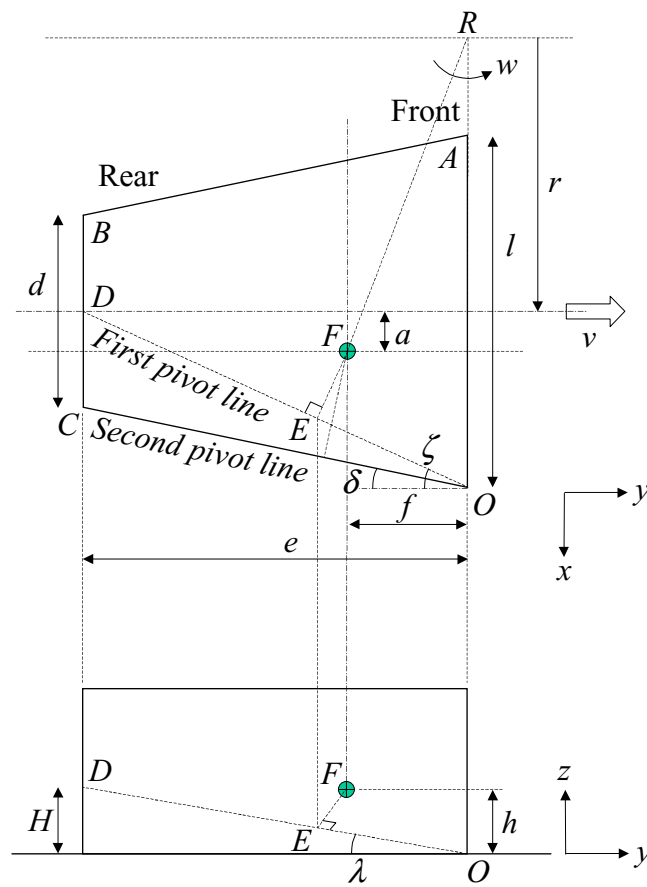
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$\lambda$  - angle of the first pivot line on the y-z plane (radian),  $0 \leq \lambda \leq \pi/2$   
 $\phi$  - incremental roll angle (radian),  $0 \leq \phi \leq \pi/2$



$$OC = \begin{pmatrix} -\frac{l}{2} + \frac{d}{2} \\ -e \\ 0 \end{pmatrix}$$

$$OD = \begin{pmatrix} \frac{l}{2} \\ -e \\ H \end{pmatrix}$$

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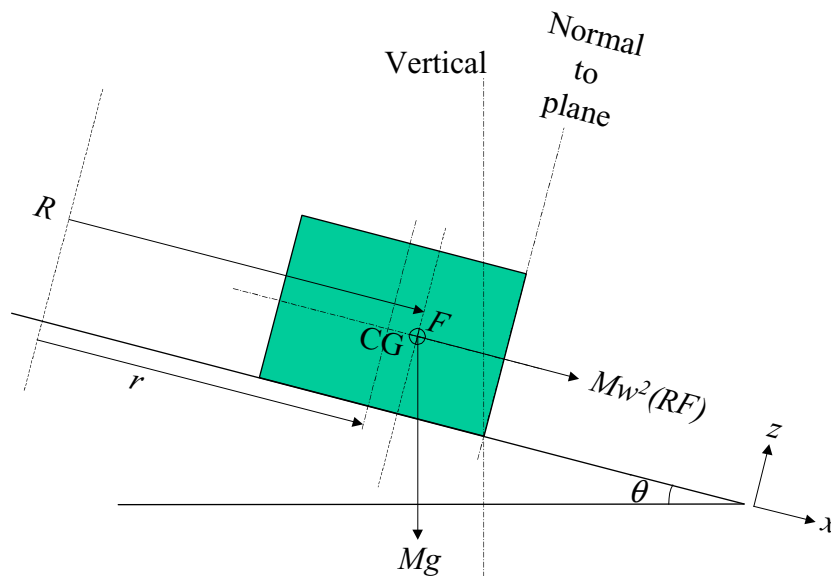
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$$OF = \begin{pmatrix} -\frac{l}{2} + a \\ -f \\ h \end{pmatrix}$$

Displacement from centre of turn to CG

$$OR = \begin{pmatrix} -r - \frac{l}{2} \\ 0 \\ h \end{pmatrix}$$

Take a coordinate system laying in parallel to the slope such that the vehicle is horizontal. At a slope greater than 0, only the direction of the weight is affected.



Weight (with the coordinate system laying flat on the slope)

$$Weight = \begin{pmatrix} M \cdot g \cdot \sin(\theta) \\ 0 \\ -M \cdot g \cdot \cos(\theta) \end{pmatrix}$$

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**Before the hinge reached its limit of turn**

Define pivot line (select OD since vehicle turns about the hinge)

$$PV = OD$$

Displacement from origin to point E

$$P = \frac{PV}{|PV|} \cdot OF$$

$$OE = P \cdot \frac{PV}{|PV|}$$

Shortest vector between the pivot line and CG of vehicle, i.e. EF

$$EF = -OE + OF$$

Centrifugal force

$$RF = -OR + OF$$

$$CForce = M \cdot \omega^2 \cdot RF$$

Moment of centrifugal force about the pivot line

$$MoF = (CForce \times EF) \cdot \frac{PV}{|PV|}$$

Moment of weight about the pivot line

$$MoW = (Weight \times EF) \cdot \frac{PV}{|PV|}$$

$$T = MoF + MoW$$

Angular acceleration of rolling

$$acc1 = \frac{T}{I + M \cdot (|EF|)^2}$$

$$acc = \frac{0 + acc1}{2}$$

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5352

**File Ref.**

5352/19306v

**Calc. No.****Calc. Title**

Analysis of the rate of roll of a truck (3 or 4 wheels with high back axle hinge) turning on a slope

**Author**

Kin Chan

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A.F. Wylie

**Date**

31 January 2000

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Using the initial velocity ( $u$ ) and average acceleration, the time required to travel a pre-defined angle can be calculated. The time of roll ( $t$ ) can be found by a series of steps of size  $\phi$ .

$$\frac{1}{2} \cdot acc \cdot t^2 + u \cdot t = \phi$$

The final velocity can be calculated using the initial velocity

$$v = \sqrt{u^2 + 2 \cdot acc \cdot \phi}$$

The new position of the CG can be re-calculated by mapping the vector space to a new space with the pivot line laying on the new y-axis. After the rotation of  $\phi$  over the new y-axis, the position vector of the CG can then be mapped back to its original vector space.

#### After the hinge reached its limit of turn

Define pivot line (select OC since vehicle turns about the wheel base)

$$PV = OC$$

Calculate OF and OG and GF for the new pivot line

Displacement from origin to point G

$$P = \frac{PV}{|PV|} \cdot OF$$

$$OG = P \cdot \frac{PV}{|PV|}$$

Shortest vector between the pivot line and CG of vehicle, i.e. GF

$$GF = -OG + OF$$

Centrifugal force

$$RF = -OR + OF$$

$$CForce = M \cdot w^2 \cdot RF$$

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Moment of centrifugal force about the pivot line

$$MoF = (CForce \times GF) \cdot \frac{PV}{|PV|}$$

Moment of weight about the pivot line

$$MoW = (Weight \times GF) \cdot \frac{PV}{|PV|}$$

$$T = MoF + MoW$$

The same calculation applies.

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**Date:** 28 / 3 / 2000**Work example**

$$M := 2500 \quad a := 0 \quad \theta := 0.1 \quad f := 0.5$$

$$I := 225 \quad h := 0.5 \quad w := 1 \quad H := 0.4$$

$$g := 9.81 \quad l := 0.9 \quad r := 10 \quad \gamma := 0.1$$

$$\alpha := 0 \quad d := 0.45 \quad e := 1.5$$

Take a coordinate system laying in parallel to the slope such that the vehicle is horizontal. At a slope greater than 0, only the direction of the weight vector is affected.

$$OC := \begin{pmatrix} -\frac{l}{2} + \frac{d}{2} \\ -e \\ 0 \end{pmatrix} \quad |OC| = 1.517$$

$$OD := \begin{pmatrix} -\frac{l}{2} \\ -e \\ H \end{pmatrix} \quad |OD| = 1.616$$

$$OF := \begin{pmatrix} -\frac{l}{2} + a \\ -f \\ h \end{pmatrix} \quad OD \cdot OF = 1.153$$

$$OR := \begin{pmatrix} -r - \frac{l}{2} \\ 0 \\ h \end{pmatrix} \quad OR = \begin{pmatrix} -10.45 \\ 0 \\ 0.5 \end{pmatrix}$$

Weight (with the coordinate system laying flat on the slope)

$$Weight := \begin{pmatrix} M \cdot g \cdot \sin(\theta) \\ 0 \\ -M \cdot g \cdot \cos(\theta) \end{pmatrix} \quad Weight = \begin{pmatrix} 2.448 \times 10^3 \\ 0 \\ -2.44 \times 10^4 \end{pmatrix}$$

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**Date:** 28 / 3 / 2000**Before the hinge reached its limit of turn**

Define pivot line (select OD since vehicle turns about the hinge)

$$PV := OD$$

$$PV = \begin{pmatrix} -0.45 \\ -1.5 \\ 0.4 \end{pmatrix}$$

Displacement from origin to point E

$$P := \frac{PV}{|PV|} \cdot OF$$

$$P = 0.713$$

$$OE := P \cdot \frac{PV}{|PV|}$$

$$OE = \begin{pmatrix} -0.199 \\ -0.662 \\ 0.176 \end{pmatrix}$$

Shortest vector between the pivot line and CG of vehicle, i.e. EF

$$EF := -OE + OF$$

$$EF = \begin{pmatrix} -0.251 \\ 0.162 \\ 0.324 \end{pmatrix}$$

$$EFi := EF$$

$$|EF| = 0.441$$

$$RF := -OR + OF$$

$$RF = \begin{pmatrix} 10 \\ -0.5 \\ 0 \end{pmatrix}$$

Centrifugal force

$$CForce := M \cdot w^2 \cdot RF$$

$$CForce = \begin{pmatrix} 2.5 \times 10^4 \\ -1.25 \times 10^3 \\ 0 \end{pmatrix}$$

Moment of centrifugal force about the pivot line

$$MoF := (CForce \times EFi) \cdot \frac{PV}{|PV|}$$

$$MoF = 8.542 \times 10^3$$

Moment of weight about the pivot line

$$MoW := (Weight \times EF) \cdot \frac{PV}{|PV|}$$

$$MoW = -5.961 \times 10^3$$

$$T := MoF + MoW$$

$$T = 2.581 \times 10^3$$

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$$acc1 := \frac{T}{I + M \cdot (|EF|)^2}$$

$$acc1 = 3.634$$

$$acc := \frac{0 + acc1}{2}$$

$$acc = 1.817$$

$$\phi := \frac{\pi}{100}$$

$$\phi = 0.031$$

$$u := 0$$

$$v := \sqrt{u^2 + 2 \cdot acc \cdot \phi}$$

$$v = 0.338$$

The time of roll can be found by a series of steps

$$z(t) := \frac{1}{2} \cdot acc \cdot t^2 + u \cdot t - \phi$$

$$root(z(t), t, 0, 5) = 0.186$$

Evaluate new position of CG using vector transformation.

This involves two rotation of the vector space about the origin, then rotate the vector EF by  $\phi$  about the pivot line and transform the vector space back to its original position.

First rotate of the system by  $\zeta$  about the z axis so that the pivot line lays on the new y-z plane

$$\zeta := atan\left(\frac{|PV_0|}{|PV_1|}\right)$$

$$\zeta = 0.291$$

$$Rot1 := \begin{pmatrix} \cos(\zeta) & \cos\left(\frac{\pi}{2} + \zeta\right) & \cos\left(\frac{\pi}{2}\right) \\ \cos\left(\frac{\pi}{2} - \zeta\right) & \cos(\zeta) & \cos\left(\frac{\pi}{2}\right) \\ \cos\left(\frac{\pi}{2}\right) & \cos\left(\frac{\pi}{2}\right) & \cos(0) \end{pmatrix}$$

$$Rot1 = \begin{pmatrix} 0.958 & -0.287 & 0 \\ 0.287 & 0.958 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$PVn := Rot1 \cdot PV$$

$$PVn = \begin{pmatrix} 0 \\ -1.566 \\ 0.4 \end{pmatrix}$$

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**Date:** 28 / 3 / 2000Then rotate of the system by  $\lambda$  about the x axis so that PVn lays on the new y axis

$$\lambda := \operatorname{atan}\left(\frac{|PVn_2|}{|PVn_1|}\right)$$

$$\lambda = 0.25$$

$$Rot2 := \begin{pmatrix} \cos(0) & \cos\left(\frac{\pi}{2}\right) & \cos\left(\frac{\pi}{2}\right) \\ \cos\left(\frac{\pi}{2}\right) & \cos(\lambda) & \cos\left(\frac{\pi}{2} + \lambda\right) \\ \cos\left(\frac{\pi}{2}\right) & \cos\left(\frac{\pi}{2} - \lambda\right) & \cos(\lambda) \end{pmatrix}$$

$$Rot2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.969 & -0.247 \\ 0 & 0.247 & 0.969 \end{pmatrix}$$

$$PVn := Rot2 \cdot PVn$$

$$PVn = \begin{pmatrix} 0 \\ -1.616 \\ 0 \end{pmatrix}$$

$$EFn := Rot2 \cdot Rot1 \cdot EF$$

$$EFn = \begin{pmatrix} -0.287 \\ 0 \\ 0.334 \end{pmatrix}$$

Checking EFn calculation:

$$OE = \begin{pmatrix} -0.199 \\ -0.662 \\ 0.176 \end{pmatrix}$$

$$OEn := Rot2 \cdot Rot1 \cdot OE$$

$$OEn = \begin{pmatrix} 0 \\ -0.713 \\ 0 \end{pmatrix}$$

$$OFn := Rot2 \cdot Rot1 \cdot OF$$

$$OFn = \begin{pmatrix} -0.287 \\ -0.713 \\ 0.334 \end{pmatrix}$$

$$EFn := -OEn + OFn$$

$$EFn = \begin{pmatrix} -0.287 \\ 0 \\ 0.334 \end{pmatrix}$$

Turn EFn by  $\phi$  about ODn

$$\alpha := \operatorname{atan}\left(\frac{EFn_2}{EFn_0}\right)$$

$$\alpha = -0.86$$

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$$EFn := \begin{bmatrix} |EFn| \cdot \cos[(\pi + \alpha) - \phi] \\ 0 \\ |EFn| \cdot \sin[(\pi + \alpha) - \phi] \end{bmatrix}$$

$$EFn = \begin{pmatrix} -0.277 \\ 0 \\ 0.343 \end{pmatrix}$$

Transform EFn back to EF

$$EF := Rot1^T \cdot Rot2^T \cdot EFn$$

$$EF = \begin{pmatrix} -0.241 \\ 0.161 \\ 0.332 \end{pmatrix}$$

$$|EF| = 0.441$$

$$OF := OE + EF$$

$$OF = \begin{pmatrix} -0.439 \\ -0.501 \\ 0.509 \end{pmatrix}$$

$$RF := -OR + OF$$

$$RF = \begin{pmatrix} 10.011 \\ -0.501 \\ 8.585 \times 10^{-3} \end{pmatrix}$$

Centrifugal force

$$CForce := M \cdot w^2 \cdot RF$$

$$CForce = \begin{pmatrix} 2.503 \times 10^4 \\ -1.252 \times 10^3 \\ 21.464 \end{pmatrix}$$

Moment of centrifugal force about the pivot line

$$MoF := (CForce \times EF) \cdot \frac{PV}{|PV|}$$

$$MoF = 8.757 \times 10^3$$

Moment of weight about the pivot line

$$MoW := (Weight \times EF) \cdot \frac{PV}{|PV|}$$

$$MoW = -5.69 \times 10^3$$

$$T := MoF + MoW$$

$$T = 3.066 \times 10^3$$

$$acc2 := \frac{T}{I + M \cdot (|EF|)^2}$$

$$acc2 = 4.317$$

$$acc := \frac{acc1 + acc2}{2}$$

$$acc = 3.976$$

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$$\phi := \frac{\pi}{100}$$

$$\phi = 0.031$$

$$v := \sqrt{v^2 + 2 \cdot acc \cdot \phi}$$

$$v = 0.603$$

The time of roll can be found by a series of steps

$$z(t) := \frac{1}{2} \cdot acc \cdot t^2 + v \cdot t - \phi$$

$$root(z(t), t, 0, 5) = 0.045$$

#### When the hinge reached its limit of turn

Evaluate new position of CG using vector transformation.

This involves two rotation of the vector space about the origin, then rotate the vector EF by  $\gamma$  about the first pivot line and transform the vector space back to its original position.

$$EFn := Rot2 \cdot Rot1 \cdot EFi$$

$$EFn = \begin{pmatrix} -0.287 \\ 0 \\ 0.334 \end{pmatrix}$$

$$EFn := \begin{bmatrix} |EF| \cdot \cos[(\pi + \alpha) - \gamma] \\ 0 \\ |EF| \cdot \sin[(\pi + \alpha) - \gamma] \end{bmatrix}$$

$$EFn = \begin{pmatrix} -0.253 \\ 0 \\ 0.361 \end{pmatrix}$$

Transform EFn back to EF

$$EF := Rot1^T \cdot Rot2^T \cdot EFn$$

$$EF = \begin{pmatrix} -0.216 \\ 0.158 \\ 0.35 \end{pmatrix}$$

$$|EF| = 0.441$$

$$OF := OE + EF$$

$$OF = \begin{pmatrix} -0.415 \\ -0.504 \\ 0.526 \end{pmatrix}$$

$$RF := -OR + OF$$

$$RF = \begin{pmatrix} 10.035 \\ -0.504 \\ 0.026 \end{pmatrix}$$

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When the back hinge turned to its limit, the pivot line will shift to OC and the vector GF has to be calculated from OC.

Define pivot line (select OC since vehicle turns about the wheel base)

$$PV := OC$$

$$PV = \begin{pmatrix} -0.225 \\ -1.5 \\ 0 \end{pmatrix}$$

$$P := \frac{PV}{|PV|} \cdot OF$$

$$P = 0.56$$

$$OG := P \cdot \frac{PV}{|PV|}$$

$$OG = \begin{pmatrix} -0.083 \\ -0.553 \\ 0 \end{pmatrix}$$

Shortest vector between the pivot line and CG of vehicle, i.e. GF

$$GF := -OG + OF$$

$$GF = \begin{pmatrix} -0.332 \\ 0.05 \\ 0.526 \end{pmatrix}$$

$$|GF| = 0.624$$

Centrifugal force

$$RF := -OR + OF$$

$$RF = \begin{pmatrix} 10.035 \\ -0.504 \\ 0.026 \end{pmatrix}$$

$$CForce := M \cdot w^2 \cdot RF$$

$$CForce = \begin{pmatrix} 2.509 \times 10^4 \\ -1.259 \times 10^3 \\ 65.446 \end{pmatrix}$$

Moment of centrifugal force about the pivot line

$$MoF := (CForce \times GF) \cdot \frac{PV}{|PV|}$$

$$MoF = 1.317 \times 10^4$$

Moment of weight about the pivot line

$$MoW := (Weight \times GF) \cdot \frac{PV}{|PV|}$$

$$MoW = -6.913 \times 10^3$$

$$T := MoF + MoW$$

$$T = 6.262 \times 10^3$$

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$$acc1 := \frac{T}{I + M \cdot (|GF|)^2}$$

$$acc1 = 5.225$$

Evaluate new position of CG using vector transformation.

This involves two rotation of the vector space about the origin, then rotate the vector GF by  $\phi$  about the pivot line and transform the vector space back to its original position.

First rotate of the system by  $\delta$  (which is  $\zeta$  in the following calculation) about the z axis so that the pivot line lays on the new y-z plane

$$\zeta := \operatorname{atan}\left(\frac{|PV_0|}{|PV_1|}\right)$$

$$\zeta = 0.149$$

$$Rot1 := \begin{pmatrix} \cos(\zeta) & \cos\left(\frac{\pi}{2} + \zeta\right) & \cos\left(\frac{\pi}{2}\right) \\ \cos\left(\frac{\pi}{2} - \zeta\right) & \cos(\zeta) & \cos\left(\frac{\pi}{2}\right) \\ \cos\left(\frac{\pi}{2}\right) & \cos\left(\frac{\pi}{2}\right) & \cos(0) \end{pmatrix}$$

$$Rot1 = \begin{pmatrix} 0.989 & -0.148 & 0 \\ 0.148 & 0.989 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$PVn := Rot1 \cdot PV$$

$$PVn = \begin{pmatrix} 0 \\ -1.517 \\ 0 \end{pmatrix}$$

Then rotate of the system by  $\lambda$  about the x axis so that PVn lays on the new y axis

$$\lambda := \operatorname{atan}\left(\frac{|PVn_2|}{|PVn_1|}\right)$$

$$\lambda = 0$$

$$Rot2 := \begin{pmatrix} \cos(0) & \cos\left(\frac{\pi}{2}\right) & \cos\left(\frac{\pi}{2}\right) \\ \cos\left(\frac{\pi}{2}\right) & \cos(\lambda) & \cos\left(\frac{\pi}{2} + \lambda\right) \\ \cos\left(\frac{\pi}{2}\right) & \cos\left(\frac{\pi}{2} - \lambda\right) & \cos(\lambda) \end{pmatrix}$$

$$Rot2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$PVn := Rot2 \cdot PVn$$

$$PVn = \begin{pmatrix} 0 \\ -1.517 \\ 0 \end{pmatrix}$$

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$$GF_n := Rot2 \cdot Rot1 \cdot GF$$

$$GF_n = \begin{pmatrix} -0.335 \\ 0 \\ 0.526 \end{pmatrix}$$

Turn  $GF_n$  by  $\phi$  about  $OC_n$ 

$$\alpha := \text{atan} \left( \frac{GF_{n2}}{GF_{n0}} \right)$$

$$\alpha = -1.003$$

$$GF_n := \begin{bmatrix} |GF_n| \cdot \cos[(\pi + \alpha) - \phi] \\ 0 \\ |GF_n| \cdot \sin[(\pi + \alpha) - \phi] \end{bmatrix}$$

$$GF_n = \begin{pmatrix} -0.319 \\ 0 \\ 0.536 \end{pmatrix}$$

Transform  $GF_n$  back to  $GF$ 

$$GF := Rot1^T \cdot Rot2^T \cdot GF_n$$

$$GF = \begin{pmatrix} -0.315 \\ 0.047 \\ 0.536 \end{pmatrix}$$

$$|GF| = 0.624$$

$$OF := OG + GF$$

$$OF = \begin{pmatrix} -0.398 \\ -0.506 \\ 0.536 \end{pmatrix}$$

$$RF := -OR + OF$$

$$RF = \begin{pmatrix} 10.052 \\ -0.506 \\ 0.036 \end{pmatrix}$$

Centrifugal force

$$CForce := M \cdot w^2 \cdot RF$$

$$CForce = \begin{pmatrix} 2.513 \times 10^4 \\ -1.265 \times 10^3 \\ 91.141 \end{pmatrix}$$

Moment of centrifugal force about the pivot line

$$MoF := (CForce \times GF) \cdot \frac{PV}{|PV|}$$

$$MoF = 1.346 \times 10^4$$

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Moment of weight about the pivot line

$$MoW := (Weight \times GF) \cdot \frac{PV}{|PV|}$$

$$MoW = -6.48 \times 10^3$$

$$T := MoF + MoW$$

$$T = 6.981 \times 10^3$$

$$acc2 := \frac{T}{I + M \cdot (|EF|)^2}$$

$$acc2 = 9.83$$

$$acc := \frac{acc1 + acc2}{2}$$

$$acc = 7.527$$

$$\phi := \frac{\pi}{100}$$

$$\phi = 0.031$$

$$v := \sqrt{v^2 + 2 \cdot acc \cdot \phi}$$

$$v = 0.915$$

The time of roll can be found by a series of steps

$$z(t) := \frac{1}{2} \cdot acc \cdot t^2 + v \cdot t - \phi$$

$$root(z(t), t, 0, 5) = 0.031$$

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## Analysis on the rate of roll of dumper truck (with mid-vehicle joint) turning on a slope

Version : 2.1

### Assumptions:

traction is large enough to stop skidding  
negligible wheel width  
front and rear track widths are equal  
constant slope during the movement of the vehicle  
relative position of the front and rear sections unchanged during rolling  
turning velocity remains constant during rolling

### Note:

all parameters are in SI units  
analysis conducted using vector manipulation

### List of symbols:

$a$  - lateral offset of CG from centre line of vehicle (m)  
 $acc$  - average acceleration (radian/sec<sup>2</sup>)  
 $d$  - length of vehicle front section (m)  
 $e$  - wheel base (m)  
 $f$  - distance of CG from the front (m)  
 $h$  - height of CG from ground (m)  
 $I$  - roll inertia of vehicle about CG  
 $l$  - track width (front and rear) (m)  
 $L$  - perpendicular distance between CG and pivot edge (m)  
 $M$  - mass of vehicle (kg)  
 $r$  - radius of turn of vehicle before the start of roll (m)  
 $R$  - radius of turn of CG after the start of roll (m)  
 $T$  - turning moment (Nm)  
 $v$  - tangential velocity at midpoint of front track of vehicle (m/sec)  
 $w$  - turning velocity of vehicle (radian/sec)  
 $x$  - moment arm of weight from CG (m)  
 $y$  - moment arm of centripetal force from CG (m)  
 $z$  - radius of turn of CG before the start of roll (m)  
 $\alpha$  - angle of roll relative to the starting position (radian),  $0 \leq \alpha \leq \pi/2$   
 $\beta$  - angle of CG from the origin relative to the -ve x-axis (radian),  $0 \leq \beta \leq \pi/2$   
 $\delta$  - as shown in figure (radian)  
 $\epsilon$  - as shown in figure (radian)  
 $\zeta$  - angle of turn (radian),  $0 \leq \zeta \leq \pi/2$   
 $\theta$  - angle of the slope (radian),  $0 \leq \theta \leq \pi/2$   
 $\sigma$  - as shown in figure (radian)  
 $\phi$  - incremental roll angle (radian),  $0 \leq \phi \leq \pi/2$



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$$\frac{|DR|}{\sin(\delta l)} = \frac{ODm}{\sin(\sigma l + \sigma 2)}$$

Angle of articulation:

$$\zeta 1 = \frac{\pi}{2} - \delta l$$

$$\delta 2 = \pi - \delta l - \sigma l - \sigma 2$$

$$\zeta 2 = \frac{\pi}{2} - \delta 2$$

Take a coordinate system laying in parallel to the slope such that the vehicle is horizontal. For worst case analysis, assume the pivot line is perpendicular to the slope. At a slope greater than 0, only the direction of the weight is affected.

Vector of pivot line before turning

$$OD = \begin{pmatrix} 0 \\ -e \\ 0 \end{pmatrix}$$

Position vector of CG before turning

$$OF = \begin{pmatrix} -\frac{l}{2} + a \\ -f \\ h \end{pmatrix}$$

Position vector of the centre of turn

$$OR = \begin{bmatrix} \left(-r - \frac{l}{2}\right) \cdot \cos(\zeta 1) \\ \left(-r - \frac{l}{2}\right) \cdot \sin(\zeta 1) \\ 0 \end{bmatrix}$$

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Position vector of pivot line during turning

$$ODn = \begin{pmatrix} 0 \\ -ODm \\ 0 \end{pmatrix}$$

Position vector of CG during turning and before rolling

When  $f \leq d$ Turn OF by  $\zeta_1$  about the z-axis

$$\beta_1 = \text{atan} \left( \frac{OF_1}{OF_0} \right)$$

$$OFn = \begin{bmatrix} \sqrt{(OF_0)^2 + (OF_1)^2} \cdot \cos[(\pi + \beta_1) + \zeta_1] \\ \sqrt{(OF_0)^2 + (OF_1)^2} \cdot \sin[(\pi + \beta_1) + \zeta_1] \\ OF_2 \end{bmatrix}$$

When  $f > d$ Turn DF by  $\zeta_2$  about the z-axis

$$DF = -OD + OF$$

$$\beta_2 = \text{atan} \left( \frac{DF_1}{DF_0} \right)$$

$$DFn = \begin{bmatrix} \sqrt{(DF_0)^2 + (DF_1)^2} \cdot \cos[(\pi + \beta_2) - \zeta_2] \\ \sqrt{(DF_0)^2 + (DF_1)^2} \cdot \sin[(\pi + \beta_2) - \zeta_2] \\ DF_2 \end{bmatrix}$$

$$OFn = ODn + DFn$$

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File Ref. 5352/19411v

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Kin Chan

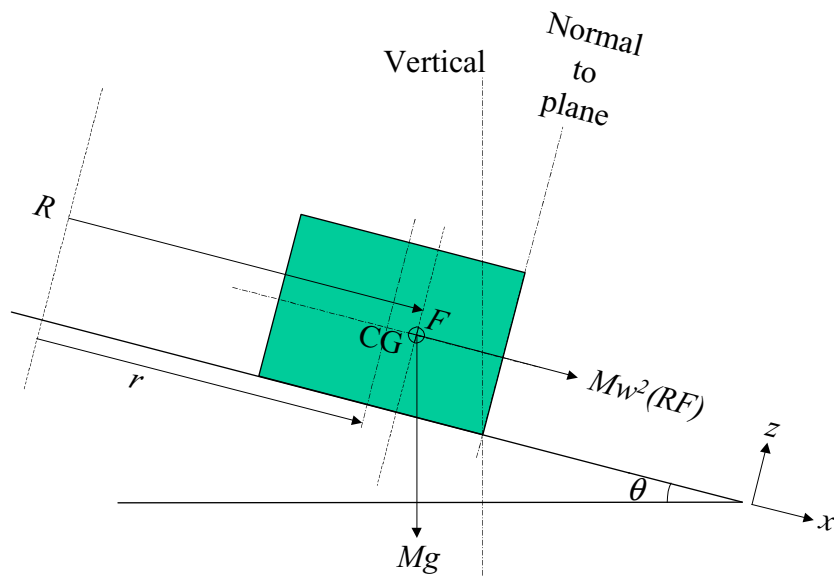
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A.F. Wylie

Date  
14 February 2000

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### Rear View of the Dumper Truck



Weight (with the coordinate system laying flat on the slope)

$$\text{Weight} = \begin{pmatrix} M \cdot g \cdot \sin(\theta) \\ 0 \\ -M \cdot g \cdot \cos(\theta) \end{pmatrix}$$

Define pivot line

$$PV = OD$$

Displacement from origin to point E

$$P = \frac{PV}{|PV|} \cdot OF$$

$$OE = P \cdot \frac{PV}{|PV|}$$

Shortest vector between the pivot line and CG of vehicle, i.e. EF

$$EF = -OE + OF$$

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Centrifugal force

$$RF = -OR + OF$$

$$CForce = M \cdot w^2 \cdot RF$$

Moment of centrifugal force about the pivot line

$$MoF = (CForce \times EF) \cdot \frac{PV}{|PV|}$$

Moment of weight about the pivot line

$$MoW = (Weight \times EF) \cdot \frac{PV}{|PV|}$$

$$T = MoF + MoW$$

Angular acceleration of rolling

$$\frac{d^2}{dt^2} \alpha = \frac{T}{I + M \cdot (|EF|)^2}$$

$$new \left( \frac{d}{dt} \alpha \right)^2 = last \left( \frac{d}{dt} \alpha \right)^2 + 2 \cdot \frac{d^2}{dt^2} \alpha \cdot \phi$$

The angle of roll can be found by a series of steps

$$\phi = \frac{d}{dt} \alpha \cdot t + \frac{1}{2} \cdot \frac{d^2}{dt^2} \alpha \cdot t^2$$

$$\frac{1}{2} \cdot \frac{d^2}{dt^2} \alpha \cdot t^2 + \frac{d}{dt} \alpha \cdot t - \phi = 0$$

Or the time required to turn an angle of  $\phi$  is

$$t = \frac{-\left(\frac{d}{dt} \alpha\right) + \sqrt{\left(\frac{d}{dt} \alpha\right)^2 - 4 \cdot \left(\frac{1}{2} \cdot \frac{d^2}{dt^2} \alpha\right) \cdot (-\phi)}}{2 \cdot \left(\frac{1}{2} \cdot \frac{d^2}{dt^2} \alpha\right)}$$

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$$t = \frac{-\left(\frac{d}{dt}\alpha\right) + \sqrt{\left(\frac{d}{dt}\alpha\right)^2 + 2 \cdot \left(\frac{d^2}{dt^2}\alpha\right) \cdot (\phi)}}{\frac{d^2}{dt^2}\alpha}$$

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**Date:** 27 / 3 / 2000**Work example**

$$M := 2500 \quad a := 0 \quad e := 0.9 \quad f := 0.4$$

$$I := 225 \quad h := 0.5 \quad \theta := 0.1$$

$$g := 9.81 \quad l := 0.9 \quad w := 1$$

$$\alpha := 0 \quad d := 0.45 \quad r := 10$$

$$\sigma_1 := \operatorname{atan}\left(\frac{d}{r}\right)$$

$$\sigma_1 = 0.045$$

$$ORm := r + \frac{l}{2}$$

$$ORm = 10.45$$

$$BRm := \sqrt{d^2 + r^2}$$

$$BRm = 10.01$$

$$\sigma_2 := \operatorname{asin}\left(\frac{e-d}{BRm}\right)$$

$$\sigma_2 = 0.045$$

$$DRm := BRm \cdot \cos(\sigma_2) + \frac{l}{2}$$

$$DRm = 10.45$$

$$ODm := \sqrt{(ORm)^2 + (DRm)^2 - 2 \cdot ORm \cdot DRm \cdot \cos(\sigma_1 + \sigma_2)}$$

$$ODm = 0.94$$

$$\frac{DRm}{\sin(\delta l)} = \frac{ODm}{\sin(\sigma_1 + \sigma_2)}$$

$$\delta l := \operatorname{asin}\left(\frac{DRm \cdot \sin(\sigma_1 + \sigma_2)}{ODm}\right)$$

$$\delta l = 1.526$$

Angle of articulation:

$$\zeta_1 := \left(\frac{\pi}{2} - \delta l\right)$$

$$\zeta_1 = 0.045$$

$$\delta_2 := \pi - \delta l - \sigma_1 - \sigma_2$$

$$\delta_2 = 1.526$$

$$\zeta_2 := \left(\frac{\pi}{2} - \delta_2\right)$$

$$\zeta_2 = 0.045$$

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Take a coordinate system laying in parallel to the slope such that the vehicle is horizontal. For worst case analysis, assume the pivot line is perpendicular to the slope. At a slope greater than 0, only the direction of the weight is affected.

Vector of pivot line before turning

$$OD := \begin{pmatrix} 0 \\ -e \\ 0 \end{pmatrix}$$

$$OD = \begin{pmatrix} 0 \\ -0.9 \\ 0 \end{pmatrix}$$

Position vector of CG before turning

$$OF := \begin{pmatrix} -\frac{l}{2} + a \\ -f \\ h \end{pmatrix}$$

$$OF = \begin{pmatrix} -0.45 \\ -0.4 \\ 0.5 \end{pmatrix}$$

Position vector of the centre of turn

$$OR := \begin{bmatrix} \left(-r - \frac{l}{2}\right) \cdot \cos(\zeta l) \\ \left(-r - \frac{l}{2}\right) \cdot \sin(\zeta l) \\ 0 \end{bmatrix}$$

$$OR = \begin{pmatrix} -10.439 \\ -0.47 \\ 0 \end{pmatrix}$$

Vector of CG from the rear wheel

$$DF := (-OD + OF)$$

$$DF = \begin{pmatrix} -0.45 \\ 0.5 \\ 0.5 \end{pmatrix}$$

Vector of pivot line during turning

$$ODn := \begin{pmatrix} 0 \\ -ODm \\ 0 \end{pmatrix}$$

$$ODn = \begin{pmatrix} 0 \\ -0.94 \\ 0 \end{pmatrix}$$

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Position vector of CG during turning and before rolling

$$q(d, f, OF, ODn, DF, \zeta 1, \zeta 2) := \begin{cases} \text{if } f \leq d \\ \left. \begin{array}{l} \beta \leftarrow \operatorname{atan}\left(\frac{OF_1}{OF_0}\right) \\ v \leftarrow \begin{bmatrix} \sqrt{(OF_0)^2 + (OF_1)^2} \cdot \cos[(\pi + \beta) + \zeta 1] \\ \sqrt{(OF_0)^2 + (OF_1)^2} \cdot \sin[(\pi + \beta) + \zeta 1] \\ OF_2 \end{bmatrix} \end{array} \right\} \\ \text{otherwise} \\ \left. \begin{array}{l} \beta \leftarrow \operatorname{atan}\left(\frac{DF_1}{DF_0}\right) \\ x \leftarrow \begin{bmatrix} \sqrt{(DF_0)^2 + (DF_1)^2} \cdot \cos[(\pi + \beta) - \zeta 2] \\ \sqrt{(DF_0)^2 + (DF_1)^2} \cdot \sin[(\pi + \beta) - \zeta 2] \\ DF_2 \end{bmatrix} \\ v \leftarrow ODn + x \end{array} \right\} \end{cases}$$

$$OFn := q(d, f, OF, ODn, DF, \zeta 1, \zeta 2) \quad OFn = \begin{pmatrix} -0.432 \\ -0.42 \\ 0.5 \end{pmatrix}$$

Weight (with the coordinate system laying flat on the slope)

$$Weight := \begin{pmatrix} M \cdot g \cdot \sin(\theta) \\ 0 \\ -M \cdot g \cdot \cos(\theta) \end{pmatrix} \quad Weight = \begin{pmatrix} 2.448 \times 10^3 \\ 0 \\ -2.44 \times 10^4 \end{pmatrix}$$

Define pivot line

$$PV := ODn$$

$$PV = \begin{pmatrix} 0 \\ -0.94 \\ 0 \end{pmatrix}$$

Displacement from origin to point E

$$P := \frac{PV}{|PV|} \cdot OFn$$

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$$OEn := P \cdot \frac{PV}{|PV|}$$

$$OEn = \begin{pmatrix} 0 \\ -0.42 \\ 0 \end{pmatrix}$$

Shortest vector between the pivot line and CG of vehicle, i.e. EF

$$EF := (-OEn + OFn)$$

$$EF = \begin{pmatrix} -0.432 \\ 0 \\ 0.5 \end{pmatrix}$$

Centrifugal force

$$RFn := (-OR + OFn)$$

$$RFn = \begin{pmatrix} 10.008 \\ 0.05 \\ 0.5 \end{pmatrix}$$

$$CForce := M \cdot w^2 \cdot RFn$$

$$CForce = \begin{pmatrix} 2.502 \times 10^4 \\ 124.874 \\ 1.25 \times 10^3 \end{pmatrix}$$

Moment of centrifugal force about the pivot line

$$MoF := (CForce \times EF) \cdot \frac{PV}{|PV|}$$

$$MoF = 1.305 \times 10^4$$

Moment of weight about the pivot line

$$MoW := (Weight \times EF) \cdot \frac{PV}{|PV|}$$

$$MoW = -9.307 \times 10^3$$

$$T := MoF + MoW$$

$$T = 3.742 \times 10^3$$

$$acc1 := \frac{T}{I + M \cdot (|EF|)^2}$$

$$acc1 = 2.845$$

$$acc := \frac{0 + acc1}{2}$$

$$acc = 1.422$$

$$\phi := \frac{\pi}{100}$$

$$\phi = 0.031$$

$$u := 0$$

$$v := \sqrt{u^2 + 2 \cdot acc \cdot \phi}$$

$$v = 0.299$$

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The time of roll can be found by a series of steps

$$z(t) := \frac{1}{2} \cdot acc \cdot t^2 + u \cdot t - \phi$$

$$root(z(t), t, 0, 5) = 0.21$$

Turn EF by  $\phi$  about ODn

$$\alpha := atan\left(\frac{EF_2}{EF_0}\right)$$

$$\alpha = -0.859$$

$$EF := \begin{bmatrix} |EF| \cdot \cos[(\pi + \alpha) - \phi] \\ 0 \\ |EF| \cdot \sin[(\pi + \alpha) - \phi] \end{bmatrix}$$

$$EF = \begin{pmatrix} -0.416 \\ 0 \\ 0.513 \end{pmatrix}$$

$$|EF| = 0.66$$

$$OFn := OEn + EF$$

$$OFn = \begin{pmatrix} -0.416 \\ -0.42 \\ 0.513 \end{pmatrix}$$

Centrifugal force

$$RFn := (-OR + OFn)$$

$$RFn = \begin{pmatrix} 10.024 \\ 0.05 \\ 0.513 \end{pmatrix}$$

$$CForce := M \cdot w^2 \cdot RFn$$

$$CForce = \begin{pmatrix} 2.506 \times 10^4 \\ 124.874 \\ 1.283 \times 10^3 \end{pmatrix}$$

Moment of centrifugal force about the pivot line

$$MoF := (CForce \times EF) \cdot \frac{PV}{|PV|}$$

$$MoF = 1.34 \times 10^4$$

Moment of weight about the pivot line

$$MoW := (Weight \times EF) \cdot \frac{PV}{|PV|}$$

$$MoW = -8.886 \times 10^3$$

$$T := MoF + MoW$$

$$T = 4.511 \times 10^3$$

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$$acc2 := \frac{T}{I + M \cdot (|EF|)^2}$$

$$acc1 = 2.845$$

$$acc := \frac{acc1 + acc2}{2}$$

$$acc = 3.137$$

$$\phi := \frac{\pi}{100}$$

$$\phi = 0.031$$

$$u := 0$$

$$v := \sqrt{u^2 + 2 \cdot acc \cdot \phi}$$

$$v = 0.444$$

The time of roll can be found by a series of steps

$$z(t) := \frac{1}{2} \cdot acc \cdot t^2 + u \cdot t - \phi$$

$$root(z(t), t, 0, 5) = 0.142$$

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**Calc. No.**

**Calc. Title**

Calculate the new CG  
position of dumper truck due  
to compression of tyres

**Author**

Kin Chan

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**Approved**

A.F.Wylie

**Date**

17 March 2000

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## **Analysis on the wheel suspension effect of dumper truck (with mid-vehicle joint) turning on a slope**

Version : 1.0

### **Assumptions:**

the weight of the truck is supported equal on the two outside wheels during roll  
negligible wheel width  
front and rear track widths are equal  
relative position of the front and rear sections unchanged during rolling

### **Note:**

all parameters are in SI units  
analysis conducted using vector manipulation

### **List of symbols:**

$a$  - lateral offset of CG from centre line of vehicle (m)  
 $acc$  - average acceleration (radian/sec<sup>2</sup>)  
 $d$  - length of vehicle front section (m)  
 $e$  - wheel base (m)  
 $f$  - distance of CG from the front (m)  
 $h$  - height of CG from ground (m)  
 $I$  - roll inertia of vehicle about CG  
 $l$  - track width (front and rear) (m)  
 $k$  - tyre stiffness (N/m)  
 $L$  - perpendicular distance between CG and pivot edge (m)  
 $M$  - mass of vehicle (kg)  
 $r$  - radius of turn of vehicle before the start of roll (m)  
 $R$  - radius of turn of CG after the start of roll (m)  
 $T$  - turning moment (Nm)  
 $v$  - tangential velocity at midpoint of front track of vehicle (m/sec)  
 $w$  - turning velocity of vehicle (radian/sec)  
 $z0$  - compression of tyre when the truck is on level ground (m)  
 $z1$  - compression of tyre when the truck is at start of roll (m)  
 $\alpha$  - angle of roll relative to the starting position (radian),  $0 \leq \alpha \leq \pi/2$   
 $\beta$  - angle of CG from the origin relative to the -ve x-axis (radian),  $0 \leq \beta \leq \pi/2$   
 $\delta$  - as shown in figure (radian)  
 $\epsilon$  - as shown in figure (radian)  
 $\zeta$  - angle of turn (radian),  $0 \leq \zeta \leq \pi/2$   
 $\theta$  - angle of the slope (radian),  $0 \leq \theta \leq \pi/2$   
 $\lambda$  - angle of tilt of CG due to tyre stiffness  
 $\sigma$  - as shown in figure (radian)  
 $\phi$  - incremental roll angle (radian),  $0 \leq \phi \leq \pi/2$

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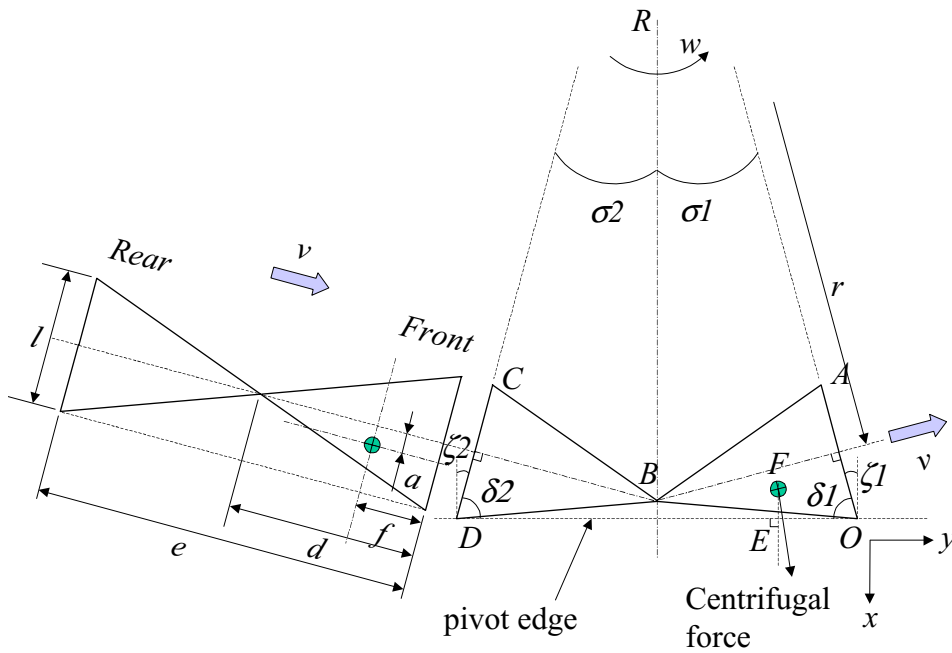
Subscripts:

1 - vehicle front section

2 - vehicle rear section

n - vectors when vehicle is turning

Top view of a dumper truck turning a corner



$$\tan(\sigma 1) = \frac{d}{r}$$

$$|OR| = r + \frac{l}{2}$$

$$|BR| = \sqrt{d^2 + r^2}$$

$$\sin(\sigma 2) = \frac{e - d}{|BR|}$$

$$|DR| = |BR| \cdot \cos(\sigma 2) + \frac{l}{2}$$

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$$ODm^2 = (|OR|)^2 + (|DR|)^2 - 2 \cdot |OR| \cdot |DR| \cdot \cos(\sigma_1 + \sigma_2)$$

$$\frac{|DR|}{\sin(\delta_1)} = \frac{ODm}{\sin(\sigma_1 + \sigma_2)}$$

Angle of articulation:

$$\zeta_1 = \frac{\pi}{2} - \delta_1$$

$$\delta_2 = \pi - \delta_1 - \sigma_1 - \sigma_2$$

$$\zeta_2 = \frac{\pi}{2} - \delta_2$$

Take a coordinate system laying in parallel to the slope such that the vehicle is horizontal. For worst case analysis, assume the pivot line is perpendicular to the slope. At a slope greater than 0, only the direction of the weight is affected.

Vector of pivot line before turning

$$OD = \begin{pmatrix} 0 \\ -e \\ 0 \end{pmatrix}$$

Position vector of CG before turning

$$OF = \begin{pmatrix} -\frac{l}{2} + a \\ -f \\ h \end{pmatrix}$$

Position vector of the centre of turn

$$OR = \begin{bmatrix} \left(-r - \frac{l}{2}\right) \cdot \cos(\zeta_1) \\ \left(-r - \frac{l}{2}\right) \cdot \sin(\zeta_1) \\ 0 \end{bmatrix}$$

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Position vector of pivot line during turning

$$ODn = \begin{pmatrix} 0 \\ -ODm \\ 0 \end{pmatrix}$$

Position vector of CG during turning and before rolling

When  $f \leq d$ Turn OF by  $\zeta_1$  about the z-axis

$$\beta_1 = \text{atan} \left( \frac{OF_1}{OF_0} \right)$$

$$OFn = \begin{bmatrix} \sqrt{(OF_0)^2 + (OF_1)^2} \cdot \cos[(\pi + \beta_1) + \zeta_1] \\ \sqrt{(OF_0)^2 + (OF_1)^2} \cdot \sin[(\pi + \beta_1) + \zeta_1] \\ OF_2 \end{bmatrix}$$

When  $f > d$ Turn DF by  $\zeta_2$  about the z-axis

$$DF = -OD + OF$$

$$\beta_2 = \text{atan} \left( \frac{DF_1}{DF_0} \right)$$

$$DFn = \begin{bmatrix} \sqrt{(DF_0)^2 + (DF_1)^2} \cdot \cos[(\pi + \beta_2) - \zeta_2] \\ \sqrt{(DF_0)^2 + (DF_1)^2} \cdot \sin[(\pi + \beta_2) - \zeta_2] \\ DF_2 \end{bmatrix}$$

$$OFn = ODn + DFn$$

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**Calculation of new CG position**

When the truck is stationary on level ground (assume all 4 tyres are compressed equally)

$$M \cdot g = 4 \cdot k \cdot z_0$$

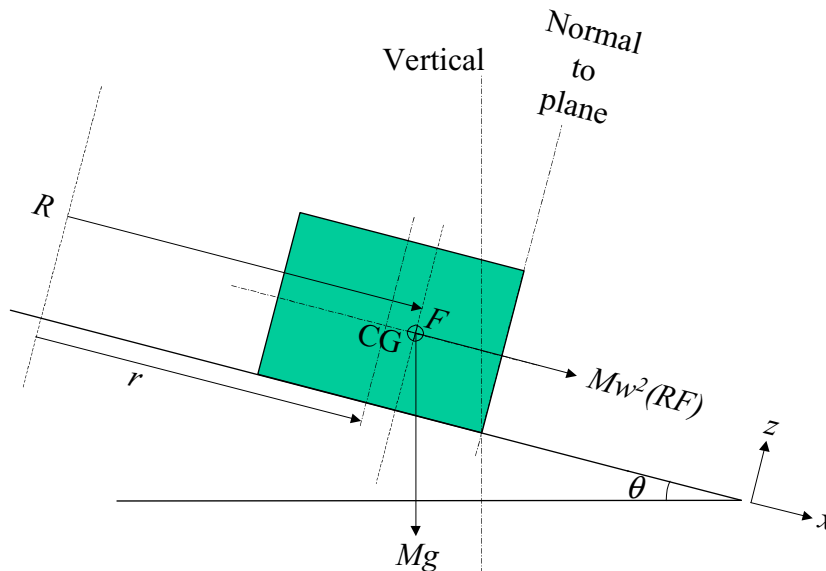
$$z_0 = \frac{M \cdot g}{4 \cdot k}$$

The height of the CG when the tyres are not compressed is then

$$TyreD = z_0 + h$$

Calculation of new CG position assuming the truck is at the start of roll and the outside tyres are compressed equally.

**Rear View of the Dumper Truck**



Weight (with the coordinate system laying flat on the slope)

$$Weight = \begin{pmatrix} M \cdot g \cdot \sin(\theta) \\ 0 \\ -M \cdot g \cdot \cos(\theta) \end{pmatrix}$$

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When the truck is turning on the slope at the critical velocity (i.e. when the inside wheels lift off) vertical force balance (all the weight supported by the two outside wheels)

$$M \cdot g \cdot \cos(\theta) = 2 \cdot k \cdot z1$$

$$z1 = \frac{M \cdot g \cdot \cos(\theta)}{2 \cdot k}$$

Therefore the tilt angle of the truck at the beginning of roll

$$\sin(\lambda) = \frac{z1}{l \cdot \cos(\zeta1)}$$

New position of the CG at the start of roll

$$OF_n = \begin{pmatrix} OF_0 \cdot \cos(\lambda) \\ OF_1 \\ tyreD - z1 + OF_0 \cdot \sin(\lambda) \end{pmatrix}$$

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**Date:** 27 / 3 / 2000**Work example**

$$M := 2870$$

$$a := 0.007$$

$$e := 1.229$$

$$f := 0.625$$

$$I := 225$$

$$h := 0.47$$

$$\theta := 0.1$$

$$k := 20 \times 10^4$$

$$g := 9.81$$

$$l := 0.906$$

$$w := 1$$

$$\alpha := 0$$

$$d := 0.615$$

$$r := 10$$

$$\sigma_1 := \operatorname{atan}\left(\frac{d}{r}\right)$$

$$\sigma_1 = 0.061$$

$$ORm := r + \frac{l}{2}$$

$$ORm = 10.453$$

$$BRm := \sqrt{d^2 + r^2}$$

$$BRm = 10.019$$

$$\sigma_2 := \operatorname{asin}\left(\frac{e-d}{BRm}\right)$$

$$\sigma_2 = 0.061$$

$$DRm := BRm \cdot \cos(\sigma_2) + \frac{l}{2}$$

$$DRm = 10.453$$

$$ODm := \sqrt{(ORm)^2 + (DRm)^2 - 2 \cdot ORm \cdot DRm \cdot \cos(\sigma_1 + \sigma_2)}$$

$$ODm = 1.282$$

$$\frac{DRm}{\sin(\delta l)} = \frac{ODm}{\sin(\sigma_1 + \sigma_2)}$$

$$\delta l := \operatorname{asin}\left(\frac{DRm \cdot \sin(\sigma_1 + \sigma_2)}{ODm}\right)$$

$$\delta l = 1.509$$

**Angle of articulation:**

$$\zeta_1 := \left(\frac{\pi}{2} - \delta l\right)$$

$$\zeta_1 = 0.061$$

$$\delta_2 := \pi - \delta l - \sigma_1 - \sigma_2$$

$$\delta_2 = 1.509$$

$$\zeta_2 := \left(\frac{\pi}{2} - \delta_2\right)$$

$$\zeta_2 = 0.061$$

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Take a coordinate system laying in parallel to the slope such that the vehicle is horizontal. For worst case analysis, assume the pivot line is perpendicular to the slope. At a slope greater than 0, only the direction of the weight is affected.

Vector of pivot line before turning

$$OD := \begin{pmatrix} 0 \\ -e \\ 0 \end{pmatrix}$$

$$OD = \begin{pmatrix} 0 \\ -1.229 \\ 0 \end{pmatrix}$$

Position vector of CG before turning

$$OF := \begin{pmatrix} -\frac{l}{2} + a \\ -f \\ h \end{pmatrix}$$

$$OF = \begin{pmatrix} -0.446 \\ -0.625 \\ 0.47 \end{pmatrix}$$

Position vector of the centre of turn

$$OR := \begin{bmatrix} \left(-r - \frac{l}{2}\right) \cdot \cos(\zeta l) \\ \left(-r - \frac{l}{2}\right) \cdot \sin(\zeta l) \\ h \end{bmatrix}$$

$$OR = \begin{pmatrix} -10.433 \\ -0.641 \\ 0.47 \end{pmatrix}$$

Vector of CG from the rear wheel

$$DF := (-OD + OF)$$

$$DF = \begin{pmatrix} -0.446 \\ 0.604 \\ 0.47 \end{pmatrix}$$

Vector of pivot line during turning

$$ODn := \begin{pmatrix} 0 \\ -ODm \\ 0 \end{pmatrix}$$

$$ODn = \begin{pmatrix} 0 \\ -1.282 \\ 0 \end{pmatrix}$$

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Position vector of CG during turning and before rolling

$$q(d, f, OF, ODn, DF, \zeta_1, \zeta_2) := \begin{cases} \text{if } f \leq d \\ \left. \begin{array}{l} \beta \leftarrow \operatorname{atan}\left(\frac{OF_1}{OF_0}\right) \\ v \leftarrow \begin{bmatrix} \sqrt{(OF_0)^2 + (OF_1)^2} \cdot \cos[(\pi + \beta) + \zeta_1] \\ \sqrt{(OF_0)^2 + (OF_1)^2} \cdot \sin[(\pi + \beta) + \zeta_1] \\ OF_2 \end{bmatrix} \end{array} \right\} \\ \text{otherwise} \\ \left. \begin{array}{l} \beta \leftarrow \operatorname{atan}\left(\frac{DF_1}{DF_0}\right) \\ x \leftarrow \begin{bmatrix} \sqrt{(DF_0)^2 + (DF_1)^2} \cdot \cos[(\pi + \beta) - \zeta_2] \\ \sqrt{(DF_0)^2 + (DF_1)^2} \cdot \sin[(\pi + \beta) - \zeta_2] \\ DF_2 \end{bmatrix} \\ v \leftarrow ODn + x \end{array} \right\} \end{cases}$$

$$OFn := q(d, f, OF, ODn, DF, \zeta_1, \zeta_2) \quad OFn = \begin{pmatrix} -0.408 \\ -0.652 \\ 0.47 \end{pmatrix}$$

**Calculation of new CG position**

When the truck is stationary on level ground (assume all 4 tyres are compressed equally)

$$z_0 := \frac{M \cdot g}{4 \cdot k}$$

$$z_0 = 0.035$$

The height of the CG when the tyres are not compressed is then

$$TyreD := z_0 + h$$

$$TyreD = 0.505$$

Calculation of new CG position assuming the truck is at the start of roll and the outside tyres are compressed equally.

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Weight (with the coordinate system laying flat on the slope)

$$Weight := \begin{pmatrix} M \cdot g \cdot \sin(\theta) \\ 0 \\ -M \cdot g \cdot \cos(\theta) \end{pmatrix}$$

$$Weight = \begin{pmatrix} 2.811 \times 10^3 \\ 0 \\ -2.801 \times 10^4 \end{pmatrix}$$

When the truck is turning on the slope at the critical velocity (i.e. when the inside wheels lift off)  
vertical force balance (all the weight supported by the two outside wheels)

$$z1 := \frac{M \cdot g \cdot \cos(\theta)}{2 \cdot k}$$

$$z1 = 0.07$$

Therefore the tilt angle of the truck at the beginning of roll

$$\lambda := \arcsin\left(\frac{z1}{1 \cdot \min(\cos(\zeta1), \cos(\zeta2))}\right)$$

$$\lambda = 0.078$$

New position of the CG at the start of roll

$$OFn := \begin{pmatrix} OFn_0 \cdot \cos(\lambda) \\ OFn_1 \\ TyreD - z1 + |OFn_0| \cdot \sin(\lambda) \end{pmatrix}$$

$$OFn = \begin{pmatrix} -0.407 \\ -0.652 \\ 0.467 \end{pmatrix}$$

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